CHAPTER IV

CHAPTER IV

Effect of Magnetic Field on Stratified Shear Flows

4.1 Introduction

Stability of stratified shear flows of an inviscid, incompressible fluid is of interest in meteorology and oceanography. Stability of stratified shear flows playa vital role in various problems of fluid dynamics, atmospheric and oceanic physics, astrophysics, and other areas of knowledge. The linear stability of a stratified plane parallel shear flow of an inviscid incompressible fluid has been studied extensively by many authors namely Drazin and Howard (1966), Miles (1958), Yih (1980) and Pellacani (1983).

Farrell and Ioannou (1993) studied transient development of perturbations in inviscid stratified shear flow. Graham (1978) studied about non-parallel shear flows of an inviscid, incompressible, stratified fluid and considered a stability analysis in terms of the possibility of complete mixing within a horizontal layer of thickness. Parhi and Nath (1991) made the linear stability analysis of a stratified shear flow of a perfectly conducting fluid in the presence of a magnetic field aligned with the flow, buoyancy forces under Boussinesq's approximation taken into account and modified Höiland's criterion for hydrodynamic case to magneto-hydrodynamic case.

Padmini and Subbiah (1995) studied the problem on linear stability of inviscid, incompressible non-parallel stratified shear flows to normal mode disturbances. In this paper, the work of Padmini and Subbiah (1995) is extended to examine the effect of uniform horizontal magnetic field. The stability of the flow is analyzed using normal mode approach and the analysis is restricted to long wave approximation.

4.2 Flow Description and Basic Equations

We consider the motion of an electrically conducting inviscid, Boussinesq stratified fluid confined between two horizontal infinite plates of variable density $\rho(x, y, z, t)$ situated at $y = \pm L$. A uniform magnetic field $\vec{B} = (B_x, 0, B_z)$ is applied



Fig 4.1: Magnetic stratified shear flow

In the present work, the following assumptions are made:

- Flow of a Newtonian fluid is considered, which is unsteady, inviscid and laminar in nature.
- Flow is between two horizontal rigid boundaries.
- ✤ No slip boundary conditions are imposed at the boundaries.
- Boussinesq approximation is applied in the momentum equation.
- All fluid properties are assumed constant except that the density is considered to vary with vertical co-ordinate z in the application of Boussinesq approximation.
- The effects of dissipation and diffusion are neglected
- Only two dimensional disturbances are considered.
- The basic flow is assumed as $\vec{q}_e = (U(y), 0, W(y))$
- A uniform magnetic field $\vec{B} = (B_x, 0, B_z)$ is applied

The governing equations of the system are

$$\nabla . \vec{q} = 0 \tag{4.1}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = \frac{-\nabla p}{\rho_0} - \frac{\rho g \hat{y}}{\rho_0} + \mu_m (\nabla \times \vec{H}) \times \vec{H} \quad (4.2)$$

$$\rho_{\rm t} + (\vec{q}.\nabla)\rho = 0 \tag{4.3}$$

$$\nabla . \vec{H} = 0 \tag{4.4}$$

with the boundary conditions

$$\vec{q} = 0$$
 on $y = \pm L$ (4.5)

where \vec{q} is the velocity vector, ρ the density, p the pressure, g the acceleration due to gravity, \hat{y} the unit vector in the vertical direction, \vec{H} the magnetic field and ρ_0 the mean density. Magnetic induction equation is given by

$$\frac{\partial \vec{H}}{\partial t} = \eta \nabla^2 \vec{H} + \nabla \times \left(\vec{q} \times \vec{H} \right)$$
(4.6)

where $\eta = \frac{1}{4\pi\sigma}$ the magnetic resistivity of the fluid. We introduce the non-dimensional quantities

$$\vec{q} = u_o \vec{q}^*,$$

$$(x, y, z) = L(x^*, y^*, z^*)$$

$$p = \rho_o u_o^2 p^*$$

$$t = \frac{Lt^*}{U_o},$$

$$\vec{H} = H_o \vec{H}^*$$

$$\rho' = \frac{\rho_o (U_0^2 N_0^2)}{Lg} \rho^* \qquad (4.6a)$$

Substituting these nondimensional quantities in equations (4.1) - (4.5) and removing asterisks for convenience, we get

$$\nabla . \, \vec{q} = 0 \tag{4.7}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} = -\nabla p - Ri\,g\hat{y} + S(\nabla \times \vec{H}) \times \vec{H} \quad (4.8)$$

$$\rho_t + (\vec{q}.\nabla)\rho = 0 \tag{4.9}$$

$$\nabla . \vec{H} = 0 \tag{4.10}$$

$$\frac{\partial \vec{H}}{\partial t} = \frac{1}{Rm} \nabla^2 \vec{H} + \nabla \times (\vec{q} \times \vec{H})$$
(4.11)

The related boundary conditions in dimensionless form are

$$\vec{q} = 0$$
 on y = ± 1 (4.12)

The condition for the flow to be at equilibrium is given by

$$\frac{-\partial p_0}{\partial y} = Ri \rho_0 \tag{4.13}$$

The dependence of the problem on the material properties has been reduced to the dimensionless parameters such as magnetic pressure number S, magnetic Reynolds number Rm and Richardson number Ri and are given by

$$S = \frac{\mu H_0^2}{\rho U_0^2}, \qquad Rm = \frac{L U_0}{\eta}, \qquad Ri = \frac{-g}{\rho} \frac{d\rho}{dy}$$
(4.14)

The perturbed flow variables are taken as U(y) + u, v, W(y) + w, $\rho_0(y) + \rho$, $p_0(y) + p$ and $(H_x + h_x, 0, H_z + h_z)$. Hence, the linearized perturbation equations for infinitesimal normal modes of the form $f(y) e^{i(kx+k|z-k\sigma t)}$, where k is the longitudinal wave number, l is the transverse wave number and σ is the complex wave velocity are obtained as

$$ik[u + lw] + \frac{\partial v}{\partial y} = 0$$

$$ik(-\sigma + U(y) + IW(y))u + v.\frac{\partial U(y)}{\partial y} = ik(-p + SH_z(lh_x - h_z))$$
$$ik(-\sigma + U(y) + IW(y))v = -\frac{\partial p}{\partial y} - Ri\rho - S\left(\left(H_x\frac{\partial h_x}{\partial y} + H_z\frac{\partial h_z}{\partial y}\right) - \frac{\partial h_z}{\partial y}\right)$$

ikhyHx+lHz

$$ik(-\sigma + U(y) + lW(y))w + v.\frac{\partial W(y)}{\partial y} = ik(-lp + SH_x(h_z - lh_x))$$

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$$ik(-\sigma + U(y) + lW(y))\rho - \frac{N^2}{N_0^2}v = 0$$

$$ik(h_{x} + lh_{y}) + \frac{\partial h_{z}}{\partial y} = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{x} = ikl(uH_{z} - wH_{x} - W(y)h_{x} + U(y)h_{z}) - H_{x}\frac{\partial v}{\partial y}$$

$$+ \frac{\partial}{\partial y} (U(y)h_y)$$

$$\left(-ik\sigma - \frac{1}{Rm} \left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2} \right) \right) h_y = ik \left((H_x + lH_z)v - h_y(U(y) + lW(y)) \right)$$

$$\left(-ik\sigma - \frac{1}{Rm} \left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2} \right) \right) h_z = ik(wH_x - uH_z + W(y)h_x - U(y)h_z) -$$

$$H_z \frac{\partial v}{\partial y} + \frac{\partial}{\partial y} (W(y)h_y)$$

$$(4.15)$$

The corresponding boundary conditions are

$$u = v = w = 0$$
 on $y = \pm 1$ (4.16)

4.3 Analysis

The analysis is restricted to long waves (i.e) k is assumed to be small and the flow is assumed to be bounded between two plates $y = \pm 1$. Basic velocity profile is taken to be U(y) = W(y) = y. Hence equation (4.15) takes the form

$$\begin{split} \mathrm{i} \mathrm{k}(\mathrm{u} + \mathrm{l} \mathrm{w}) &+ \frac{\partial \mathrm{v}}{\partial \mathrm{y}} = 0\\ \mathrm{i} \mathrm{k} \big((1+\mathrm{l}) \mathrm{y} - \sigma \big) \mathrm{u} + \mathrm{v} = \ \mathrm{i} \mathrm{k} \big(-\mathrm{p} + \mathrm{SH}_{\mathrm{z}} (\mathrm{l} \mathrm{h}_{\mathrm{x}} - \mathrm{h}_{\mathrm{z}}) \big)\\ \mathrm{i} \mathrm{k} \big((1+\mathrm{l}) \mathrm{y} - \sigma \big) \mathrm{v} = \ - \frac{\partial \mathrm{p}}{\partial \mathrm{y}} - \ \mathrm{Ri} \ \rho - \mathrm{S} \left(\Big(\mathrm{H}_{\mathrm{x}} \frac{\partial \mathrm{h}_{\mathrm{x}}}{\partial \mathrm{y}} + \mathrm{H}_{\mathrm{z}} \frac{\partial \mathrm{h}_{\mathrm{z}}}{\partial \mathrm{y}} \Big) - \\ \mathrm{i} \mathrm{k} \mathrm{h} \mathrm{y} \mathrm{H} \mathrm{x} + \mathrm{l} \mathrm{Hz} \end{split}$$

$$\begin{split} & \mathrm{i} k \big((1+l) y - \sigma \big) w + v = \ \mathrm{i} k \big(-lp + SH_x (h_z - lh_x) \big) \\ & \mathrm{i} k \big((1+l) y - \sigma \big) \rho - \frac{N^2}{N_0^2} v = 0 \end{split}$$

$$ik[h_{x} + lh_{y}] + \frac{\partial h_{z}}{\partial y} = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{x} = ikl\left((wH_{x} - uH_{z}) + y(h_{z} - h_{x})\right) - H_{x}\frac{\partial v}{\partial y} + y(h_{z} - h_{z})\right) - H_{x}\frac{\partial v}{\partial y} + y(h_{z} - h_{z})\right)h_{y} = ik\left\{(H_{x} + lH_{z})v - yh_{y}(1+l)\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\} - H_{z}\frac{\partial v}{\partial y} + y(h_{z} - h_{z})\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\} - H_{z}\frac{\partial v}{\partial y} + y(h_{z} - h_{z})\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\} - H_{z}\frac{\partial v}{\partial y} + y(h_{z} - h_{z})\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\}$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^{2}(1+l^{2}) + \frac{\partial^{2}}{\partial y^{2}}\right)\right)h_{z} = ik\left\{(wH_{x} - uH_{z}) + y[h_{x} - h_{z}]\right\}$$

By assuming the series expansions with respect to k in the form $f = f_0 + kf_1 + k^2f_2 + \cdots$ where $f = (u, v, w, \sigma, \rho, h_x, h_y, h_z)$ and substituting into equation (4.14) and collecting the coefficients of like powers of k, we get O (k⁰)

$$iu_0 + ilw_0 + \frac{\partial v_0}{\partial y} = 0$$
$$iT(y)u_0 + v_0 = -ip_0$$

$$-\frac{\partial \mathbf{p}_0}{\partial \mathbf{y}} - Ri\,\rho_0 = 0$$

 $iT(y)w_0 + v_0 = -ilp_0$

$$iT(y)\rho_0 - \frac{N^2}{N_0^2}v_0 = 0$$

$$ih_{x0} + ilh_{z0} + \frac{\partial h_{y0}}{\partial y} = 0$$

$$(4.18)$$

$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{x0}}{\partial y^2} \right) = -i l w_0 H_x + i l u_0 H_z - H_x \frac{\partial v_0}{\partial y}$$

$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{y0}}{\partial y^2} \right) = \ [iH_x + ilH_z] v_0$$

$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{z0}}{\partial y^2} \right) = i w_0 H_x - i u_0 H_z - H_z \frac{\partial v_0}{\partial y}$$
(4.19)

where $T(y) = (1 + l)y - \sigma_0$ $O(k^1)$

$$iu_1 + ilw_1 + \frac{\partial v_1}{\partial y} = 0$$

$$\begin{aligned} -i\sigma_{1}u_{0} + iT(y)u_{1} + v_{1} &= -i\left(p_{1} + SH_{z}(lh_{x0} - h_{z0})\right) \\ &- \frac{\partial p_{1}}{\partial y} - Ri\rho_{1} = SH_{x}\left(\frac{\partial h_{x0}}{\partial y} + \frac{\partial h_{z0}}{\partial y}\right) \\ &- i\sigma_{1}w_{0} + iT(y)w_{1} + v_{1} = -i\left(lp_{1} + SH_{x}(h_{z0} - lh_{x0})\right) \\ iT(y)\rho_{1} - i\sigma_{1}\rho_{0} - \frac{N^{2}}{N_{0}^{2}}v_{1} &= 0 \\ &ih_{x1} + ilh_{z1} + \frac{\partial h_{y1}}{\partial y} = 0 \\ &- \frac{1}{Rm}\left(\frac{\partial^{2}h_{x1}}{\partial y^{2}}\right) = i(H_{x} + lH_{z})u_{1} - iT(y)h_{x0} + h_{y0} \end{aligned}$$

$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{y1}}{\partial y^2} \right) = i(H_x + lH_z) v_1 - iT(y) h_{y0}$$
$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{z1}}{\partial y^2} \right) = i(H_x + lH_z) w_1 - iT(y) h_{z0} + h_{y0} \quad (4.21)$$

 $O(k^2)$

$$\begin{split} & \mathrm{i} u_2 + \mathrm{i} \mathrm{l} w_2 + \frac{\partial v_2}{\partial y} = 0 \\ & \mathrm{i} (T(y) u_2 - \sigma_2 u_0 - \sigma_1 u_1) + v_2 = -\mathrm{i} \big(p_2 + \mathrm{SH}_z (\mathrm{l} h_{x1} - h_{z1}) \big) \end{split}$$

$$iT(y)v_{0} = -\frac{\partial p_{2}}{\partial y} - \text{Ri} \rho_{2} - \text{SH}_{x}\frac{\partial h_{x1}}{\partial y} - \text{SH}_{x}\frac{\partial h_{z1}}{\partial y}$$

$$i(T(y)w_{2} - \sigma_{2}w_{0} - \sigma_{1}w_{1}) + v_{2} = -i(lp_{2} + \text{SH}_{x}(h_{z1} - lh_{x1}))$$

$$i(T(y)\rho_{2} - \sigma_{2}\rho_{0} - \sigma_{1}\rho_{1}) - \frac{N^{2}}{N_{0}^{2}}v_{2} = 0$$

$$ih_{x2} + ilh_{z2} + \frac{\partial h_{y2}}{\partial y} = 0$$

$$-\frac{1}{Rm}(\frac{\partial^{2}h_{x2}}{\partial y^{2}}) - i(\frac{1}{Rm}(1 + l^{2}) - i\sigma_{1})h_{x0} = i((H_{x} + lH_{z})u_{2} - T(y)h_{x1}) + h_{y1}$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y2}}{\partial y^2}\right) = i(H_x + lH_z)v_1 - iT(y)h_{y0}$$

$$-\frac{1}{Rm} \left(\frac{\partial^2 h_{z2}}{\partial y^2}\right) - i \left(\frac{1}{Rm} (1+l^2) - i\sigma_1\right) h_{z0} = i \left((H_x + lH_z)w_2 - T(y)h_{z1}\right) + h_{y1}$$
(4.23)

The boundary conditions (4.16) reduces to

 $u_0 = u_1 = u_2 = 0$, $v_0 = v_1 = v_2 = 0$, $w_0 = w_1 = w_2 = 0$ (4.24) Equation (4.18) can be solved to obtain

$$v_0 = A(T(y))^{m_1} + B(T(y))^{m_2}$$

where $m_{1,2} = \frac{N_0(1+l) \pm \sqrt{N_0^2(1+l)^2 - 4 \operatorname{Ri} N^2(1+l^2)}}{2 N_0(1+l)}$

A and B are arbitrary constants of integration. To determine the arbitrary constants, we impose the boundary condition that the velocity should vanish at the boundaries (i.e) $v_0 = 0$ at $y = \pm 1$, yields

$$\begin{vmatrix} ((1+l) - \sigma_0)^{m_1} & ((1+l) - \sigma_0)^{m_2} \\ (-(1+l) - \sigma_0)^{m_1} & (-(1+l) - \sigma_0)^{m_2} \end{vmatrix} = 0$$

From which the value of σ_0 is obtained as

$$\sigma_0 = (1+l) \frac{\frac{2n\pi i}{m_1 - m_2}}{\frac{2n\pi i}{1 - e^{\frac{2n\pi i}{m_1 - m_2}}}}$$
(4.25)

The solution of equations (4.18) and (4.19) is given by

$$u_0 = C_5 T(y)^{m_1 - 1} + C_6 T(y)^{m_2 - 1}$$

$$v_{0} = T(y)^{m_{1}} + BT(y)^{m_{2}}$$

$$w_{0} = C_{7}T(y)^{m_{1}-1} + C_{8}T(y)^{m_{2}-1}$$

$$p_{0} = C_{1}T(y)^{m_{1}} + C_{2}T(y)^{m_{2}}$$

$$\rho_{0} = C_{3}T(y)^{m_{1}-1} + C_{4}T(y)^{m_{2}-1}$$

$$h_{x0} = -Rm(C_{13}T(y)^{m_{1}+1} + C_{14}T(y)^{m_{2}+1})$$

$$h_{y0} = -Rm(C_9T(y)^{m_1+2} + C_{10}T(y)^{m_2+2})$$

$$h_{z0} = -Rm(C_{17}T(y)^{m_1+1} + C_{18}T(y)^{m_2+1})$$
(4.26)

By solving equation (4.20) the second approximation of σ can be obtained as

$$\sigma_1 = \frac{S Rm C_{46}}{Ri C_{47} - C_{48}} \tag{4.27}$$

Solving differential equations (4.20) and (4.21) with the corresponding boundary conditions, we obtain

$$\begin{split} & u_{1}(y) = \sigma_{1}((-RiC_{85} + C_{86})T(y)^{m_{1}-2} + (-RiC_{87} + C_{88})T(y)^{m_{2}-2}) \\ & T(y)^{m_{1}-1} + (-SRmC_{80} + \sigma_{1}(RiC_{81} - C_{82}))T(y)^{m_{2}-1} + \\ & SRm(C_{83}T(y)^{m_{1}} + C_{84}T(y)^{m_{2}}) \\ & v_{1}(y) = \sigma_{1}((-RiC_{26} + c_{20})T(y)^{m_{1}-1} + (-RiC_{27} + c_{21})T(y)^{m_{2}-1}) \\ & + (-SRmC_{43} + \sigma_{1}(RiC_{44} - C_{45}))T(y)^{m_{1}} \\ & + (-SRmC_{36} + \sigma_{1}(RiC_{37} - C_{38}))T(y)^{m_{2}} \\ & + SRm(C_{28}T(y)^{m_{1}+1} + C_{29}T(y)^{m_{2}+1}) \\ & w_{1}(y) = \sigma_{1}((-RiC_{97} + C_{98})T(y)^{m_{1}-2} + (-RiC_{99} + C_{100})T(y)^{m_{2}-2}) \\ & + (-SRmC_{89} + \sigma_{1}(RiC_{90} - C_{91}))T(y)^{m_{1}-1} \\ & + (-SRmC_{89} + \sigma_{1}(RiC_{90} - C_{94}))T(y)^{m_{2}-1} \\ & + SRm(C_{95}T(y)^{m_{1}} + C_{96}T(y)^{m_{2}}) \\ & \rho_{1}(y) = \sigma_{1}((-RiC_{57} + C_{58})T(y)^{m_{1}-2} + (-RiC_{59} + C_{60})T(y)^{m_{2}-2}) \\ & + (-SRmC_{49} + \sigma_{1}(RiC_{50} - C_{51}))T(y)^{m_{1}-1} \\ & + (-SRmC_{52} + \sigma_{1}(RiC_{50} - C_{51}))T(y)^{m_{2}-1} \\ & -SRm(C_{55}T(y)^{m_{1}} + C_{56}T(y)^{m_{2}}) \\ & p_{1}(y) = \sigma_{1}((-RiC_{73} + C_{74})T(y)^{m_{1}-1} + (-RiC_{75} + C_{76})T(y)^{m_{2}-1}) \\ & + (-SRmC_{65} + \sigma_{1}(RiC_{66} - C_{67}))T(y)^{m_{2}} \\ & + SRm(C_{71}T(y)^{m_{1}+1} + C_{72}T(y)^{m_{2}+1}) \\ & h_{x1}(y) = Rm(\sigma_{1}((C_{115} - RiC_{116})T(y)^{m_{1}} + (C_{117} - RiC_{118})T(y)^{m_{2}}) \\ & + (SRmC_{122} + \sigma_{1}(-RiC_{123} + C_{124}))T(y)^{m_{2}+1} \\ \end{split}$$

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$$+SRm(C_{125}T(y)^{m_{1}+2}C_{126}T(y)^{m_{2}+2}) +Rm(C_{127}T(y)^{m_{1}+4} + C_{128}T(y)^{m_{2}+4})) h_{y1}(y) = Rm(\sigma_{1}((RiC_{109} - C_{110})T(y)^{m_{1}+1} + (RiC_{111} - C_{112})T(y)^{m_{2}+1}) +(SRmC_{101} + \sigma_{1}(-RiC_{102} + C_{103}))T(y)^{m_{1}+2} +(SRmC_{104} + \sigma_{1}(-RiC_{105} + C_{106}))T(y)^{m_{2}+2} -SRm(C_{107}T(y)^{m_{1}+3} + C_{108}T(y)^{m_{2}+3}) + +RmC113T(y)m1+5+C114T(y)m2+5 h_{z1}(y) = Rm(\sigma_{1}((C_{129} - RiC_{130})T(y)^{m_{1}} + (C_{131} - RiC_{132})T(y)^{m_{2}}) +(\sigma_{1}(C_{135} - RiC_{134}) - SRm C_{133})T(y)^{m_{1}+1} +(\sigma_{1}Rm(C_{138} - RiC_{137}) - SRm C_{136})T(y)^{m_{2}+1} - SRm(C_{139}T(y)^{m_{1}+2} + C140Tym2+2+RmC141T(y)m1+4+C142T(y)m2+4$$
(4.28)

From equation (4.22), the third approximation of σ can be obtained as

$$\sigma_{2} = \frac{\sigma_{1} S Rm(C_{233} - RiC_{238}) - Ri \sigma_{1}^{2}(C_{234} - Ri C_{239}) - \sigma_{1}^{2}C_{235} + S^{2}Rm^{2}C_{240}}{C_{236} - RiC_{237}}$$

For the sake of brevity the constants are given in Appendix A

4.4 Results and Discussion

In this section, the influence of wave number k, magnetic Reynolds number *Rm*, Brunt - Vaisala frequency N, magnetic pressure number S, Richardson number *Ri* and transverse wave number 1 on the stability of non parallel stratified shear flow confined between the plates $y = \pm 1$ is examined. In the presence of a magnetic field the manner of the onset of instability depend in an extremely complicated way on the relevant parameter S and R_m

In Figures (4.1) – (4.7) we have depicted the frequency as a function of a magnetic Reynolds number. From the analytical expressions derived for σ_0 in the previous section, it can be noted that there exists infinite number of modes both stable and unstable corresponding to the values of n. We have presented few of these modes in Figure (4.7).

From Figures (4.1) – (4.6) it can be seen that when the Brunt - Vaisala frequency N^2 is small, we have stable modes and as N^2 increases, the flow becomes

unstable. In all the cases, increasing R_m increases the magnitude of $Imag(\sigma)$ thereby increasing the region of stability or instability accordingly.

From Figures (4.8) and (4.9) it can be deduced that in the absence of magnetic field the system oscillates between stable and unstable state due to increase in the transverse wave number 1. When the magnetic field is present the growth rate decays as 1 increases thereby making the system stable.

Figures (4.10) - (4.22) show the dependence of growth rate on the longitudinal wave number k. Increase in k like R_m , increases the magnitude of Imag (σ) and hence amplifies the region stability/instability.

All the above mentioned figures suggest that the stability of the system is greatly influenced by Brunt- Vaisala frequency and hence the effect of Brunt - Vaisala frequency is studied in Figures (4.13) – (4.15). From Figures (4.13) and (4.14) we can observe that for very small values of N², the system is stable and then the system becomes unstable as N² increases. The enlarged region of instability is given in Figure (4.15). The velocity as a function of y is given in Figures (4.16) – (4.22). Increasing l increases the velocity when N²<1 and decreases when N²>1.

4.5 Conclusion

We have investigated the stability of inviscid, stratified electrically conducting non parallel shear flow. The fluid was considered to be in a state of non parallel flow with the basic velocity profile (U(y),0,W(y)). The governing equations were derived. These equations reduce to those obtained by Padmini and Subbaiah (1995) when $R_m =$ 0. The stability of the flow was analysed using normal mode approach and the analysis was restricted to a long wave approximation.

From the results obtained in the previous section, the following conclusions are made.

- The effect of the magnetic Reynolds number is to increase the magnitude of the growth rate and hence increases the region of stability or instability.
- The stability of the system is greatly affected by the Brunt- Vaisala frequency N². The systems remains stable for small N² and tend to be unstable as N² increases.



Figure 4.1. Growth rate as a function of N L=1.0, Ri=0.01, Rm=1.0, K=0., S=1.0



Figure 4.2. Frequency as a function of magnetic Reynolds number($N^2=0.1,n=2$)



Figure 4. 3. Frequency as a function of magnetic Reynolds number $(N^2=0.1,n=2)$



Figure 4.4. Frequency as a function of magnetic Reynolds number $(N^2=0.4, n=2)$



Figure 4.5. Frequency as a function of magnetic Reynolds number $(N^2=1.0, n=2)$



Figure 4.6. Frequency as a function of magnetic Reynolds number $(N^2=1.0,n=2)$



Figure 4.7. Frequency as a function of magnetic Reynolds number $(N^2=1.0)$



Figure 4.8. Frequency as a function of $l(N^2=0.1)$



Figure 4.9. Frequency as a function of $l (N^2=1, Rm = 0.0)$



Figure 4.10. Frequency as a function of $l (N^2=1, Rm = 10.0)$



Figure 4.11. Frequency as a function of k ($N^2=0.1$, Rm = 10.0)



Figure 4.12. Frequency as a function of k ($N^2=1$, Rm=0.0)



Figure 4.13. Frequency as a function of k ($N^2=1$, Rm = 10.0)



Figure 4.14. Frequency as a function of N^2 (k =0.1, Rm =10.0)



Figure 4.15. Frequency as a function of N^2 (k =0.2, Rm =10.0)



Figure 4.16. Frequency as a function of N^2 (k =0.2, Rm =10.0)



Figure 4.17. Velocity as a function of y $N^2=0.1$



Figure 4.18. Velocity as a function of y $N^2=1.5$



Figure 4.19. Velocity as a function of y



Figure 4.20. Velocity as a function Rm = 10.0, Ri = 0.1, k = 0.1, l = 1.0, N = 0.01



Figure 4.21. Velocity as a function of y S = 10.0, Ri = 0.1, k = 0.1, l = 1.0, N = 0.01



Fig 4.22. Velocity as a function of y S = 10.0 *Rm* = 10.0, *Ri* = 0.1, k = 0.1, N= -1.0



Fig 4.23. Velocity as a function of y S = 10.0 *Rm* = 10.0, *Ri* = 0.1, k = 0.1, N = 1.0