CHAPTER II

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Review of Literature

2.1 Introduction

One of the most basic and challenging problems in fluid mechanics is to reach an understanding of the various physical mechanisms involved in the process of transition from laminar to turbulent flow. It is now well known that in many instances initial process is one of instability with respect to infinitesimal disturbances. From a practical point of view, the overriding goal of hydrodynamic stability theory is to predict the transition from laminar to turbulent flow and the ability to control transition would open significant possibilities to the design engineering.

2.2 Theoretical Frameworks

The mathematical side of the subject, especially in terms of quantity of papers, has been dominated by the normal mode approach to linear stability.

Shear flow was first introduced and analysed by Euler (1755). The instability due only to the shearing motion of the fluid was found by Lord Rayleigh (1880, 1916). Rayleigh (1880) considered constant wind-shear with constant wind at the top and at the bottom in a fluid of constant density (R_i=0). Using half the depth of the shear layer as the characteristic length and the constant velocity at the top of the shear layer as a characteristic velocity, he found all non-dimensional wave-numbers between 0 and 0.639 to be unstable. Also, amplifying waves were all stationary.

Rayleigh (1913) investigated the stability of straight horizontal flow having a velocity shear in the direction perpendicular to the direction of the straight flow. He did not consider the rotation of the earth. He considered the fluid to be homogeneous and incompressible. He concluded that the stability of this flow depended on the type of the horizontal shear. The flow would be unstable if the shear vorticity has a mathematical maximum in the region. To arrive at this conclusion, Rayleigh tried to approximate the continuous variation of velocity by a number of belts, each with a linear variation of velocity in the direction normal to the velocity direction.

Rayleigh (1916) considered basic rotating flow of an inviscid fluid moving with angular velocity ω(r), an arbitrary function of the distance r from the axis of rotation. By a simple physical argument, Rayleigh derived his celebrated criterion for stability: a necessary and sufficient condition for the stability of the flow with respect to axis symmetric perturbations is that the square of the circulation either increases or remains constant with increase of r, i.e $\frac{\partial}{\partial r}\left(2\pi r^2\omega\right)^2 \geq 0$. For convenience of further applications, this condition is also written as $\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^2\omega\right)^2 \geq 0$

Goldstein (1931) considered the stability of a shear layer, within which both the velocity and density vary linearly, and outside which the velocity and density are constant. Thus he introduced density gradient into the type of flow considered by Rayleigh (1880). Solberg (1936) considered the stability of an axisymmetric baroclinic vortex. Because the disturbances are axisymmetric in that case, the instability has been called *symmetric instability*.

Charney's (1947) pioneering the work on baroclinic instability is well-known. This analysis was analytical, based on quasi-geostrophic model, continuous in the vertical. In his classical paper, he analysed U having constant vertical shear $\frac{\partial U}{\partial Z}$ in a continuous stratified atmosphere. He used quasi-geostrophic approximation in a β-plane and Charney's work has been the basis for stability analysis of synoptic scale systems in metrology.

Kuo (1949) extended the result of Rayleigh to zonal currents on the rotating earth; He considered the shear flow on the rotating earth and treated continuous variation of velocity in the direction normal to the velocity direction. In this type of flow, there is exchange of kinetic energy between the basic linear flow and the sinusoidal perturbation.

The fully viscous, linear, non-hydrostatic inertial stability problem has been solved exactly by Kuo (1954) for the special case of a neutrally stratified fluid confined above and below by rigid, free-slip boundaries. Solutions were obtained for horizontally unbounded disturbance as well as for motions confined by two vertical free-slip walls. The possibility of oscillatory instability is not considered, although it is shown that amplifying oscillatory motions are not possible when the Prandtl number is unity. Kuo finds that the flow becomes unstable when a normalized vertical shear parameter exceeds a critical value dependent on the Taylor number(T_0). The following table compares some of the significant earlier works done on shear flow instability,

	Linear Or Non Linear	**Equilibrium Flow**	**Boundaries**	**Criterion for Instability**	**Wavelength of Maximum Growth**
Rayleigh (1916)	L	Homogeneous, incompressible and inviscid circular vortex	None	$\frac{\partial M^2}{\partial r} < 0$	-
Solberg (1936)	L	Inviscid, Boussinesq circular vortex with constant vertical shear and static stability	None	$\left(\frac{\partial M^2}{\partial r}\right)_\theta < 0$	-
Kuo (1954)	L	Viscous nonhydrostatic zonal flow with constant shear and neutral stratification	Rigid free-slip boundaries at bottom and top	$\frac{\bar{U}_z}{\tilde{\eta}}(1+\sigma) > F(T_0)$	0(H)
Stone (1966)	L	Inviscid hydrostatic boussinesq zonal flow with constant shear and static stability	Rigid top and bottom	$1 - \frac{1}{Ri} - \frac{\bar{U}_y}{f} < 0$	0
McIntyre (1970)	L	Viscous hydrostatic Boussinesq zonal flow with constant shear and static stability	None	$Ri < (1+\sigma)^2 / 4\sigma$	∞ or $o\left(v^{\frac{1}{2}} f^{-\frac{1}{4}} \bar{U}_z^{-\frac{1}{4}}\right)$
Yanai	NL	Inviscid zonal	Rigid free-	-	-

and Tokioka (1969)	(Numerical integration)	flow	slip at top and bottom		
Walton (1975)	NL (weakly)	Viscous hydrostatic boussinesq zonal flow with constant shear and static stability	Rigid at top and bottom	$4\frac{\sigma}{(1+\sigma)^2}Ri <$ $F(T_0)$	$o\left(T_0^{-\frac{1}{6}}\bar{U}_z f^{-1} H\right)$

Miles and Howard (1964) generalized Rayleigh's (1880) problem, considered the stability of a shear layer within which both the velocity and the density vary linearly and outside of which the velocity and density are constant. As found by Rayleigh, they confirmed that for R_i=0, the band of unstable wave-numbers lies between 0 and 0.639. Further they found that instability starts at R_i=0.25, increases as R_i decreases to less than 0.25 and becomes maximum at $R_i = 0$. The non-dimensional wave number of maximum growth rate is 0.415. This corresponds to dimensional wave length of (15.14h) where h is half the thickness of the shear layer. For maximum growth, the wave length is 7.57 times the depth of the shear layer.

Nitta and Yanai (1969) considered profiles of westerly and easterly zonal currents with y-variation given by $sin^2(\frac{\pi y}{2b})$. In both the cases, the basic wind is maximum at y=b and vanishes at y=0,2b . Based on linear theory they have found that for moderate horizontal shears of the zonal current, the most preferred scale of the unstable wave in the easterlies is larger than that in the westerlies.

Busse and Chen (1981a) showed that the disturbance of maximum growth of the symmetric instability is not quite x-independent. The general problem of stability of an inviscid fluid with continuously varying density distribution in a direction normal to the mean flow was first attacked by Taylor (1931) and Goldstein (1931). Drazin (1958) observed that a velocity distribution that varies as the hyperbolic tangent of the transverse co-ordinate can be handled analytically. Menkes (1959) investigated the effects of density variation in the absence of gravity on the stability of a horizontal shear layer between two streams of uniform velocities. The density is assumed to decrease exponentially with height and velocity is represented by U(y) = tanh y. Qualitative agreement with experimental evidence is obtained.

The importance of doubly diffusive effects for the symmetric instability was discovered by McIntyre (1970) who demonstrated their strongly destabilizing influence on baroclinic shear flows. Thus the stability of a baroclinic shear flow depends on the Prandtl number and the Prandtl number dependence of the critical Richardson number was originally derived by Kuo (1956). The stability of an infinite fluid layer was considered by Magen *et al* (1985) subject to horizontal flow and arbitrary initial vertical temperature and salinity distributions. They outlined a method for the constructions of a general stability chart and showed that an increase in the Reynolds number cannot increase the stability domain.

Johnsons' (1963) work was concerned with the stability of an inviscid, homogeneous unbounded rotating shear flow. Emanuel (1979) gave a variational solution of the linear stability problem of a bounded, viscous, Boussinesq rotating fluid with constant vertical and horizontal shears. Brunsvold and Vest (1973) studied the stability of a layer of Newtonian fluid confined between two horizontal disks which rotate with different angular velocities.

Walton (1984) examined the stability of a zonal shear flow when the Ekman number is small. Nagata (1985) examined the stability of rotating shear flows by extending Busses' (1968) analysis of the non-magnetic case to fluids of high magnetic diffusivity. In Busses' theory, the viscous effects limiting the instability arise not in the shear layer itself but in the Ekman layers generated on the end walls by the perturbation.

Banerjee and Jain (1972) opened up an aspect of investigation in the stability problem of heterogeneous shear flows that was new and asserted that there must exist a necessary condition of instability in the form of a unification of Mile's and Howards theorems such that it will not only yield Miles theorem but also reduce Howards semi-circle bound on the range of c_r and c_i of unstable flows .

Stern (1969) and Linden (1974) have analysed the effects of shear on salt-finger instability and found that salt-fingers tend to form two-dimensional sheets in a shear flow. Hsu (1974) considered the influence of weak shear on thermohaline instability in general. He determined which modes of instability were preferred in different regimes and whether the heat and salt fluxes where enhanced or inhibited. In a review paper, Huppert and Turner (1981) summarized important applications of doubly diffusive convection theory to geophysical phenomena.

The stability of stratified shear flows of an inviscid, incompressible fluid is of interest in meteorology and oceanography. The linear stability of a stratified plane, parallel shear flow of an inviscid, incompressible fluid has been extensively studied by many authors, namely Drazin and Howard (1966), Pellacani (1983) and Yih (1980). However, the stability of non- parallel shear flows has not been studied sufficiently. Recently interest has been shown in the analysis of linear stability of non-parallel flows of incompressible boundary layers and has shown that the inclusion of cross- flow leads to the appearance of standing wave instability models.

Grosch and Jackson (1991) have analysed the stability of non- parallel flows of incompressible mixing layers and have found that the inclusion of cross- flow enhances mixing especially at supersonic speeds. The study of stability of compressible shear flows is important in the context of the projected use of scramjet engine for the propulsion of hypersonic aircraft.

Graham (1978) made a study of a non-parallel shear flow with a constant local density gradient in an inviscid, incompressible density stratified fluid. The shear was caused by maintaining a fixed resultant horizontal velocity and rotating the direction of the resultant velocity vector uniformly with increasing height. He found that for the case where each horizontal fluid plane exhibits a translational velocity of the same magnitude and where shear is caused by rotating the velocity vector with increasing height, stability strongly increases with increasing layer thickness.

Padmini and Subbaiah (1995) studied the problem of linear stability of non-parallel stratified shear flows to normal mode disturbances. It was shown that for a non-parallel flow, two components of Reynold's stress transfer energy from the basic flow to the disturbance while it was well known that only one component of the Reynolds's stress was important for parallel flows. Generalizations of Miles' formula for the vertical variation of Reynold's stress and Eliassen-Palm theorem were obtained.

Dunkerton (1997) studied a steady, non-parallel flow with vertical profiles of horizontal velocity and static stability. He noted that the shear production of perturbation kinetic energy in transverse instability is positive at intermediate or large R (R = non-dimensional rotation rate ie. R = f/w <1). For R approaching unity, shear instability takes precedence over convective instability.

Hart (2000) studied several aspects of thermal convection in the presence of a lateral shear associated with either a shear line or a circular vertex.

Adam (1980) used a rigorous approach by Barston to stability of Lagrengian systems to establish both rectangle and semicircle theorems for plane parallel flow along a horizontal but otherwise arbitrary magnetic field, permeating a perfectly electrically conducting incompressible fluid under gravity. The radius of the semicircle is reduced by magnetic effects and stable stratification. A Richardson criterion for stability against constant shear flow is also derived. The analogous problem for a compressible fluid is also discussed, and for a certain class of disturbances a 'semi-dumbell' theorem is established which is considerably stronger than the semicircle theorem. Possible astrophysical applications are discussed.

Since the appearance of a paper by Howard (1961) on eigenvalue bounds for plane parallel flow of an inviscid incompressible stratified fluid, many subsequent papers extending and generalising the results have been published (see e.g. Eckart (1963), Adam (1980), among many others).

The stability of parallel shear flow of a compressible, inviscid fluid to infinitesimal perturbations has received considerable attention in the scientific literature. Results for fluids having continuous velocity profiles have been obtained by several authors (e.g., Mack (1965); Blumen (1970); Blumen, Drazin, and Billings (1975)).

Lucas (1981) considered the stability of plane parallel flow of an inviscid, compressible, perfectly conducting fluid permeated by a tranverse magnetic field. He constructed a Liapunov functional for the plane parallel flow of an inviscid, compressible, perfectly conducting fluid permeated by a transverse magnetic field. Sufficient conditions for stability to infinitesimal perburbations are obtained. The L_2-norms of certain system variables are shown to be bounded uniformely in time.

Leiv Storesletten (1982) considered an inviscid compressible and stratified fluid subject to an external force field depending on the vertical coordinate z. In this work it was shown that a shear in the horizontal direction always gives rise to instabilities, even if the flow is stratified in a (statically) stable way. Moreover, it was shown that all shear flows are unstable if the external force field vanishes. This result was found to be in agreement with that obtained for homogeneous incompressible fluids by Landahl(1980).

2.3. Numerical Studies

The stability of parallel shear flow of a compressible inviscid fluid to infinitesimal perturbations has received considerable attention in the scientific literature. One of the initial investigations on stability was done by Kelvin (1880). Kelvin stated that for homogeneous, incompressible, non- viscous fluid, filling a finite fixed space, and possessing a given amount of vorticity, the condition for steady motion is that the energy is a thorough maximum, or a thorough minimum or a minimax. Out of a number of such steady- state configurations, those configurations are stable in which the energy is a thorough maximum or a thorough minimum. The ultimate condition of steady state equilibrium is that in which potential energy an absolute minimum is.

In particular, Landau (1944), Miles (1958), Fejer and Miles (1963) and Fejer (1964) investigated the stability of the plane interface between two compressible fluids in the presence of uniform magnetic fields.

Rayleighs' inflection point theorem states that a necessary condition for instability is that $U''(y)$ must vanish somewhere between the walls $y = y_1$ and $y = y_2$. Velocity profile $U = \tanh(y)$ which represents the exemplary case of a profile with an inflection point has been widely used in the literature. Michalke (1964) integrated numerically the Rayleigh stability equation for amplified disturbances of the hyperbolic-tangent velocity profile. He showed that in the case of amplified disturbances two concentrations of vorticity occur within a disturbance wavelength, while in the neutral case only one maximum of vorticity exists.

Yanai and Tokioka(1969) performed a numerical experiment in order to simulate meridional motions in an axially symmetric vortex. In this experiment, the nonlinear inviscid equations of motions are integrated in a domain bounded above and below by rigid boundaries. The results are in accord with the linear theory but the horizontal wavelength is limited by the numerical grid size.

Blumen (1971) investigated the stability of a barotropic shear flow $\overline{U} = U \tanh(y/L)$ of a stably stratified fluid of three dimensional disturbances by linear analysis.

The stability of a layer of Newtonian fluid confined between two horizontal disks which rotate with different angular velocities was studied by Brunavold and Vest (1973). Both iso thermal and adversely stratified fluids are considered for small shear rates at low to moderate Taylor numbers. The linearized formulation of the stability problem is given a finite difference representation and the resulting algebraic eigenvalue problem is solved using efficient numerical techniques.

For the specific situations of fluids having continuous velocity profiles, representative contributions include the work of Blumen (1970), Blumen *et al* (1975), Lucas (1981) and Parhi and Nath (1991). Vyas (1992) studied the effect of magnetic field on the stability of an inviscid compressible fluid by using progressing wave expansion method.

The problem of the stability of plane parallel shear flows in rotating systems is complex not only because of the presence of the coriolis force also, due to the absence of Squires' theorem. Howard and Drazin (1964) considered the stability of basic steady parallel flow of incompressible, inviscid homogeneous fluid rotating with variable coriolis parameter, governed by β - plane approximation. He showed that barotropic instability may be considered as a cause of the infrequent occurrence of easterly jets in the atmosphere and oceans. Stability of a baroclinic zonal current to symmetric perturbations on a meridionally unbounded f-plane was considered by Walton (1975). The lower boundary was at rest, but the upper one moves with a constant velocity in keeping with velocity of the zonal current. It was found that for the monotonic mode zonal momentum is convected polewards. The possible implications of this result for the dynamics of Jupiter's atmosphere were also discussed.

Roger and Lance (1960) have obtained complete numerical solutions over a wide range of conditions with the fluid at infinity in solid rotation or at rest. Baroclinic instability of weakly non-parallel zonal flows was investigated numerically by Merkine and Balgovind (1983).

Churilov and Shukhman (1988) studied the character of the evolution of weakly unstable disturbances of a small amplitude in a stratified shear flow that was initiated by Churilov and Shukhman (1987) and Brown *et al*(1981). Shukhman (1989,1991) also considered the stability of a cylindrical mixing layer in an incompressible and compressible fluid at large Reynolds number. It is shown that the stability is defined by the shear width parameter D, namely the model is unstable when $D < D_{crit} = 1/2$.

Over the last hundred years or so, there have been innumerable studies of the problem of instability of plane parallel shear flow. Comprehensive reviews of the works on the shear instabilities are summarized by Orzag and Patera (1983) and Brown (1980).

Uchida. and Ohya. (2001) investigated the stably stratified flows over a two-dimensional hill in a channel of finite depth numerically at a Reynolds number of 2000. They detected a flow unsteadiness due to the periodic shedding of upstream advancing columnar disturbances with mode n = 1 with a clockwise circulation.

Graham Pullan, John Denton and Michael Dunkley (2003) made a study of shear layers shed by aircraft wings roll up into vortices. They used a simple numerical experiment to model the shear layer instability and the effects of trailing edge shape. It was found that the latter had a strong effect on shear layer roll up.

2.4. Observational/ Experimental Works

The instability of shear flows is extensively invoked for explaining various phenomena in hydrodynamics, the physics of the atmosphere and ocean, geophysics and so on. The environment setting in which a monsoon depression or Africa wave is imbedded exhibits strong easterly shear (Mishra and Salvekar (1980); Burpee (1972)). Shear and rotation in combination induce thermohaline instability. The possible instability of a doubly diffusive system due to such added effects may be important in a wide range of planetary and stellar phenomena, as well as in engineering problems dealing with centrifugal separators and other devices (Worthem (1983)). Several authors have proposed hydrodynamical models based on instabilities of zonal shear flow for explanations of the global motion structure in atmospheres of large planets (e.g., Dicke (1964); Moore and Spiegel (1964); Kirbiyik and Smith (1976); Yavorskaya and Belyaev (1982); Stone (1967); Hide (1966); Eady (1949); Bennetts and Hoskins (1979)).

Townsend (1958) was the first to discuss shear flow in a thermally diffusing medium, in connection with turbulence in the Earths' upper atmosphere. Stone (1967) analysed the baroclinic stability theory to the dynamics of Jovian atmosphere. He investigated the hypothesis that the zonal motion in the Jovian atmosphere are thermal winds and that the latitudinal cloud bands are caused by baroclinic instabilities under non geostropic conditions. The stability theory is modified to take into account deep atmospheric effects. Comparison with observations indicates that the hypothesis is feasible although very speculative. It has long been known that the atmosphere of Jupiter is in a state of motion. Two fundamental observations lead to this conclusion: First the observation that the rate of rotation is different at different latitudes, implying the existence of zonal motions and second the observation that the clouds are present implying the existence of vertical motion. During the past century much data has been gathered about these motions, but there has been a singular lack of theoretical work attempting to explain them. Various energy sources are available to drive these motions, but so far only very tentative discussions of Jovian dynamics have been presented. (Eg. Hide (1966)) and no attempt has been made to explain how such energy source could lead to the observed motions or to put such suggestions on quantitative basis. Such a lack is unfortunate because theoretical studies may point to the need for observations different from the kind now being made;

Faller (1963) carried out an experimental study concerning the stability of the steady laminar boundary layer flow of a homogeneous fluid which occurs in a rotating system when the relative flow is slow compared to the basic speed of the rotation. This boundary layer flow became unstable above the critical Reynolds number Re_c = UD / γ =125$\pm$ 5 where U is the tangential speed of flow, D = $(\gamma/\Omega)^{1/2}$ is the characteristic depth of the boundary layer, γ is the kinematic viscosity, and Ω is the basic speed of rotation.

Zahn (1974) has applied Townsends' method to derive physical criteria appropriate in stellar situations. Jones (1977) proposed a criterion for the onset of shear instability in a stably stratified radiating star. The stability to non-axisymmetric perturbations of differentially rotating stellar interiors in the presence of toroidal and poloidal magnetic fields was investigated by Frizbe and Schubert (1970). The stability of rotating baroclinic star has been studied by Raghavachar (1984) in the presence of poloidal and toroidal magnetic fields taking into account the diffusive effects.

Some test experiments were carried out in an attempt to reproduce on a laboratory scale the astrophysical phenomena of magnetohydrodynamics which include the works of Amaghishi *et al* (1991), Lehnert (1955) and Nezlin (1986). In an experimental study Gregory *et al* (1955) determined a critical Reynolds number for the onset of shear instability. Faller (1963) made an experimental study of the instability of the laminar Ekamn boundary layer.

Hide and Titman (1967) reported an experimental study of detached rotatingshear layers under very simple conditions. Malcolm (1970) presented an experimental investigation of instability and post-critical flow in an electrically driven shear layer. Hart (1981) reported experiments with finite amplitude baroclinic waves arising from instability of a two layer f-plane shear flow.

Mal (1931) performed a series of laboratory experiments with controlled variation of temperature and vertical shear in fluids and obtained a variety of forms resembling photographic picture of clouds observed in the atmosphere from the ground and from the few aircraft ascents, available at that time.

The fact that frontal cloud and precipitation are frequently concentrated in bands approximately parallel to the front has been highlighted in many observational studies (e.g. Elliott and Hovind (1964,1965); Nozumi and Arakawa (1968); Browning and Harrold (1969); Kreitzberg and Brown (1970)). One occurrence of rainbands has been the object of a radar-synoptic study by Browning *et al*.(1973) and a dropsonde investigation below 4 km by Roachan and Hardman (1975). The presence of mid-level convection, near 600 mb, has been commented on in the above papers and in Houze *et al* (1976). Kreitzberg and Brown (1970) stressed the importance of this convection and stated that it occurs due to differential advection.

The observational studies have indicated that the spacing of bands is of the order of 80-300 km and that the length scale along the bands is much larger. The bands make only a small angle with temperature contours and, indeed, Elliott and Hovind (1964) related their orientation to the vertical shear in the region of convection. Theoretical possibilities for the origin of the bands include Ekman layer instability in the frontal region, a gravity wave generated at the front, and a natural scale for the convection which is itself caused by the differential advection (Brown *et al*. 1981).

Asnani and Keshavamurty (1974) have suggested that, moist adiabatic process and distinctly different vertical velocity inside the cloud area, as different from dry adiabatic process and vertical velocity in the cloud-free area, give effective static stability parameter in the thermodynamic equation, which is much smaller than the conventional dry static stability parameter used in the baroclinic model.

In order to study the possible importance of symmetric baroclinic instability in the formation of frontal rain bands, Bennetts and Hoskins (1979) investigated the conditional symmetric instability as a possible explanation for frontal rain bands with the inclusion of the effects of latent heat release.

Kelvin Helmholtz instability in geophysical situations were discussed by many authors. (Reiss and Corona (1977); Melatos and Peralta (2010); Nasr-Azadani *et al*.(2013); Nasr-Azadani and Meiburg (2014))

Motivated by the above interesting characteristics of shear flow instabilities, in the following chapters we have discussed the stability of parallel and non parallel shear flows to include the effect of uniform magnetic field. The stability of the flow is analysed by the normal mode approach. The analysis is restricted to long wave approximations.