Chapter I

CHAPTER I

INTRODUCTION

1.1 Motivation and Introduction

Fluid mechanics is the study of mechanics of fluids based on fundamental laws of motion. The large number of applications of fluid mechanics has made it one of the most vital and fundamental subjects in the field of almost all engineering and applied scientific studies. The flight of birds in the air, design of aeroplanes and ships, action of fishes in water are based on the theory of fluid mechanics. In most of the fluid mechanics problems, some approximations and simplified assumptions are taken regarding nature of the fluids and flow boundaries. When we make an attempt to study a realistic physical situation, the problem becomes more complicated and the mathematical tools available are insufficient to solve the problems in its original nature.

Hence, it is necessary to make suitable assumptions and approximations which not only simplify the mathematical formulation of the problem, but also in agreement with the physical needs of the problem. The actual flow conditions or boundary conditions may be slightly different than those taken in the theoretical analysis. If these small changes lead to large deviation in the flow variables, the theoretically obtained flow cannot be realized physically. Further, due to some disturbances inherently present in the flow, we must analyze whether these disturbances grow or decay with time. To analyze this theoretically, the investigation of the stability of fluid flows becomes essential.

During the past few decades, the study of electrically conducting fluid flows in the presence of magnetic or electric fields have become very important because of their wide applications. The motion of an electrically conducting fluid under a magnetic field gives rise to induced electric currents on which mechanical forces are exerted by the magnetic field. The flows of electrically conducting fluids in the presence of a magnetic field are called the hydromagnetic flows. Hydromagnetic flows are more complex than the hydrodynamic flows.

Stratification and Rotation are the distinctive features in geophysical fluid dynamics. Stratified fluid is a fluid with density variations in the vertical direction. It consists of fluid particles of various densities. These fluid particles, due to density variation will tend to arrange themselves under gravity. So, the higher density fluid particles are found below the fluid particles with lower density. This type of vertical layering introduces a gradient of properties in vertical directions, which affects the velocity field. Hence the vertical rigidity induced by the effect of rotation will be attenuated by the presence of stratification. Because of stratification, certain degree of decoupling is induced between various fluid masses (of different densities). Stratified systems typically contain more degrees of freedom than homogeneous systems and we anticipate that the presence of stratification permits the existence of additional types of fluid motions. One example is a system of two superimposed fluids in a channel with lighter fluid on the top. In this case, changes in density takes place with height, as illustrated in Figure. 1.1.

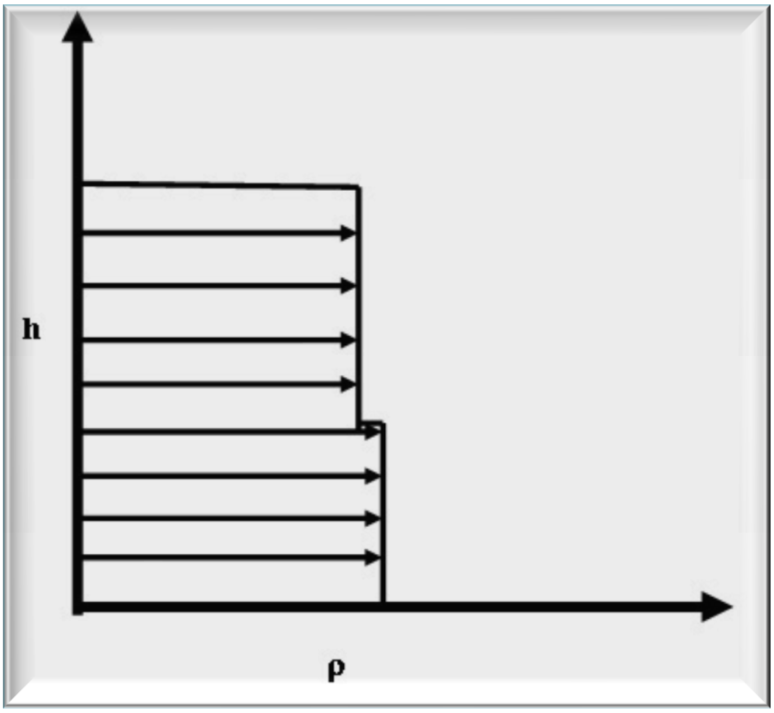


Figure 1. 1. Density-height curve in a two fluid system

In day to day life layered systems of stratified fluids occur in many instances. Some of the examples include warm water lying above cold water, fresh water above salt water, interface between air and water. In oceans and seas, variations in temperature of water and salinity at the surface and also below the surface govern stratification, due to advection and adiabatic processes.

Stratification in water occurs when water masses with different properties like salinity (halocline), oxygenation (chemocline), density (pycnocline) and temperature (thermocline) form layers that act as barriers to mixing of water. These layers are arranged according to density, which is a function of salinity and temperature. It also creates barrier to nutrient mixing between layers. This can affect the primary production in an area by limiting photosynthetic processes. The stratification is commonly referred as a two-layer structure that consists of an upper and a lower layer. A transitional middle layer exists between the upper and lower layers, which are known as halocline and thermocline, respectively (see Figure 1.2).

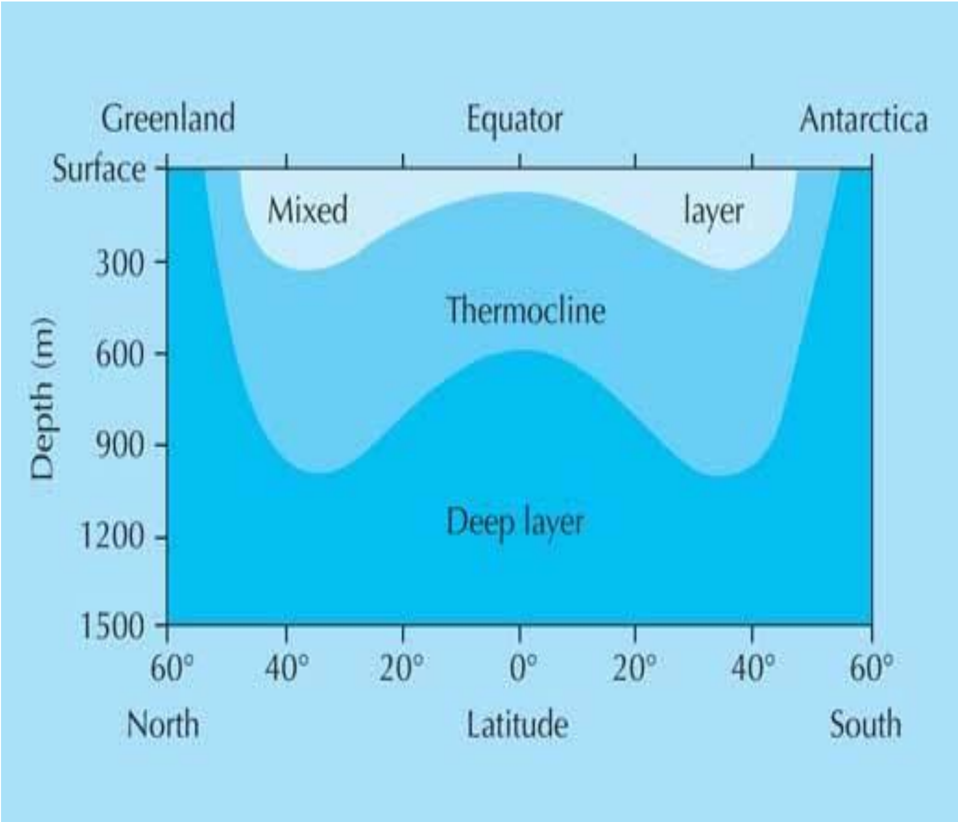


Figure 1.2 Oceanic stratification

Baltic Sea is a brackish sea located in northern Europe from 53° N to 66° N latitude and from 20° E to 26° E longitude, connected to the Atlantic Ocean via the Danish Straits. The Baltic Ice Lake was born 13,000 years ago and its present brackish state emerged 7000 years ago. For 2000 years, the salinity has been close to the present level (mean salinity: 7 parts per thousand). The Baltic Sea is a shallow sea that consists of a series of basins interconnected through narrow sills (see Figure 1.3). The Baltic Sea is highly stratified by strong vertical salinity and temperature gradients.

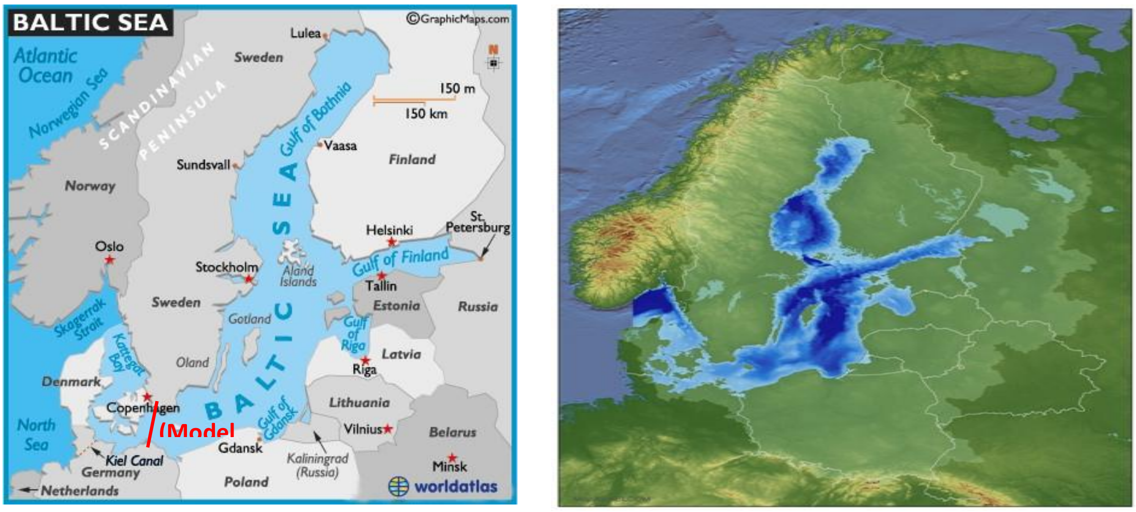


Figure 1.3 Baltic Sea and its nature

Frequently, however, density varies continuously, as in the oceans and atmosphere. Density variations profoundly affect the motion of water and air. Wave phenomena in air flow over mountains and the occurrence of Smog (Figure 1.4) are examples of stratification effects in the atmosphere.

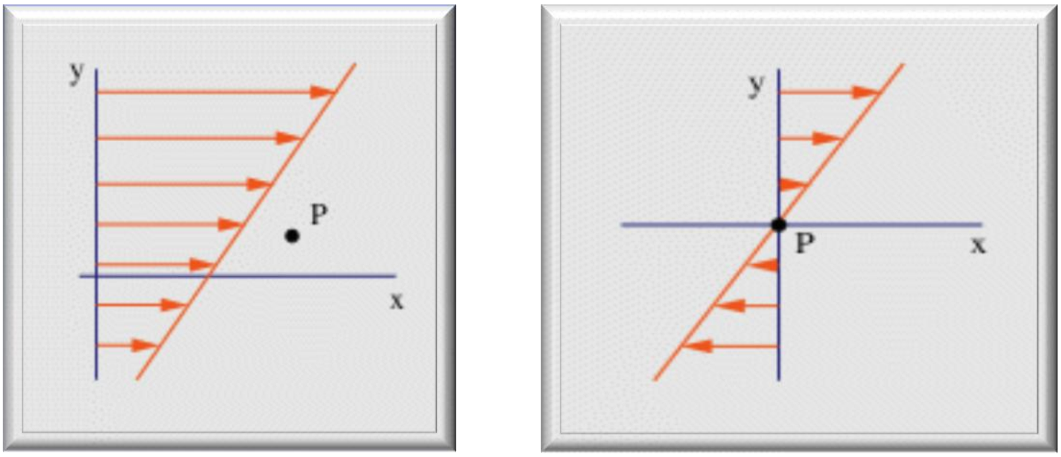
Figure 1.4. Smog over Los Angeles - brown haze covering the basin and making the skyscrapers of downtown Los Angeles barely visible.

Shear flows of magnetized plasmas are routinely observed in the solar atmosphere, in interplanetary space and in planetary magnetospheres. They are also ubiquitous models of remote astrophysical objects like the interacting stellar winds in binary stellar systems. Studying stability of such flows is paramount for understanding physical processes in space. The simplest shear flow is a tangential MHD discontinuity. Mixing is a very common nature in the environmentally prevalent flows with both vertical velocity and density variation. Examples of mixing include the thermocline, the lutocline, planetary boundary layers, river mouths, etc.

One of the classical astrophysical problem involving shear instability is the problem of Sun’s magnetic field. In solar magneto hydrodynamics interaction between magnetic fields and solar instabilities, stratification, compressibility, effects due to the buoyancy and magnetic buoyancy are all important.

1.2 Shear flows

Shear occurs whenever adjacent fluid layers move parallel to one another, with different speeds. In fluid mechanics, shear flow refers to a type of fluid flow which is caused by forces, relatively than to the forces themselves.



a) The fluid is moving to the right and the magnitude of the fluid velocity increases linearly with y

(b) Fluid is moving with velocity at point P

Figure 1.5. Sketch depicting the velocity profile of a simple shear flow

Shear flows are ubiquitous in nature and can occur on any scale. Flows pumped through pipes by some pressure gradient along the pipe (called Poiseuille flows) are present everywhere in natural or engineered systems. Examples include blood flow through the body, from small capillaries to arteries, fluid flows through underground river systems, magma flows and pyroclastic flows through volcano chimneys, water owing through a hose, a kitchen faucet, oil in a car engine, in a pipeline, etc. Many astrophysical and geophysical phenomena that are found in the ocean, in the atmospheric wind patterns, in the surface and subsurface flows of the

Sun, giant planets, other stars and in the orbital motion of gas in accretion disks can be explained by the study of shear flows driven by differential pressure gradient.

1.3 Practical examples of shear layers

Wind shear defines a change in the speed of wind and/or direction over a short distance. It can occur either horizontally or vertically and is most often associated with strong temperature conversions or density gradients. Wind shear occurs at high as well as at low altitudes.

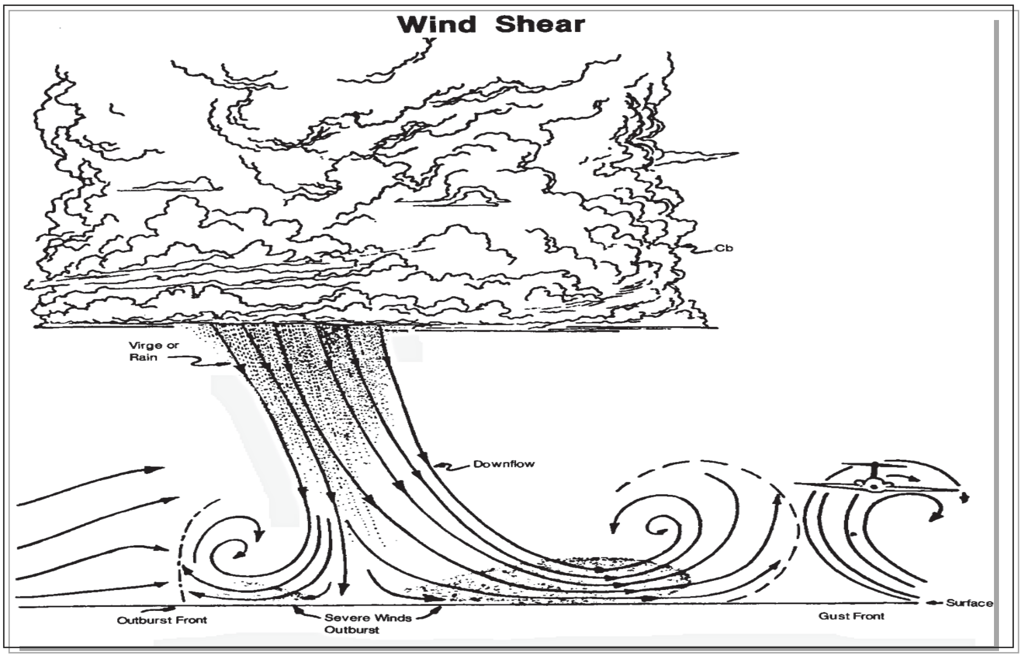


Figure 1.6 Low-altitude wind shear

The effects of stratification are critical to the occurrence of many environmental phenomena. It is well known, for example, that stratified flows past long mountain ranges may be blocked and thus may lead to serious air pollution problems.

Figure 1.7. Some examples of stratified shear layers

Kelvin-Helmholtz instability is a hydrodynamic instability in which inviscid and incompressible fluids are in relative and irrotational motion. Here, the density and velocity profiles in each fluid layer are uniform, but discontinuity arises at the (plane) interface between the two fluids. This tangential discontinuity in the velocity induces vorticity at the interface and at last the interface becomes an unstable vortex sheet that rolls up into a spiral. The heavier fluid parcels from lower denser fluid are lifted up and the lighter fluid parcels from lighter upper fluid are pushed down so overall potential energy is increased. KHI in the atmosphere is shown in Figure. 1.8

Figure 1.8: Kelvin–Helmholtz instability in atmosphere rendered visible by clouds, known as fluctus, over Mount Duval in Australia

1.4 Stability Analysis

Every system in nature is subject to small perturbations. The system is given small disturbances (perturbations) and reactions of the system of these perturbations are studied. If the disturbances gradually die down, the system is said to be stable. If the perturbations grow with time i.e. the system never reverts to its initial position, it is said to be unstable. If the system neither departs from its disturbed state nor tends to return to its initial position, the system is said to be in neutral equilibrium. Further if at the onset of instability, there is an oscillatory motion with growing amplitude, the instability is termed as over stability. Instability of the system even for a single mode of disturbance will qualify the system to be unstable whereas the system cannot be termed as stable unless it is stable with respect to every possible disturbance to which it is subjected.

Stability can also be defined as the ability of a dynamical system to be immune to small disturbances. The concept of stability in the mathematical study of a physical system has had a long and fruitful history. Real situations show that for the practical use of many technical systems, stability properties can be a decisive criterion. Some examples where stability properties are important include: engineering structures (bridges, plates, shells structures under pressure loading or unloading by flowing fluids), vehicles moving at high speed, truck-trailer combinations, railway trains and hydrodynamics problems. Over the past decades, engineers have approached many of their stability problems using linearized stability analysis.

The hydrodynamic stability of fluid flows is important in different fields, such as aerodynamics, astrophysics, mechanics, atmospheric sciences, oceanography and biology. The central issue of the stability analysis is to understand the underlying reasons for the breakdown of laminar flow and its subsequent transition to turbulence.

1.5 Origin and Development of linear stability analysis

The linear stability analysis in fluid dynamics probably dates back to 19^{th} century. Researchers like Rayleigh, Taylor, and later, Chandrasekhar and many others carried out stability analysis with one-dimensional base flow.

In the 1980s, increased power of computer led to the possibility of calculating two-dimensional base flows numerically. The corresponding eigenvectors are two-dimensional and homogeneous or trigonometric in the third direction. At around the same time, techniques were developed for approximating certain two-dimensional base flows, called weakly non-parallel, as a sequence of one-dimensional flows, each located at different stations, then studying the stability of each one dimensional flows separately and assembling and interpreting the results, leading to classifications such as absolute and convective, local and global. The weakly non-parallel approach led to important advances in understanding, but is not applicable to strongly two-dimensional or three dimensional flows.

In local stability analysis, cross-stream slice through a flow evolving slowly in the stream-wise direction is taken and the velocity and density profiles of that slice are calculated.

Global linear instability analysis is concerned with the temporal or spatial development of small-amplitude perturbations superposed upon laminar steady or time-periodic three-dimensional flows, which are inhomogeneous in two (and periodic in one) or all three spatial directions. Global instabilities are found in the fuel injectors of rocket engines and aircraft gas turbines.

Absolute-convective linear stability analysis investigates about the stability of the flow, where infinitesimal perturbations grow both in time and space.

During 21st century, the numerical computation of three-dimensional base states and of their stability became attainable. At around the same time, some researchers in the weakly non-parallel community turned to the full and exact computation of two- or three-dimensional base states and their stability. These researchers used the term “global" to differentiate this from the weakly non-parallel approach. This leads to number of problems.

- ♦ It leads to historical inaccuracy. New researchers often date the advent of two or three dimensional analysis of stability to the time at which the term “global" began to be used and mistakenly attribute its development to the authors who initiated the use of this term.
- ♦ It leads to confusion. New researchers often believe that “global stability analysis" comprises a specialized theory, rather than referring merely to the numerical solution of equations.
- ♦ It prefers the weakly non-parallel approach, describing full and exact analysis as a special or competing technique. It is as though the WKB approach to solving differential equations was treated as fundamental, and other non-WKB techniques were given a special name.

It unnecessarily overloads the word “global", which already has other uses in fluid dynamics. For example, it is used to describe a type of bifurcation that does not involve eigenvalue crossing as well as an oceanographic circulation.

1.6 Formal approach and Stability techniques

The hydrodynamic equations i.e., the equations of mass conservation, momentum, energy and state, inspite of their complexity, allows some simple patterns of flow as basic solutions. However, the problem of discussing the stability of a hydrodynamic system may not be tractable mathematically. Even in the case of simplest possible flows, the resulting differential equations are of higher order, often having variable coefficients and sometimes singular as well. Therefore, the discussion of stability of the flows has been concerned mainly to simple problems only, for example, static fluid layer, flow between parallel plates, couette and spiral flow between coaxial cylinders etc. Here we have briefly given the energy method and method of normal mode approach which are most widely used in the existing literature.

(i) Energy Method

To analyze the stability of a flow by this method, the kinetic energy of the perturbations is calculated. The perturbations in Kinetic energy increases with time, the flow becomes unstable and if it decays with time, then the flow becomes stable. This method is global in nature and thus restricted in applications since the kinetic energy of the whole system is calculated. To investigate the stability of the flow by this method, (i.e.,) when the fluid is confined within rigid boundaries, sometimes the vorticity of the perturbations is considered rather than their kinetic energy. So

$$w = \int (M^2 + N^2 + O^2)\, dv,$$

where (M, N, O) are the vorticity components of perturbations, and the integration is taken over whole of the flow domain. The basic flow is stable or unstable accordingly $\frac{dw}{dt}$ is negative or positive. Now the perturbations must vanish at the boundaries as these are taken to be rigid. But there cannot be a non-trivial irrotational flow which vanishes at the boundary. Therefore, the velocity components of the perturbations must also tend to zero. Hence the flow is stable.

This method is used mostly in the non-dissipative systems. Now a days, researchers use this technique for dissipative systems also. This method is more useful to do the non-linear stability analysis.

(ii) Normal Mode Technique

It is the most important technique that is used so far widely to determine the linear stability of a system, because its applications are wider. In this method, the perturbations are assumed to be small in magnitude and the nonlinear terms in the perturbation variables and (or) their derivatives are neglected as compared to the linear terms in the governing equations of the system.

In this technique, the perturbation terms are assumed as the regular functions of space variables and hence the Fourier analysis is possible. Thus in this method, the perturbation terms are expressed into Fourier components, called normal modes, and analysis will be considered about these modes which decay or grow with time. When all the modes decay with time, the flow becomes unstable because once in a while this mode will influence over the whole flow.

In the theory of linear stability, the exponential dependence of the function 'f' on 't' is considered. Then

$$f(x,y,z,t) = \sum \phi(y,k_X,k_Z,n)\, e^{i(k_X x + k_Z z + nt)}$$

where $\vec{k}(k_X, 0, k_Z)$ is called the wave number vector, $k = |\vec{k}|$, the wave number and the summation is taken over all k_X and k_Z, n is complex wave velocity. If the real part of n is positive, then the perturbations grow exponentially with time and the flow will be unstable in this case. If the real part of n is negative, then the perturbations decay exponentially with time and the flow will be stable.

1.7 Dimensionless numbers

The nondimensionalization of the governing equations of fluid flow is important for both theoretical and computational reasons. Nondimensional scaling provides a method for developing dimensionless groups that can provide physical insight into the importance of various terms in the system of governing equations. Dimensionless forms also allow us to present the solution in a compact way. Some of the important dimensionless numbers used in this thesis are given below.

1. 7. 1 Richardson number

Richardson number is the ratio of buoyancy force to the viscous force. It is defined as

$$Ri = \frac{g\beta L^2}{\rho U^2}$$

where U is a typical velocity scale of the flow

g represents the acceleration due to gravity

ρ is the density of the fluid

L is the typical length scale of the flow

If the Richardson number is less than unity, buoyancy is not important in the flow. If it is greater than unity, buoyancy is dominant (because of the insufficient kinetic energy to homogenize the fluids).

1 . 7. 2 Magnetic pressure number

Magnetic pressure number represents the ratio of the magnetic pressure to the dynamic pressure of the fluid. It is defined as

$$S = \frac{\mu_m H_0^2}{\rho U^2}$$

where μ_m is the magnetic permeability

H_0 is the strength of the magnetic field

ρ is the density of the fluid

U is a typical velocity of the flow

1. 7. 3 Brunt Vaisala frequency

Brunt–Väisälä frequency, or buoyancy frequency, is the angular frequency at which a vertically displaced parcel will oscillate within a statically stable environment. It is the oscillation frequency of a parcel displaced vertically in an incompressible fluid and released.

It is defined by

$$N^2 = -\frac{g}{\rho}\left(\frac{d\rho}{dz}\right)$$

where g is the acceleration due to gravity

ρ is the density of the fluid

when, $N^2 > 0$, the system is stable

$N^2 = 0$ represents the neutral stability

$N^2 < 0$, the system is unstable

1. 7. 4 Magnetic Reynolds number

It provides an estimate of the relative effects of advection or induction of a magnetic field by the motion of a conducting medium. It is defined by

$$Rm = \frac{UL}{\eta}$$

where U is a typical velocity scale of the flow

L is a characteristic length scale of the flow

η is the magnetic diffusivity

For $Rm << 1$, diffusion is relatively important, hence the magnetic field will tend to reduce in the direction of purely diffusive state, determined by the boundary conditions rather than the flow.

For $Rm >> 1$, advection is relatively important on the length scale L. Flux lines of the magnetic field are then transported with the fluid flow, where gradients are concentrated into regions of short length scale that diffusion can balance advection.

1. 7. 5 Rotation number

Rotation number is the ratio of Coriolis force to the bulk flow inertia force. The rotation number is defined by

$$\tau = \frac{\Omega L}{U_0}$$

where Ω is the angular velocity of rotation

L is the characteristic length

U_0 is the characteristic velocity

1. 7. 6 Hall current parameter

The Hall parameter is defined as the ratio of cyclotron to collision frequencies.

$$M = \left(\frac{H_0}{4\pi N e\eta}\right)^2$$

where N is the electron number density

E is charge of an electron

H_0 is the magnetic field strength

The Hall Effect in ionized gas is significantly different from the Hall effect in solids (where the Hall parameter is always much less than unity)

1. 7. 7 Hartmann number

Hartmann number (Ha) is the ratio of electromagnetic force to the viscous force. It is defined as

$$Ha = B\ L\sqrt{\frac{\sigma}{\mu}}$$

where B is the magnetic field

L is the characteristic length

σ is the electrical conductivity

μ is the dynamic viscosity

1.8 Brief Outline of the thesis

The theme of this thesis is to study the linear stability of magneto hydrodynamic stratified shear flows of an inviscid, incompressible fluid under specified conditions. The work in the present thesis is divided into nine chapters.

- Introduction
- Literature Review
- Hydromagnetic effect on the stability of plane couette flow of an inviscid, incompressible, non-parallel stratified shear flow
- Hall effect on linear stability of non-parallel stratified shear flow of inviscid incompressible fluid
- Effect of varying magnetic field on linear stability of parallel stratified shear fluid
- Hydromagnetic stability of plane Couette flow of a parallel stratified shear fluid
- Effect of rotation on the linear stability of parallel stratified shear flows
- Stability of stratified rotating non parallel shear flow
- Summary

Chapter I highlights the introductory concepts of the stability analysis and shear flows. Basic preliminaries relevant to the thesis are given. Chapter II is devoted to review existing literature relevant to the problems considered in this thesis.

In Chapter III, we have extended the work of Padmini and Subbiah (1995) to study the effect of applied magnetic field on stratified nonparallel shear flows. The analysis is restricted to long wave approximations. A good qualitative agreement of the results obtained with those results obtained by Padmini and Subbaiah (1995) is found in the case of vanishing Hartmann number.

In Chapter IV magnetic field is assumed to be strong enough to produce Hall current. Restricting the analysis to long wave approximations we have found analytical expressions to calculate growth rate and stream functions. The results obtained were validated with the results obtained by Padmini and Subbaiah (1995) for vanishing Hall parameter.

In Chapter V, the effect of varying magnetic field on a stratified shear flow through a channel is studied. The fluid is considered to be in a state of parallel flow with the basic velocity profile (U(y), 0, 0) and induced magnetic field (H(y), 0, 0). Analytical solutions are found for growth rate and velocity using regular perturbation technique. Effect of various parameters such as Brunt-Vaisala frequency, magnetic parameter and wave number on the growth rate of the small disturbances is studied numerically.

The aim of Chapter VI is to investigate the effect of uniform magnetic field on the stability of a stratified flow through an infinite channel with linear shear velocity profiles. Asymptotic solutions have been obtained for velocity and growth rate using perturbation techniques.

An effect of rotation on the linear stability of parallel stratified shear flows is investigated in Chapter VII. Governing equations for the flow are solved and numerical analysis is carried out to study the effect of various nondimensional parameters on growth rate and on velocity and the results are depicted graphically.

In Chapter VIII the stability of inviscid, rotating stratified non parallel shear flow is investigated. The fluid was considered to be in a state of non parallel flow with the basic velocity profile (U(z), V(z), 0). The governing equations were derived. These equations reduce to those equations obtained by Farrell and Ioannou (1993) for vanishing rotation number. The stability of the flow was analyzed using normal mode approach and the analysis was restricted to long wave approximations.

Chapter IX presents a brief summary of the findings obtained from the above mentioned works.

In all the above mentioned problems, the set of nonlinear equations which includes equation of continuity, equation of motion, equation of state and Maxwell's equations is considered. Appropriate boundary conditions for the geometry are also specified. The governing equations are solved using perturbation technique. Neglecting higher order terms of wave number, we obtain the linearized equations governing the dynamics of the perturbations. These linearized equations are studied via normal mode analysis. Analytical solutions are found for eigen functions and eigen values using long wavelength approximations. In general, the eigen values are complex. Closed form solutions are obtained wherever possible or solutions are found numerically using the software Matlab 7.14. The numerical results of the flow characteristics are presented graphically.