Chapter II

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REVIEW OF LITERATURE

The notion of stability is as old as the civilized world and has a very clear intuitive meaning. Stable and unstable situations can be met everywhere – in mechanical motion, in technical devices, in medical treatment (stable or unstable state of the patient), in currency exchange and so on. Stability theory has been of interest to mathematicians and astronomers for a long time and has had a stimulating impact on these fields. If deviations describing response of the system from a given regime (state of equilibrium) lie within prescribed limits, the system is called stable. Prescribed limits, disturbances and response are specified in each case in different ways.

The hydrodynamic stability of fluid flows is important in different fields, such as aerodynamics, mechanics, astrophysics, oceanography, atmospheric sciences and biology. In this thesis, we mostly deal with magneto hydrodynamic stability and rotation in parallel and nonparallel flows. The problem of hydrodynamic stability is of fundamental importance in fluid mechanics. Its main concerns are to find when and how laminar flows break down, their subsequent development and their consequent transition to turbulence.

The pioneering works in the field were made in the nineteenth century by Helmholtz (1868), Kelvin (1871), Rayleigh (1879) and Reynolds (1883). Since then the development of hydrodynamic stability has been steady and substantial. For the clear and wider description of the fundamental ideas involved, the physical mechanisms, the methods used, and the results obtained, we can refer to the books written by Lin (1955), Chandrasekhar (1961), Drazin and Howard (1966), Joseph (1976), Drazin and Reid (1981), Yih (1980), Turner (1973) and Shivamoggi (1986). In particular the stability of inviscid incompressible homogeneous shear flows between two infinite horizontal rigid walls was initiated by Rayleigh (1880). The method of infinitesimal normal mode disturbances was adapted by him. He obtained the celebrated Rayleigh’s inflexion point theorem of hydrodynamic stability which states that a necessary condition for instability is the existence of an inflexion point of the basic velocity profile.

Fjørtoft (1950) extended the Rayleigh’s criterion by establishing that the instability occurs only for a maximum in the shear rate. An important result in this area is the Howard's semicircle theorem (Howard, (1961)) which ensures that the complex wave velocity of unstable modes must lie inside a semicircle in the upper half plane and the diameter of the semicircle will be the range of the basic velocity profile. Wooler (1961) examined the stability of plane parallel flow when $Rm \ll 1$ and found that three dimensional disturbances are most stable.

Asai (1970) investigated the influence of variable vertical shear flow of an unstably stratified plane parallel fluid using perturbation analysis. He concluded that, in general shear flow is responsible for the formation of longitudinal convection roll, and a variable shear affects characteristics of thermal instability of a constant shear flow slightly. Collyer (1970) considered small perturbations of a stably stratified fluid of density $\rho(z)$ of an inviscid, incompressible, parallel shear flow *U(z)*. It is proved that for any combination of velocity and density profiles, the flow is unstable if $Ri < \frac{1}{4}$ at any level, unless the unstable wavelengths cannot be excited due to restrictions prescribed by the boundary conditions.

Dudis (1972) analysed the effect of saturation on the stability of stably-stratified shear layer. Critical Richardson numbers were found to decrease about 15% at ground level, with the inclusion of saturation effect. At the tropopause level (20 Km), a saturated atmosphere was shown to have a critical Richardson number 2% less than the dry atmosphere. In both cases the adiabatic lapse rates are obviously decreased by saturation and the net effects reduce critical temperature gradients. Pellacani (1983) performed a mathematical analysis of the stability problem of stratified fluids in horizontal sheared motion. He showed that for a given disturbance stability is ensured for values of the Richardson number which depend on the ratio of the components of the horizontal wave vector of the perturbation itself.

Graham (1978) considered the non-parallel shear flows of an incompressible, inviscid, density-stratified fluid. He showed that if each horizontal fluid plane has a translational velocity of the same magnitude and shear is produced by rotation of the velocity vector with increasing height, then 'stability' increases strongly with increasing layer of thickness.

El-Hady and Nayfeh (1979) discussed the effect of the non parallelism of the mean flow on the stability characteristics for two dimensional subsonic and supersonic flows. Results calculated by the nonparallel stability theory are in better agreement with the supersonic experimental data of Laufer and Vrebalovich (1960) and Kendall (1975) than the results calculated by the parallel theory of Mack (1975). Later, El-Hady (1980) presented a compressible linear stability theory for three dimensional disturbances in nonparallel three dimensional boundary-layer flows considering the normal velocity component as well as stream wise and span wise variations of the basic flow.

Jain and Kochar (1983) analyzed the steady plane parallel flow of an incompressible, inviscid fluid of variable density under gravity. It is shown that the complex wave velocity for any unstable mode lies in a semi ellipse-type region whose minor axis depends on the stratification, while its major axis synchronizes with the diameter of Howard’s semicircle. Fujimura and Kelly (1988) investigated the linear stability of unstably stratified shear flows between two horizontal parallel plates. The eigen value problem was solved numerically by making use of the expansion method in Chebyshev polynomials to obtain the critical Rayleigh numbers accurately in the Reynolds number range (0.01, 100).

Mobbs and Darby (1989) presented a general method to test the stability of a stratified, parallel shear flow. They have undertaken the stability analysis for several idealized velocity profiles to validate the method. Barston (1991) made a study on the linear stability of inviscid incompressible plane parallel flow using normal mode analysis. He obtained a class of constants of motion for the time-dependent form of the Rayleigh stability equation, which provides stability criteria for velocity profiles with multiple inflexion points.

Farrell and Ioannou (1993) investigated the transient development of perturbations in inviscid stratified shear flow. A general lower bound on the energy growth is obtained by an optimal perturbation in a stratified flow over a given time interval.

Churilov (2004) employed a two-layer model of a stably stratified medium to study the stability of flows without inflection points on a monotonic velocity profile increasing from zero to a maximum value U_0. Salhi and Cambon (2007) obtained the solution of the Euler equations with Boussinesq approximation by considering unbounded flows subject to spatially uniform density stratification and shear rate that are time dependent. Alexandros Alexakis (2009) performed numerical simulations of stratified shear flow instabilities in two dimensions in the Boussinesq limit. He suggests that turbulent mixing processes in strongly stratified environments cannot be excluded.

Long-wave instability and growth rate of the inviscid shear flows is studied by Liang Sun (2011). The new upper bound for neutral wave number, imaginary part of the complex phase velocity and growth rate was obtained in this work. The transient dynamics of the linearized Euler–Boussinesq equations governing parallel stratified shear flows is presented and analyzed by Camassa and Viotti (2013). They presented by means of rigorous asymptotic methods, a new class of exact solutions for linearized perturbations in stratified parallel shear flows. Ganesh and Subbiah (2013) obtained the series solution and a perturbation formula for the extended Rayleigh problem of hydrodynamic stability and proved the existence of a neutrally stable eigen solution with wave number $k_s > 0$.

Burde *et al* (2007) studied the stability of some unsteady three dimensional flows via separation of variables in the linearized equations for the flow perturbations. Hirota and Morrison (2016) studied the linear stability of parallel, inviscid stably stratified shear flow assuming smooth strict monotonic profiles of shear flow and density, the local Richardson number is positive everywhere. Jose *et al* (2015) investigated the short-time response of disturbances in a density-varying Couette flow without viscous and diffusive effects analytically. They demonstrated the algebraic instabilities in stably stratified shear flows by considering a sequence of scenarios - zero, weak and strong ($Ri < 0.25$) background stratification.

Recently, Facchini *et al* (2018) presented the stability analysis of a plane Couette flow which is stably stratified to the horizontal shear in the vertical direction orthogonally. By using of numerical simulations, they concluded that the observed pattern is a signature of the same instability predicted by the linear theory with the slight modification due to streamwise confinement.

'The presence of magnetic field induces a viscous drag on a conducting fluid and also imparts some degree of rigidity. The imposition of a magnetic field thus inhibits instability. The intuitive way to approach stability is by determining whether or not it departs far from equilibrium when it is subjected to an arbitrary small disturbance and is then left to itself. In this thesis, the role of magnetic fields which directly cause the instabilities is considered. Stuart (1954) analyzed the stability of the flow of an electrically conducting liquid between parallel walls in the presence of a parallel magnetic field. He found that, a magnetic field exerts a stabilizing influence on the flow.

Lock (1955) investigated the stability under small disturbances of the two-dimensional laminar motion of an electrically conducting fluid under a transverse magnetic field. It is found that the dominating factor is the change in shape of the undisturbed velocity profile caused by the magnetic field, which depends on the Hartmann number (*Ha)***.** Miles (1961) considered some small perturbations of a parallel shear flow *U(y)* in an inviscid, incompressible fluid of variable density$\rho_0(y)$**.** It is proved that sufficient conditions for stability are $U'(y) \neq 0$ and $J(y) > \frac{1}{4}$ throughout the flow, where $J(y) = \frac{-g\rho_0'(y)}{\rho_0(y)\left(U'(y)\right)^2}$ is the local Richardson number. Viswanadha Sarma (1961) studied the effect of uniform vortices present in the free stream on the interaction of the flow and magnetic fields for the case of two-dimensional flow of an inviscid, incompressible perfectly conducting fluid. It is found that the flow field and the magnetic field have a logarithmic singularity on the boundary as m approaches unity.

The stability of couette flow of a conducting fluid between two parallel plates in the presence of tranverse magnetic field was examined by Kakutani (1964). He found that the magnetic field may be destabilizing because of carrying curvature of the basic velocity profile. Gupta (1963) investigated the stability of horizontal layer of a perfectly conducting fluid with continuous density and viscosity stratification in the presence of horizontal magnetic field. It is found that when the coefficient of viscosity μ_0 satisfies$\frac{d^2\mu_0}{dz^2} < 0$, instability might arise in the form of oscillation with increasing amplitude. He also showed that a sufficiently strong magnetic field may stabilize a potentially unstable configuration. Kent (1966) obtained a sufficient condition $U_0'' > \left(\frac{A_0'}{U_0'}\right) A_0''$ for instability against small oscillations of symmetric laminar flows of an inviscid, incompressible, perfectly conducting magnetic fluid flowing between parallel walls in the presence of a symmetric magnetic fluid parallel to the flow.

Hunt (1966) made an analysis of the stability of plane parallel flows of fluids with finite viscosity and conductivity under the action of uniform parallel magnetic field. It is shown that three dimensional disturbances are most stable, disagreeing with the conclusion derived by Michael (1953) and Stuart (1954). Kent (1968) examined the effect of magnetic field on the stability of plasmas or magnetofluids with velocity gradients. It is shown that a constant magnetic field stabilizes some velocity profiles but destabilize others. Engevik (1973) considered the stability analysis of shear flow in a stratified, incompressible, inviscid fluid and presented a new method to study the effect of variation of certain physical quantities on the stability.

Sen Gupta and Gupta (1975) studied the stability of the flow of an electrically conducting fluid permeated by a magnetic field in a channel formed with two vertical plates which is rotating about a vertical axis. He showed that the magnetic field exerts a strong stabilizing influence on the flow. Rudraiah (1978) investigated the hydromagnetic stability of an adiabatic perfectly conducting nonviscous fluid. It is shown that the necessary condition for instability is $J < \frac{1}{4P} + \frac{g^2}{C^2(W')^2}$. The stability characteristics of Helmholtz velocity profile in a stratified Boussinesq fluid in the presence of a rigid boundary with magnetic field is studied by Sathya Narayanan (1983). New unstable modes in addition to the Kelvin-Helmhotz mode are found by him. Further, he proved that the wavelengths of these unstable modes are close to the wavelengths of internal Alfven gravity waves in the atmosphere.

The stability of dissipative Magneto hydrodynamic shear flow in a parallel magnetic field is analyzed by Lerner and Knobloch (1985). He obtained a strong decay bounds for linearized perturbations to an unbounded, plane Couette flow in a parallel magnetic field and found that finite conductivity and molecular viscosity shows stabilizing effect. Parthi and Nath (1991) obtained the linear stability of a stratified shear flow on a perfectly conducting bounded fluid in the presence of magnetic field aligned with the flow, considering buoyancy forces under Boussinesq approximation. He obtained a new upper bound for the real and imaginary parts of the complex wave velocity for growing perturbations.

Gupta (1992) investigated the stability of a stratified parallel flow varying in two directions of an incompressible conducting fluid permeated by a uniform aligned magnetic field. He proved that complex wave speed of an unstable mode lies in the upper half of a semi-circle whose diameter decreases with increasing magnetic field.

He also found that a strong enough magnetic field can completely stabilize flows with unstable density stratification.

Later, Takashima *et al* (1999) applied the linear stability theory to the problem on the onset of buoyancy driven instability in a horizontal layer of electrically conducting fluid heated from below in the presence of vertical magnetic field and obtained a necessary and sufficient condition for overstability. Hughes and Tobias (2001) considered the linear stability of an ideal, plane-parallel magnetohydrodynamic shear flow with velocity $\vec{u} = (U(z), 0, 0)$ and a variable magnetic field $\vec{B} = (B(z), 0, 0)$ and obtained a new sufficient condition

$$\frac{1}{2}(U_{max} + U_{min} + 2U_0) - \left(\frac{1}{4}(U_{max} - U_{min})^2 - (B^2)_{min}\right)^{\frac{1}{2}} > (((U + U_0)^2 - B^2)_{max})^{\frac{1}{2}},$$

where U_0 is some constant for the stability of the flow. Leonovich and Mishin (2005) solved the shear flow stability problem of compressible electrically conducting fluid in a magnetic field. They made a comparative analysis to study the influence of different boundary conditions on shear flow stability.

Ruderman and Brevdo (2006) presented the results of the stability analysis of a simple shear flow of an incompressible fluid with a piecewise linear velocity profile in the presence of a magnetic field. He obtained the dispersion equation governing the normal-mode stability of the flow and analyzed its properties. In this paper the author studied stability of two cases: (i) magnetic-free flow in the presence of gravity and the flow stability is controlled by the Rayleigh number (*R)* and (ii) magnetic flow without gravity and identified that the control parameter is the inverse squared Alfvénic Mach number (*H)* for this case.

The stability of stratified shear flow of an inviscid, incompressible fluid confined between two rigid planes in the presence of cross flow under a parallel magnetic field applied in the direction of the main flow is investigated by Naresh Kumar Dua et al. (2011). It has been shown that the cross flow with velocity *(U(y), 0, V(y))* supports the internal gravity wave when $N^2 + Sk^2 > 0$, and the flow is unstable when $N^2 + Sk^2 < 0$.

In the presence of strong electric field, the electric conductivity is found to be affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced in the direction normal to both electric and magnetic fields. This phenomenon is termed as Hall Effect. The Hall current is likely to be important in flows of laboratory plasmas as well as in many geophysical and astrophysical situations.

Gupta (1967) studied the Hall current effect on thermal instability of a horizontal layer of electrically conducting fluid. Thermal instability of Rivlin-Erickson fluid in the presence of uniform horizontal magnetic field including Hall current effect is considered by Sharma (2000). For the case of stationary convection, Hall currents quicken the onset of convection, magnetic field delays the onset of convection, whereas the kinematic viscoelasticity has no effect on the onset of convection. He also considered the case of over stability and arrived the sufficient condition for the non-existence of overstability.

Veena Sharma and Kamal Kishor (2001) considered the thermosolutal convection in Rivlin-Erickson elastico-viscous fluid in the presence of uniform horizontal magnetic field including Hall Effect with varying gravity. The case of overstability is considered and sufficient conditions for the non-existence of overstability are also obtained.

Rapid rotation and strong density stratification characterize the dynamics of geophysical fluids, the atmosphere and in particular the ocean. Paul Mathews and Stephen Cox (1997) presented the linear theory for thermal convection, restricted their attention to a layer of fluid rotating about a horizontal axis and plane Couette flow driven by differential motion of the horizontal boundaries. They found the preferred orientation of the convection rolls, for different orientations of the rotation vector with respect to the shear flow and proved that the preferred roll orientation depends only on their ratio, the Rossby number for moderate rates of shear and rotation. Nonlinear numerical simulations of the convection are also carried out and proved that these are consistent with the linear stability theory.

The linear stability of rotating inviscid stratified horizontal plane Couette flow in a channel is studied for the case of strong rotation and stratification by Vanneste and Yavneh (2007). They concluded that the flow is unconditionally stable (i.e) unbalanced instabilities, associated with linear resonances between Kelvin and inertia–gravity waves, occur for arbitrarily small Rossby numbers. They also showed that the result obtained is relevant to the stability of astrophysical accretion disks and of Taylor–Couette flows.

Gupta and Singh (2012) analyzed the Rayleigh- Taylor instability of rotating stratified viscoelastic fluids in the presence of variable magnetic field. Results show that the presence of magnetic field stabilizes certain wave number band, whereas in the absence of the magnetic field, rotation and for non- viscoelastic fluid the system is unstable for all wave numbers.

Present thesis is devoted to investigate the linear stability of stratified shear flows with rotation and magnetic effect. Macroscopic approach is adopted throughout the work and the normal mode technique is used to determine the stability or instability of fluid flows. Long wave approximation is used to enhance the mathematical tractability of the problems.