

LINEAR STABILITY ANALYSIS OF MAGNETO HYDRODYNAMIC STRATIFIED SHEAR FLOWS

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Submitted by

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Conclusion

CONCLUSION

Shear flow instabilities are seen frequently in geophysical and astrophysical flows. Occurrence of shear instabilities can be observed in the rows of cyclones, anticyclones near Saturn's *Ribbon*, near 41°S on Jupiter and Jupiter's anticyclonic Great Red Spot. The meteorologists recognized significant role of shear flows in monsoons. *Motivated by the applications in real life we have attempted to carry out a detailed analysis of shear flow instability which will throw light on various atmospheric phenomena.*

The main aim of this thesis is to investigate stability of stratified shear flows between two plates. Here, we present a consolidated narration of the results obtained in this dissertation.

Chapter I presents various preliminary concepts required for the study.

Chapter II summarizes the significant earlier contributions related to the present study.

In Chapter III, the hydro magnetic stability of stratified shear flow of an inviscid fluid confined between two rigid plates is discussed. Magnetic field is applied perpendicular to the plates in the direction of y - axis.

Some important findings are

- ◆ With the increase in longitudinal wave number (k) the system is stable for various Ha and unstable for various l . The system is stable for various Richardson number (Ri) and becomes unstable when Brunt Vaisala frequency (N^2) increases.
- ◆ Increase in Hartmann number (Ha) longitudinal and transverse wave number (k and l) and Brunt Vaisala frequency (N^2), the nature of the system will be stable with increasing Richardson number (Ri).
- ◆ The flow becomes stable when Hartmann number (Ha) and Richardson number (Ri) increases for parallel flow.
- ◆ In the absence of applied magnetic effect, the system becomes stable with the increase in transverse wave number (l), Brunt-Vaisala frequency (N^2) and Richardson number (Ri) when longitudinal wave number (k) increases.

In Chapter IV, The effect of Hall current on the linear stability of inviscid, incompressible nonparallel stratified shear flow of a perfectly conducting fluid is

analyzed. A theory for non-parallel stratified shear flow is developed formally and applied in detail for three dimensional Cartesian coordinate system for $\lambda > 0$. The following conclusions can be drawn from the study.

- ◆ The flow field is stable with the increase in Magnetic Reynolds number, Hall parameter, transverse wave number and Magnetic pressure number.
- ◆ Increase in Richardson number destabilizes the field of flow.
- ◆ The system becomes unstable for various Hall parameter, Magnetic Reynolds number, Magnetic pressure number and longitudinal wave number with the increase in Brunt Vaisala frequency.
- ◆ Increase in wave number, Magnetic Reynolds number and Brunt – Vaisala frequency results in the stability of the system, the system becomes unstable with the increase in transverse wave number.

Chapter V discusses about the linear stability analysis of an inviscid, incompressible, parallel stratified shear fluid in the presence of varying magnetic field. When $Rm = 0$, the governing equations coincide with those obtained by Padmini and Subbiah (1995). The salient features of the present study are listed as

- ◆ Richardson number (Ri) plays a crucial role in the stability of parallel stratified shear flows. Increase in Richardson number (Ri) destabilizes the flow as Magnetic Reynolds number (Rm) increases.
- ◆ Increase in wave number (k) increases the growth rate (σ) for varying Magnetic pressure number (S) and Magnetic Reynolds number (Rm) when $\lambda > 0$ thereby destabilizes the flow.
- ◆ Increase in transverse wave number (l) destabilizes the fluid flow. The flow become unstable with the increase in Brunt Vaisala frequency (N^2) when $\lambda > 0$.
- ◆ Growth rate (σ) increases for varying Magnetic pressure number (S) and Magnetic Reynolds number (Rm) thereby destabilizes the flow. In the case of an increase in transverse wave number (l) the growth rate (σ) decreases and the flow becomes stable when $\lambda < 0$.

In Chapter VI, we have analyzed the effect of magnetic field on the linear stability of an idealized stratified shear flow using series expansion method. Some important findings of the study are

- ◆ Increase in wave number increases the growth rate for varying Magnetic pressure number and Magnetic Reynolds number when $\lambda > 0$ thereby destabilizes the flow.
- ◆ As Magnetic Reynolds number increases, the flow is unstable with the increase in Richardson number and Magnetic pressure number.
- ◆ Increasing Brunt – Vaisala frequency, stabilizes the fluid flow.
- ◆ Increase in Magnetic Reynolds number, Magnetic pressure number and Richardson number stabilize the fluid flow with the increase in wave number when $\lambda < 0$.
- ◆ Growth rate decreases for varying wave number and thereby stabilizes the flow with the increase in Magnetic Reynolds number and Magnetic pressure number ($\lambda < 0$).

In Chapter VII, we study the linear stability analysis of an inviscid, incompressible parallel stratified shear fluid. The fluid is assumed to be rotating about a vertical axis with constant angular velocity Ω . Analytical expressions were found to calculate the growth rate (σ) for long waves. These expressions are evaluated numerically for a linear base flow (i.e) $U(z) = z$. Significant conclusions drawn from the study are as follows.

- ◆ Due to the increase in rotation number (τ) and transverse wave number (l) growth rate decreases thereby making the flow stable.
- ◆ Increase in Brunt vaisala frequency (N^2) and Richardson number (Ri) decreases the growth rate thereby stabilizes the flow pattern.
- ◆ Due to the increase in rotation number (τ), transverse wave number (l) and Richardson number (Ri), the growth rate of disturbance decreases, thereby contributes more to the flow stability with the increase in Brunt Vaisala frequency (N^2).
- ◆ The rotation number (τ), wave number (k and l) and Brunt Vaisala frequency (N^2) contributes more to the stability of the flow with increasing Richarson number (Ri).

In Chapter VIII, the influence of various dimensionless parameters on the inviscid, incompressible Boussinesq rotating fluid over the nonparallel stratified shear flow is analyzed. The rotation effect is exhibited through the non dimensional number τ . The

corresponding results for parallel flow without rotation can be obtained by setting $\tau = 0$ and $l = 0$. It is worth mentioning that these results are in good agreement with Farrell and Ioannou (1993).

The following conclusions are made from the study

- ◆ The system becomes stable with the increase in Rotation number (τ), transverse wave number (l) and Brunt – Vaisala frequency (N^2) for $\lambda > 0$.
- ◆ Initially there is a transition, with the increase in transverse wave number (l) and Rotation number (τ). Later it stays stable, as the Brunt –Vaisala frequency (N^2) increases ($\lambda > 0$).
- ◆ Increase in Ri and l increases the growth rate and makes the system stable. Increase in N^2 increases the growth rate and destabilizes the flow field ($\lambda < 0$).
- ◆ Growth rate increases with the increase rotation number (τ) thereby stabilizes the flow field ($\lambda < 0$).

In all the cases, linear shear velocity profile is taken as basic flow nature of stability is analyzed using normal mode analysis and we have restricted the analysis to long wave approximation.

Suggestions for Future Research

- ◆ The current work can be extended by considering nonuniform magnetic field with temperature and concentration gradient.
- ◆ The mathematical model of the problems involves rigid flat boundaries. In future, the problems with porous walls and wavy walls can be considered and it may produce some interesting results close to the engineering industry. Much more work can be done with unbounded flows.
- ◆ The problem in this thesis can be extended with Hall Effect and ion-slip effect which will produce good results.
- ◆ In this work, a generalized linear stability analysis is applied to idealized stratified shear flows, which are relevant to atmospheric dynamics. Nonlinear stability of these problems may also be studied.
- ◆ The theoretical analysis of this thesis is restricted to normal mode analysis of small oscillations. This can be further developed to find more general results of energy method and non-linearity. Application of chebyshev-tau method can also be considered.