ABSTRACT

Shear instabilities, a common feature in nature are detected in planetary atmospheres and in large scale astrophysical objects. The geostropic wind must have vertical wind shear in the presence of a horizontal temperature gradient. The process of cyclogenesis is regarded as a manifestation of the amplification of an infinitesimal perturbation superposed on a basic state current which contains vertical shear.

Hence the analysis of this thesis is motivated by the desire to understand the effect of various parameters like rotation number, wave number, magnetic Reynolds number and Richardson number on the stability of parallel and non parallel shear layers. This thesis comprises of an introductory chapter, review of literature and six main chapters.

Chapter III demonstrates the effect of Richardson number on the stability of inviscid stratified shear layers in the case of arbitrary velocity profile by employing normal mode approach to small oscillations. This analysis is restricted to long wave approximations. We have found that Richardson number plays a very significant role in determining the stability of such problems.

In Chapter IV, the work of Padmini and Subbiah (1995) is extended to examine the effect of uniform horizontal magnetic field.

In Chapter V, the onset of shear instability in an inviscid Boussinesq stratified fluid which is rotating about a vertical axis with the angular velocity Ω is for asymptotically small wave numbers. In this study viscous friction is neglected and the mathematical efforts are focused on a basic flow which is linear.

Chapter VI deals with linear stability problem of nonparallel shear flows of an inviscid, incompressible fluid and examine the effect of rotation on the stability of shear flows. We have found analytical expressions to calculate the growth rate of three dimensional disturbances using perturbation techniques. The analysis is restricted to long wave approximations. The analytical expressions derived involve arbitrary velocity profile of the basic flow.

Chapter VII aims at understanding the effect of three-dimensional disturbances on the stability of basic non parallel flows of inviscid homogeneous fluid rotating about the z-

axis with a general velocity profile (U(z), V(z), 0). Expanding all the physical quantities in terms of the parameter α and assuming that the disturbances are small, asymptotic formulas are obtained for determination of the stability characteristics up to order α^2. We have illustrated the use of small α formulas to give quantitative numerical results with an example.

Since we find only few published analytical works considering the stability of the base flow in long inclined cavities with axial temperature gradient, in chapter VIII we have investigated the stability of buoyancy driven shear flows in inclined long cavities with end wall temperature difference. Analytical expressions are found for the growth rate and stream function as a function of wave number and the stability of the flow is discussed for different angle of inclinations and a wide range of Prandtl number.

The main purpose of the thesis is to give some formulas and techniques by which the instability characteristics of parallel and non parallel shear flows can be determined analytically in the case of more or less arbitrary velocity profile and to discuss qualitatively the nature of the instabilities of such flows. At the end of each chapter, the use of the formulas is illustrated with examples to quantitative numerical results. Mathematical efforts are focused on the linear velocity profile and tanh z profile.

CHAPTER I

CHAPTER I

Introduction

1.1 Motivation

In some meteorological services of the world, there is a rule that when tropical cyclonic storm(wind speed $\geq$ 34 kts) is known to exist within its area of jurisdiction, the chief forecaster has to stay in office, on duty, all the twenty-four hours till the system weakens below the limit of intensity. There are historical reasons for this stringent rule. Observations keep coming in the forecasting office and the chief forecaster has to assess the situation continuously about the change in intensity of the system, change in its expected track and about the locations which have to be exactly on the warning list at a time. Over-warning may create a ridiculous situation for the meteorological services where people may come to disbelieve what the meteorologist says, under- warning may cost thousands of lives. Some tropical cyclones have cost more than 3,00,000 human lives within a few hours. A cyclone may leave behind it, thousands of human beings and cattle dead, the local wells near the shore filled with saline sea water, railroads shattered, telegraph and telephone wires snapped, the hutments erased and the survivors slowly dying of disease and hunger. It can cause almost as much human misery as an atomic bomb. The Fig 1.1 shows the devastating effects of hurricanes and Table 1.1, taken from Anthes (1982) shows the hurricanes during the period 1901-1979 with human death toll of more than 1000.


Figure 1.1: Photographs showing the devastating effects of the hurricanes during 1970 and 1977 in Bangladesh and AndhraPradesh, India

Table 1.1: Table showing the hurricanes during the period 1901-1979 with human death toll of more than 1000

YEAR	DEATHS	LOCATION
08.09.1900	6000	Galveston, TX
12-17.09.1928	4000	W. Indies, FL
03.09.1930	2000	San Domingo
15-16.10.1942	11000	Bengal, India
25-27.09.1953	1300	Vietnam, Japan
27.09.1954	1218	Japan
17-19.09.1959	2000	Far East
26-27.09.1959	4466	Honshu, Japan
04-08.10.1963	6000	Cuba, Haiti
13.11.1970	300000	Bangladesh
19-20.09.1974	2000	Honduras
19.11.1977	10000	Andhra Pradesh, India

(Source: Anthes (1977))

Figure 1.2: Photographs showing the devastating impact of Superstorm Sandy

A number of meteorological departments had started functioning around 1875, some of these being in the tropical region. The chief forecasters in these tropical meteorological services saw fronts everywhere, even at the centers of tropical cyclones. Soon after the introduction of barometer in tropical latitudes, it was realized that daily pressure variations in tropical regions were quite different from those in extra – tropical regions, the barometric variations are essentially caused by the successive passages of extra tropical cyclones, their period being of the order of 5 days and amplitude being of the order of 25 millibars (hPa). The meteorologists recognized that the large scale broad and extensive westerlies of extra tropical latitudes were baroclinically unstable and this baroclinic instability gave rise to intensification of an initially weak wave perturbation. *Hence a detailed analysis of shear flow instability will throw light on various atmospheric phenomena.*

1.2 Shear Flows

Variation of velocity in the direction normal to the direction of the velocity itself is called shear. The simplest example of a shear layer is the mixing layer separating two nearly parallel streams at different velocities. Another example of a layer with a strong shear is the boundary layer between a stream and a solid surface. Free shear layers are those not adjacent to a solid surface, the jet being an example.

Consider the stationary flow of a liquid located between two, infinitely, parallel planes, normal to the y direction separated by a distance 'a'. One of the plates is kept fixed, while the other moves parallel to itself at a constant velocity V_0 in the x direction. The fluid is 'dragged' along by the moving plate. Under stationary conditions (i.e. after a sufficiently long time has elapsed since the boundary wall was set in motion), we observe that the velocity of the fluid varies linearly from 0 to V_0

The shear flow in the solid body is the gradient of a shear force through the body while in a fluid it is the flow induced by such a force gradient. Consider the following sketch.

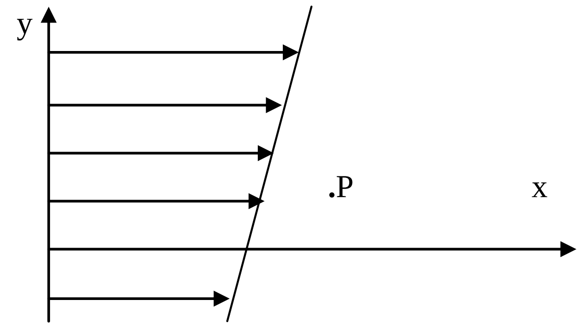

Figure 1.3: Velocity profile in a simple shear flow

The above sketch depicts the velocity profile in a simple shear flow. The fluid is moving to the right and the magnitude of the fluid velocity increase linearly with y. The velocity can be written as $u = cy$, $v = w = 0$, where c is a constant determining the slope of the profile.

A steady shear flow is the condition under which a fluid is sheared continuously in one direction during the duration of a rheometric experiment. A shear is the relative movement of parallel adjacent layers. In simple shear there is no change in the dimension normal to the plane of shear and the relative displacement of successive parallel layers of a material body are proportional to their distance from the reference layer. The type of flow used in most rheometric measurement of our fluids can be approximated by the simple shear.

1.3 Examples of Shear Layers

The simplest example of a shear layer is the mixing layer separating two nearly parallel streams at different velocities as seen in (Figure 1.4)

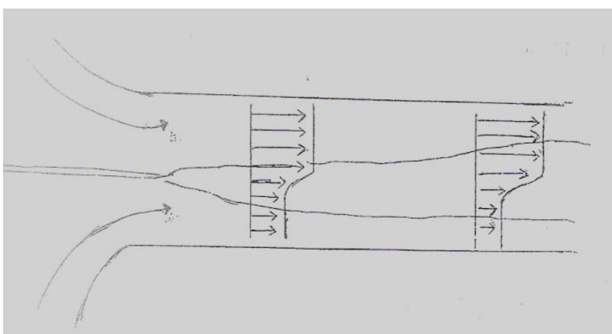

Fig 1.4: Mixing layer between parallel streams

Another example of a layer with a strong shear is the boundary layer between a stream and a solid surface (Figure 1.5) which is the commonest type of shear layer. A jet (Figure 1.6) also has a large shear; so does the flow in a pipe (Figure 1.7) where the velocity is again zero at the solid surface.

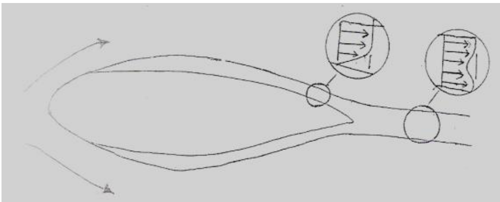

Fig 1.5: Boundary layer and wake of airfoil

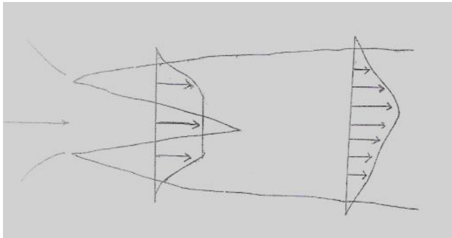

Fig 1.6: Merging mixing layer in jet

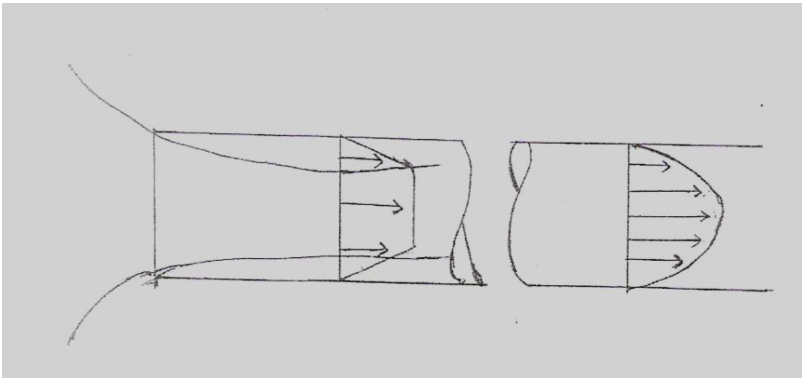

Fig 1.7: Merging boundary layers in pipe flow

1.4 Practical Examples of Shear Layers

There exists complicated shear layers too. Consider a vertical take-off aircraft near the ground, supported by the thrust of its jet engines. (Figure 1.8). There is a boundary layer on the inside circumference of each jet nozzle, and this boundary layer becomes an annular mixing layer as the exhaust leaves the nozzle. The mixing layer grows in thickness until it engulfs the whole cross section of the exhaust flow, which is therefore called a jet. When the jet hits the ground, it is sharply deflected and spreads out radically in a flow called a wall jet. If there is a wind blowing, the part of the wall jet that blows upwind will eventually separate from the ground and be blown backwards towards the aircraft. Reinjection of hot exhaust gas is undesirable for jet engines, particularly, if dust particles have been sourced off the ground, so the aircraft designer is interested in that transport of heat and foreign matter by the shear layer.

The flow inside a jet engine or any other type of turbo machine is very complicated because each blade has two boundary layers which merge at the downstream end of the blade to from a wake. The wake of each blade then interacts with the flow over blades further downstream, boundary layers grow on the dust walls as well, further complicating the flow over the blades. Jet engines combustion chambers use jet flows to mix the reactants. Extremely complicated flow patterns result, and the amount of smoke and other pollutants emitted by the engine depends critically on the completeness of the mixture.

Once the aircraft is moving fast enough to be supported by its wings, the behavior of the boundary layers on wings and fuselage determines the drag and limits the lift. The boundary layer can greatly affect the lift at given incidence, even at incidence well below the stall or condition of maximum lift. Figure (1.6) is only one example of the importance of shear layers in engineering. Examples in the environmental sciences, additional to the planetary boundary layer mentioned above, include jet streams in atmosphere and ocean, river flows and buoyant plumes.

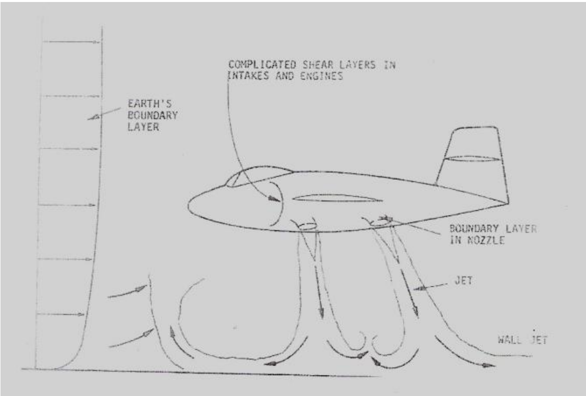

Fig 1.8: Practical Example of shear layer

1.5 Stability Principles

We are all familiar with the simple concept of stable, unstable and neutral equilibrium of bodies at rest. Steady state is a state of equilibrium. If a small perturbation given to the body has a tendency to grow in magnitude so as to take the body further and further away from its equilibrium state, we say that the equilibrium of the body was unstable. If the small perturbation has a tendency to decrease in magnitude so as to bring the body back to its equilibrium state, then we say that the equilibrium of the body is stable. If the perturbation has tendency neither to grow nor to decrease in magnitude, then the equilibrium of the body is neutral.

In the case illustrated above, the kinetic energy of the perturbation grows, decays or remains constant. This kinetic energy of the perturbation must come from or go to the energy of the body in the equilibrium position. It can be shown that in stable equilibrium configuration, the body has a minimum of potential energy. Any displacement of the body from this configuration increases its potential energy; and the tendency of the body is to come back to its original configuration of minimum potential energy.

The idea of equilibrium stated above for a body at rest is also applicable to a body in motion. A cyclist cannot balance himself on a bicycle at rest for more than few seconds without his feet touching the ground. But with bicycle in motion, he can balance himself for hours together without his feet touching the ground. The motion gives some stability to the system.

The earth is revolving round the sun. Certainly, there are perturbations in this orbiting motion of the earth. But the system is in stable type of motion. Perturbations do not tend to grow.

The atmosphere as a whole is in a state of stable equilibrium, within a stable and robust climate which does not change appreciably with time, inspite of several perturbations.

In meteorology, two separate types of stability have come to be recognized: static stability and dynamic stability. Static stability is considered with respect to small vertical displacements and the play of buoyancy forces in the disturbed state in the presence of gravity. If a parcel of air which is displaced vertically upwards, dry adiabatically, finds itself to be heavier than its new environment, it tends to return back to its original position; equilibrium is stable. If, in the new position, the parcel finds itself to be lighter than its environment, it goes further upwards; the perturbation grows, the equilibrium is unstable. In neutral equilibrium the parcel in its new position has the same density as its environment and it has neither acceleration nor declaration upwards. This reasoning about the static stability of the atmosphere with respect to dry adiabatic process is simplified if we bring in the concept of potential density or potential temperature (θ) . The equilibrium is stable if potentially colder (heavier) air is in the lower levels and potentially warmer (lighter) air is in the upper levels; in other words, potential temperature increases with height.

Except in very thin layers, particularly near hot ground in the day time, the atmosphere is in stable equilibrium with respect to dry adiabatic vertical displacements. Due to this static stability, we have the well -known Brunt -Vaisala oscillations and gravity waves in the atmosphere.

The real atmosphere has horizontal as well as vertical wind shears. Natural forces operating in the atmosphere tend to create these horizontal and vertical wind shears. It is, however, seen that when these wind shears exceed certain critical values, then the equilibrium configuration of the atmosphere in motion breaks down; the same natural forces which took the atmosphere to a state of critical wind shears and hence on the verge of dynamic instability, and actually right into the state of dynamic instability, tend to bring the atmosphere back into a configuration of stability, by reducing the wind shears. The same natural forces push the atmosphere into instability and the same natural forces restore the atmosphere to a state of stability. This play goes on and the atmosphere manifests corresponding changes in winds and weather on different scales of space and time.

When the unstable wind shears are essentially in the horizontal, the instability process is called barotropic instability; when the unstable wind shears are essentially in the vertical and associated vertical motions are dry -adiabatic, then the instability process is called baroclinic instability. Combination of horizontal and vertical shears gives what is known as " Combined barotropic –baroclinic instability ".

Dynamic instability of fluids in motion, called hydrodynamic instability, is of great importance and has interested many eminent Mathematicians and Physicists during the last 200 years. Hydro dynamic instability of the atmosphere is little more difficult to handle than the hydrodynamic instability of classical homogeneous incompressible fluid. Although a number of eminent scholars investigated the hydrodynamic instability of the atmosphere towards the end of 19th century and also later on. When heavier fluid lies on the top of lighter fluid gravitational forces will tend to cause instability. Related to this is the instability at the interface between a liquid and a gas, when there is an acceleration in the direction going from the liquid to the gas.

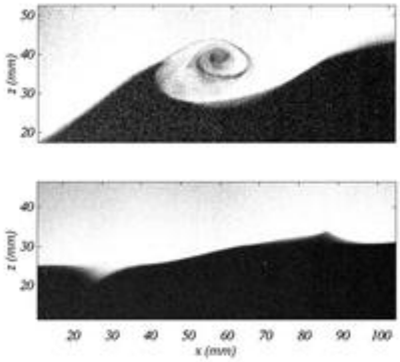

Fig 1.9: Photos showing Kelvin-Helmholtz instability (upper) and Holmboe's instability (Lower)

Figure 1.9 on the above shows two photos of different fluid instability - the upper one is often called Kelvin-Helmholtz instability; the lower one is called Holmboe's instability. These types of instability occur in both the ocean and atmosphere, where they are an important source of turbulence and mixing. ***Kelvin-Helmholtz instability can occur when velocity shear is present within/ between two fluids.***

Fig 1. 10: A KH instability rendered visible by clouds over Mount Duval in Australia

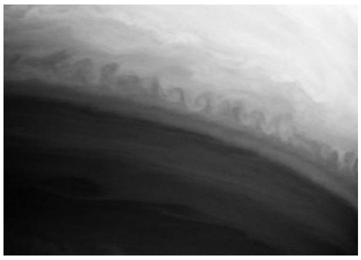

Fig 1. 11: A KH instability on the planet Saturn, formed at the interaction of two bands of the planet's atmosphere

Fige 1. 12: Kelvin–Helmholtz instability clouds in San Francisco

Fig 1.13: Complicated shear layer

1.6 Some Important Results

Here we state without proof some important results for the stability of inviscid flows.

Squire's Theorem

If a growing 3D disturbance can be found at a given Reynolds number, then a growing 2D disturbance exists at a lower Reynolds number.

Rayleigh's Inflexion Point Theorem

Suppose that u_B and Du_B are continuous in $y_1 < y < y_2$. Rayleigh's inflexion point theorem then states that a necessary (though not sufficient) condition for inviscid instability is that the base state possesses an inflexion point $D^2 u_B = 0$ somewhere in the domain $y_1 < y < y_2$. If a base state lacks an inflexion point, therefore, we can conclude it to be stable, for inviscid fluids.

From this theorem we can conclude that

- a necessary condition for inviscid instability is the presence of an inflexion point;
- the absence of an inflexion point necessarily confers (inviscid) stability.

Fjortoft's Theorem

A necessary condition for instability is that

$u_B''(u_B - u_c) < 0$ somewhere in the fluid

Where u_c is the flow speed at the inflexion point (i.e., $u_c = u_B(y_c)$ *with* $u_B''(y_c) = 0$).

Tollmien's Result

For a symmetrical profile in a channel, or for a boundary layer, the existence of an inflexion point $u_B''(y_c) = 0$ is not only necessary but also a sufficient condition for instability. The inviscid flow sketched as follows is thus linearly unstable.

Howard's Semi-circle Theorem

All unstable waves have $c = c_r + ic_i$ satisfying

$$\left[c_r - \frac{1}{2}(u_{\max} + u_{\min})\right]^2 + c_i^2 \le \frac{1}{4}(u_{\max} - u_{\min})^2$$

Where, $$u_{\max} = \max u_B(y)$$

$$u_{\min} = \min u_B(y)$$

Thus, all unstable modes lie in the shaded semi-circle, centered on $c_r = \frac{1}{2}(u_{\max} + u_{\min})$, $c_i = 0$ and of radius $\frac{1}{2}(u_{\max} - u_{\min})$

1.7 Formal Approach and Techniques to Stability Theory

The equations of fluid dynamics viz., the equations of mass conservation, momentum, energy and state, despite their complexity, allow some simple patterns of flows as stationary solutions. However, the problem, though conceptually very simple, is not very easy mathematically. The resulting differential equations even in case of simplest possible flows are of higher order; therefore, the discussion of the stability of the flows has been confined mainly to simple problems only. Either of the three methods, namely perturbation method, normal mode method and the energy method is generally adopted to examine the stability or instability of a system.

1.7.1 Perturbation Method

This is a very suitable and appropriate method for investigating stability problems. In this method, the system is imagined to undergo a specific small displacement. Additional forces so developed, if tend to increase the displacement thereby increasing the deformation of the system still further, the system is said to be unstable.

In linear stability theory we take the perturbations to be arbitrarily small and so we neglect those terms in the governing equations, which are the products of the perturbations and for their derivatives, as compared to linear terms. Thus we get a system of homogeneous linear differential equations with homogeneous boundary conditions. Therefore in linear stability theory, the perturbations either grow exponentially or decay exponentially or the magnitude of the perturbations remains constants, if the perturbations grow exponentially then the system is said to be unstable and if these decay exponentially then the system is said to be stable, and if there magnitude remains constant then the system is said to be in marginal state.

1.7.2 Energy Method

Here we closely examine the change of sign in the potential energy of the system with respect to all disturbances. If we observe a negative sign then the potential energy characterizing the system decreases for some disturbance and the kinetic energy is available for motion to depart away from the equilibrium state and the system becomes unstable. This method gives the condition of stability but not the dispersion relation, by which growth rate of instability can be obtained. Therefore, it is not useful in dissipative systems such as finitely conducting or viscous systems etc. This method presents a global outlook and unlike the normal mode approach, takes non-linearity of the system into consideration.

In this method, we calculate the variation of energy of disturbances with time and the conclusions depend on whether the energy decreases or increase as time goes on. The theory admits an arbitrary form of the superimposed motion and demands only, that it should be compatible with the equation of continuity. Since the pattern of disturbances changes in course of time, the energy could momentarily increase and subsequently die away. Thus unless a flow is unstable or stable with all disturbances, no conclusive answer can be expected.

This method is global in nature as we calculate the kinetic energy of whole of the system and thus is restricted in applications. In a viscous fluid it can hardly be expected that the motions at very small length scales be not damped out. Consequently, this method gives a surest limit for the stability of the flow, but it is crude in giving the unstable limits and also it gives very little information about the local behavior of the perturbations.

For a better physical understanding of the instability of a given flow, it is still important to examine the mechanism of energy balance. The key mechanism is shifted in the shape of two components of the velocity of oscillations by viscous forces at the solid boundary. In this process the Reynold stress converts energy from the basic flow to the disturbances.

Energy is converted from the basic f low to the disturbances by the action of Reynolds stress and dissipated into heat by viscosity. To maintain a sustained oscillation, the Reynolds stress, again must have proper sign. Energy relations have been used very often for the study of fully developed turbulence, and the discussions are obviously applicable to finite disturbances of fairly general type.

The energy principle technique generally depends upon a variational formulation of the equation of motion. Rayleigh first used it in the calculation of the frequencies of vibrating systems. Its advantages lies in the fact that if one seeks solely to determine stability, and not growth rates or oscillation frequencies, it is necessary only to discover whether there is any perturbation which decreases the potential energy from its equilibrium value.

1.7.3 Normal Mode Analysis

For determining the stability of a hydro dynamical system by normal mode analysis, the linearized perturbation equations are set up first in a single perturbation variable by eliminating the remaining variables from the linear equations derived from the equations of conservation of mass, momentum and energy, retaining only the linear terms in perturbed quantities. These equations are then solved either analytically or with the help of vibrational procedure or through an integral equation under a set of appropriate boundary conditions. This leads to the dispersion relation in the parameters determining the stability of a system. The dispersion relation thus obtained is quite complex and an analytical interpretation is not always possible. Therefore, in order to determine the effect of a particular physical parameter on the growth rates, we analyze the change by varying that parameter while keeping the other parameters fixed. An increase in growth rate implies the destabilizing influence of that particular parameter and a decrease in growth rate shows stabilizing influence of the parameter.

For the investigations in any stability analysis to be complete, it is assumed that the perturbations can be resolved into dynamically independent wave-like components, each component satisfying the linearized equations of motion and the boundary conditions separately. The essential point here is that the disturbances, in all cases, must be expanded in all possible forms of time function constituting the time behavior of the quantities in the system, i.e. in terms of some suitable sets of normal modes which must be complete for such an expansion to be possible. Thus if A'(x, y, z, t) is a typical wave component describing a disturbance, we expand it in the manner,

$$A'(x, y, z, t) = A(z) \exp [i (\alpha x + \beta y + ct)],$$

where $k= [\alpha^2 + \beta^2]^{1/2}$ is the real wave number of the disturbance and c is a constant to be determined and in general, is complex. It is to be remembered that the real parts are to be taken to get physical quantities, this being permissible for the linear problems. Further, since the perturbation equations are linear, the reaction of the system to a general disturbance can be determined if we know the reaction of the system to disturbances of all assigned wave numbers. In particular, the stability of the system will depend on its stability to disturbances of all wave numbers and instability will follow from the instability with respect to even one wave number.

The assumption that a disturbance can be represented by wave components, according to the method of normal modes, serves to separate the variables and reduces the linearized equations of motion from partial to ordinary differential equations. The final process consists of solving the set of coupled, homogeneous, ordinary linearized differential equations governing the amplitude A(z), subject to appropriate boundary conditions of the problem under investigation. Indeed, the requirement that the equation allows a non-trivial solution satisfying the various boundary conditions leads directly to a characteristic value problem for c. In general, the characteristic value for c will be complex, whose real and imaginary parts will apart from various modes, depend on the physically significant parameters involved in the system.

Now, if the subscript k is attached to c in order to emphasize the fact that different values of c correspond to various modes appropriate to a particular problem (distinguished by k) we have,

(i) $C_k^{(r)} < 0$ for all k => stability

(ii) $C_k^{(r)} > 0$ for at least one k => instability

(iii) $C_k^{(r)} (R_1, R_2, R_3, \ldots\ldots\ldots\ldots, R_j) = 0$ => the marginal state with respect to disturbance belonging to k.

Condition (iii) provides a locus in (R_1, R_2, …………, R_j) space, which separates states which are stable from those, which are unstable with respect to the disturbance belonging to the particular mode k. Also if $C_k^{(r)} = 0$ implies $C_k^{(i)} = 0$ for every k, then the Principle of Exchange of Stabilities (PES) validate the marginal state and the instability sets in through stationary cellular convection. But if $C_k^{(r)} = 0$ implies $C_k^{(i)} \neq 0$ even for at least one k, then the PES is not valid at the marginal state and we have the case of over stability.

1.8 Nondimensional Parameters

In analyzing and discussing the results of the linear stability theory, it is often convenient to combine the various parameters into certain nondimensional combination of numbers.

Some of the important nondimesional numbers are given below.

1.8.1 Reynolds Number R_e

In fluid mechanics, the Reynolds number R_e is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.

They are used to characterize different flow regimes, such as laminar or turbulent flow: laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce random eddies, vortices and other flow instabilities.

Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho VL}{\mu} = \frac{VL}{\gamma} = \frac{QL}{\gamma A}$$

where:

V - the mean fluid velocity (SI units: m/s)

L - characteristic linear dimension, (traveled length of fluid, or hydraulic radius when dealing with river systems) (m)

μ -dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/m·s) ν - the kinematic viscosity ($\nu = \mu / \rho$) (m^2/s)

ρ - the density of the fluid (kg/m^3)

Q - the volumetric flow rate (m^3/s)

A -the pipe cross-sectional area (m^2).

Reynolds Number may also be defined as,

$R_e = UL/\Upsilon$

where,

L - is the dimension of the system

U - is the measure of the velocities which prevail in the stationary flow.

ϒ - is the kinematic Viscosity.

1.8.2 Richardson Number R_i

Richardson number denoted by R_i measures the ratio of buoyancy forces to the inertial force.

$$R_i = \frac{-g}{\rho}\frac{d\rho}{dz}$$

where,

g is the acceleration due to gravity.

ρ is the density of the fluid.

1.8.3 Magnetic Reynolds Number R_m

The Magnetic Reynolds number R_m is given by

$R_m = UL/\eta$

where,

L - is the dimension of the system

U - is the measure of the velocities which prevail in the stationary flow.

η- is the Magnetic resistivity.

The magnetic Reynolds number is the ratio of the fluid flux to the magnetic diffusivity and is one of the important parameters in Magneto Fluid Dynamics. The Magnetic Reynolds number determines the diffusion of the magnetic field along the streamlines similar to the ordinary Reynolds number, which determines the diffusion of vortices along the streamlines.

The magnetic Reynolds number is a measure of the effect of the flow on the magnetic field. If it is very small compared to unity, the magnetic field is not distorted. When it is very large, the magnetic field moves with the flow and is called frozen.

1.8.4 Prandtl Number Pr

The Prandtl number Pr is a dimensionless number approximating the ratio of momentum diffusivity (kinematic viscosity) and thermal diffusivity. It is named after the German physicist Ludwig Prandtl.

It is defined as:

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{viscous\, diffusion\ rate}{thermal\ diffusion\ rate} = \frac{C_p \mu}{k}$$

where:

ν - kinematic viscosity, $\nu = \mu / \rho$, (SI units : m^2/s)

α - thermal diffusivity, $\alpha = k / (\rho c_p)$, (SI units : m^2/s)

μ - dynamic viscosity, (SI units : Pa s)

k- thermal conductivity, (SI units : W/(m K))

c_p - specific heat, (SI units : J/(kg K))

ρ - density, (SI units : kg/m^3).

Note that whereas the Reynolds number is subscripted with a length scale variable, Prandtl number contains no such length scale in its definition and is dependent only on the fluid and the fluid state. As such, Prandtl number is often found in property tables alongside other properties such as viscosity and thermal conductivity.

Typical values for Pr are:

around 0.7-0.8 for air and many other gases,

around 0.16-0.7 for mixtures of noble gases or noble gases with hydrogen

around 7 for water (At 20 degrees Celsius) around 1 × 10^{25} for Earth's mantle

between 100 and 40,000 for engine oil,

between 4 and 5 for R-12 refrigerant

around 0.015 for mercury.

1.8.5 Magnetic Pressure Number S

The Magnetic pressure number S is given by

$$S = \frac{\mu \ H_o^{\ 2}}{\rho u^2}$$

where

μ - Magnetic permeability

H - Magnetic field strength

ρ - Density of fluid

u - Velocity of fluid

Magnetic pressure number is the ratio of the magnetic pressure to the dynamic pressure and is a measure of the effect of magnetic field on the fluid. Only when S is the order of unity, will the flow be influenced noticeably by the magnetic field and if it is very small, all the magnetic effects can be disregarded.

In engineering problems it is difficult to obtain R_m,>>1 because of the low electrical conductivity of the useful fluids whereas S can be easily varied from unity to 10. In astrophysical problem R_m is very large due to the characteristically great length. The electrical current is controlled by the magnetic field rather than by the electrical conductivity when the R_m is small.

1.8.6 Brunt-Vaisala Frequency N

Brunt-Vaisala frequency or buoyancy frequency is the angular frequency at which a vertically displaced parcel will oscillate within a statically stable environment. It can be used as a measure of atmospheric stratification.

N= $\sqrt{\frac{g \ d\theta}{\theta \ dz}}$ where θ is the potential temperature, g is the local acceleration of gravity and z is the geometric height.

1.8.7 Froude Number

It is a dimensionless quantity used to indicate the influence of gravity on fluid motion. It is generally expressed as $Fr=v/(gd)^{1/2}$ in which d is the depth of the flow, g is the gravitational acceleration , v is the celerity of a small surface wave. When Fr is greater than 1, small surface waves can move downstream , when Fr is less than 1they will be carried upstream , and when Fr is equal to 1, the velocity of the flow is just equal to the velocity of surface waves.

1.8.8 Rotation Number

It is a dimensional quantity which is the ratio of Coriolis force and inertial force and is defined by

$$\tau = 2\,\Omega\, d/u_0$$

where

Ω	-	angular velocity of rotation
d	-	length scale
u_0	-	velocity of the flow

1.9 Outline of the Thesis

Given the wide range of applications of study of shear layers, the main objective of this dissertation is to study the stability of shear layers under specified conditions.

The broad outline of the thesis is as follows

- Introduction
- Literature Review
- Transient development of perturbations in inviscid stratified parallel shear flows
- Effect of magnetic field on the stability of an electrically conducting, inviscid, stratified shear flows
- Effect of rotation on the stability of an electrically conducting, inviscid, stratified shear flows
- Effect of rotation on the linear stability of nonparallel shear flows
- Linear stability of nonparallel rotating unbounded shear flows

❖ The stability of buoyancy driven shear flows in inclined long cavities with end wall temperature difference is investigated
❖ Summary

In Chapter I we have presented introductory aspects of the stability analysis and shear flows. In chapter II, significant earlier contributions related to the problems studied in the thesis are summarized.

Chapter III extends the study of Farrell and Ioannou (1993) to a stratified flow through a channel with arbitrary shear velocity profile subject to long wave approximation. Analytical solutions were found for frequency and velocity stream functions.

Chapter IV presents effect of uniform magnetic field on the stability of a stratified flow through an infinite channel with arbitrary shear velocity profile. Asymptotic solutions have been obtained for Velocity and growth rate using Perturbation techniques. The effect of various parameters such as Brunt Vaisala frequency, Magnetic parameter, wave number on the growth rate of the small disturbances is studied numerically.

In Chapter V we have investigated the stability of inviscid, rotating stratified electrically conducting parallel shear flow. The fluid was considered to be in a state of non parallel flow with the basic velocity profile (U(z),0,0). The governing equations were derived. These equations reduce to those obtained by Farrell and Ioannou (1993) for vanishing rotation number.

The stability of the flow was analyzed using normal mode approach and the analysis was restricted to a long wave approximation.

In Chapters VI and VII an attempt is made to study the effect of rotation on the stability of inviscid, stratified electrically conducting non parallel bounded and free shear flows. The results obtained were validated with the results obtained by Padmini and Subbaiah (1995) for vanishing rotation number.

In Chapter VIII we made an attempt to study the analytical solutions of stability of buoyancy driven shear flows in inclined long cavities with end to end wall temperature difference. Growth rate is calculated numerically for different non-dimensional numbers.

In all the above mentioned problems, we specify the full system of nonlinear equations which includes equation of continuity, equation of motion and equation of state. The boundary conditions for the geometry are also specified. A basic laminar flow field and pressure field that form a time-independent solution of the nonlinear equations and boundary conditions is found. We then subject the base state to a small perturbation involving a control parameter λ. Then we try to find out answers for the following questions

- For any value of the control parameter λ, is the basic laminar state linearly stable or unstable, *i.e.* do the perturbations decay or grow in time?
- What is the threshold value of λ at which the laminar state first becomes unstable?
- At the onset of instability, what is the spatial form of the unstable perturbations, and how fast do they grow?

To answer these questions, we substitute the perturbed forms into the governing equations and expand these equations about the base state in increasing powers of the perturbation's amplitude δ. Neglecting terms $O(\delta^2)$ and higher, we then have a linearised equation set governing the dynamics of the perturbations. These linearised equations are usually studied via a normal mode analysis.

Using normal mode approach, the eigen values and corresponding eigen functions are calculated. In general, the eigen values are complex. Closed form solutions are obtained wherever possible or solutions are found numerically using the software Mathematica 8.0. The numerical results of the flow characteristics are presented graphically.