

Chapter III

CHAPTER III

HYDROMAGNETIC EFFECT ON THE STABILITY OF PLANE COUETTE FLOW OF AN INVISCID INCOMPRESSIBLE NON PARALLEL STRATIFIED SHEAR FLOW

3.1 Introduction

The problem on stability of stratified shear flows is one of the classical problems in the theory of hydrodynamic stability. The linear stability problem, over its long history, outturn a number of elegant and meaningful results, reviewed in, for example, the monographs of Lin (1955) and Drazin & Reid (1981). Stability of stratified shear flows of an unsteady, inviscid, incompressible fluid is of great interest in meteorological and oceanographical studies. The linear stability of a stratified plane, parallel shear flow of an inviscid incompressible fluid has been broadly studied by many authors namely Drazin and Howard (1966), Miles (1961), and Pellacani (1983). Kochar and Jain (1979) showed that for any unstable mode complex wave velocity must lie in a certain semi-ellipse, instead of a semicircle, whose major axis coincides with the diameter of Howard's semicircle, while its minor axis depends on stratification. Farrell and Ioannou (1993) examined transient development of perturbations in inviscid stratified shear flow using matrix variational methods.

Problems of magneto hydrodynamic stability with the growth or decay of small disturbances in a system with magnetic field are of obvious importance. Lock (1955) discussed the stability of a hydromagnetic flow in the presence of uniform magnetic field between two parallel plates. However, the stability of non-parallel shear flows has not been studied appreciably. In recent years, interest has been shown in the analysis of linear stability of non-parallel flows.

Graham (1978) studied about non-parallel shear flows of an inviscid, incompressible, density stratified fluid and considered a stability analysis interms of the possibility of complete mixing within a horizontal layer of thickness.

The linear stability of stratified shear flow of a perfectly conducting fluid in the presence of magnetic field aligned with the flow and buoyancy forces under Boussinesq's approximation has been studied by Parthi and Nath (1991). Padmini and Subbiah (1995) studied the problem of linear stability of an inviscid, incompressible

non-parallel stratified shear flows to normal mode disturbances. Burde et al (2007) investigated the stability of a class of unsteady non-parallel incompressible flows via separation of variables.

Hinvi *et al* (2013) investigated the inviscid instability of an electrically conducting fluid affected by a parallel magnetic field. The effects of various dimensionless parameters on the temporal stability of the fluid, in a Couette horizontal porous channel flow in the presence of a uniform transverse magnetic field is studied by Monwanou *et al* (2017).

In this chapter, the work of Padmini and Subbiah (1995) is extended by considering the effect of applied magnetic field. The hydromagnetic stability of stratified shear flow of an inviscid, incompressible fluid confined between two rigid planes is discussed. The stability of plane Couette flow is discussed using normal mode analysis and the analysis is restricted to long wave approximation.

3.2 Mathematical formulation

Consider the motion of an electrically conducting inviscid, Boussinesq stratified fluid confined between two horizontal infinite plates of variable density $\rho(x, y, z, t)$ situated at $y = \pm L$. Let $2L$ be the distance between the parallel plates. The origin is taken at a point midway between the plates. The steady primary flow is taken parallel to the x -axis with y -axis normal to the plates. Applied magnetic field is taken perpendicular to the plates in the direction of y -axis.

The assumptions made for the present problem are

- Flow of unsteady, inviscid, incompressible Newtonian fluid is considered.
- Fluid is flowing between two horizontal infinite rigid plates at a distance $2L$.
- No slip boundary conditions are imposed at the boundaries.
- Boussinesq approximation is applied in the momentum equation.
- All the fluid properties are assumed constant except the density variation which takes place with vertical coordinate 'y' due to Boussinesq approximation.
- The basic velocity profile is assumed as $\vec{q}_e = (U(y), 0, W(y))$
- Viscous dissipation effect is neglected.
- Induced magnetic field is neglected in comparison with the applied one.

Based on the above mentioned assumptions the physical configuration of the problem is shown in Figure. 3.1.

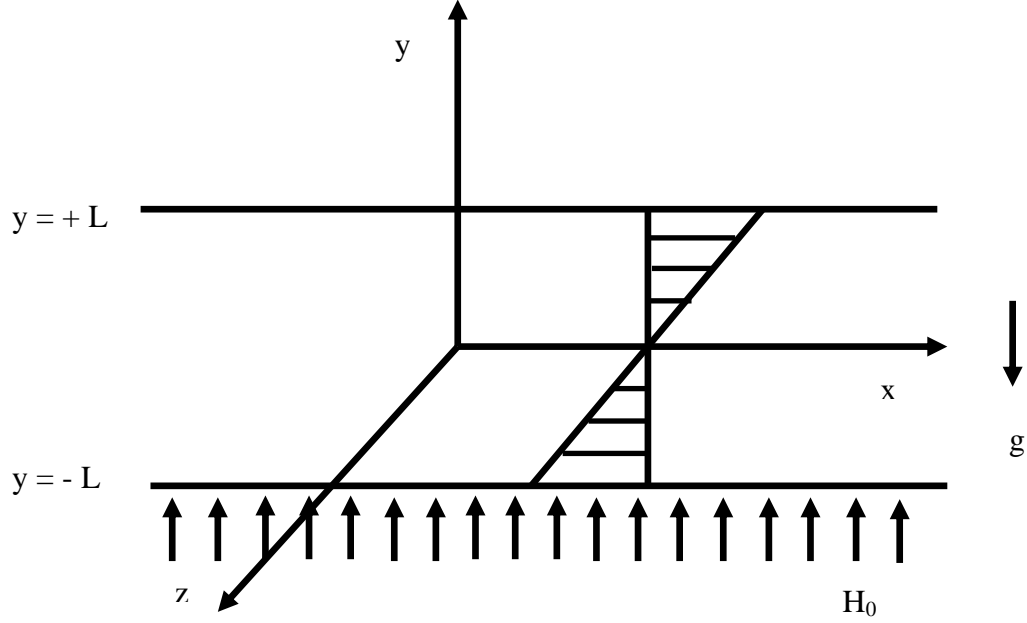


Figure 3.1 Physical configuration of the problem

The governing equations of the system are

$$\nabla \cdot \vec{q} = 0 \quad (3.1)$$

$$\rho \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = -\nabla p - \rho g \hat{y} + (\vec{J} \times \vec{B}) \quad (3.2)$$

$$\frac{\partial \rho}{\partial t} + (\vec{q} \cdot \nabla) \rho = 0 \quad (3.3)$$

$$\nabla \times \vec{B} = \mu_e \vec{J} \quad (3.4)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3.5)$$

$$\nabla \cdot \vec{B} = 0 \quad (3.6)$$

$$\nabla \cdot \vec{J} = 0 \quad (3.7)$$

with the boundary conditions

$$\vec{q} = 0 \quad \text{on } y = \pm L \quad (3.8)$$

where \vec{q} is the velocity vector, ρ is the density, p is the pressure, g is the acceleration due to gravity, \hat{y} is the unit vector in the vertical direction, \vec{B} is the magnetic field, \vec{E} is the electric field, \vec{J} is the current density vector, σ is the electrical conductivity, μ_e is the magnetic permeability and t is the time.

Ohm's law is given as

$$\vec{J} = \sigma (\vec{E} + \vec{q} \times \vec{B}) \quad (3.9)$$

Let us take $\vec{B} = (0, B_0, 0)$ (3.10)

$$\vec{E} = (E_x, E_y, E_z) \quad (3.11)$$

$$\vec{J} = (J_x, 0, J_z) \quad (3.12)$$

where B_0 is a constant. We assume that no applied and polarization voltage exists. (i.e $\vec{E} = 0$) then equations (3.9) - (3.11) gives

$$\vec{J} = \sigma B_0 (-w, 0, u) \quad (3.13)$$

and equation (3.7) yields

$$\sigma B_0 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = 0 \quad (3.14)$$

We introduce the following non dimensional quantities

$$t = \frac{L t^*}{U_0}, \quad p = \rho_0 U_0^2 p^*, \quad \rho = \frac{\rho_0 U_0^2 N_0^2}{L g} \rho^*, \quad \vec{H} = H_0 \vec{H}^* \quad \text{and} \quad (x, y, z) = L(x^*, y^*, z^*)$$

where L is the characteristic length, U_0 is the characteristic velocity, $N^2 = -\frac{g}{\rho_0} \left(\frac{d\rho}{dy} \right)$ is the Brunt – Vaisala frequency, N_0 is the typical value of this frequency in the flow domain and H_0 is constant magnetic field.

Substituting these nondimensional quantities in equations (3.1) – (3.3) and removing asterisks for convenience, we get

$$\nabla \cdot \vec{q} = 0 \quad (3.15)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -\nabla p - Ri \rho \hat{j} + Ha' (\vec{q} \times \vec{H}) \times \vec{H} \quad (3.16)$$

$$\frac{\partial \rho}{\partial t} + (\vec{q} \cdot \nabla) \rho = 0 \quad (3.17)$$

The related boundary conditions in dimensionless form are given by

$$\vec{q} = 0 \quad \text{on } y = \pm 1 \quad (3.18)$$

The fluid is in a state of plane non-parallel flow characterized by a horizontal shear layer with basic velocity $\vec{q} = (U(y), 0, W(y))$, $\rho_0 = \rho_0(y)$ and $p_0 = p_0(y)$ which satisfies the governing equations and boundary conditions provided

$$\frac{-\partial p_0}{\partial y} = Ri \rho_0 \quad (3.19)$$

where $U(y)$, $W(y)$, $\rho_0(y)$ and $p_0(y)$ are continuously differentiable functions of y in the flow domain. The dependence of the problem on the material properties has been reduced to the dimensionless parameters such as Hartmann number (Ha') and Richardson number (Ri) and are given by

$$Ha' = LB_0 \sqrt{\frac{\sigma}{\mu}}, \quad Ri = \frac{g\beta L^2}{\rho_0 U_0^2} \quad (3.20)$$

3.3 Stability analysis

Following the usual terminology of linear stability analysis, let the disturbed flow be written as a steady basic flow plus a time dependent disturbance, assumed small. The perturbed flow variables are taken as $(U(y) + u, v, W(y) + w), \rho_0(y) + \rho$ and $p_0(y) + p$. Hence, the linearized perturbation equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.21)$$

$$\frac{\partial u}{\partial t} + U(y) \frac{\partial u}{\partial x} + v \frac{\partial U(y)}{\partial y} + W(y) \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} - Ha' u \quad (3.22)$$

$$\frac{\partial v}{\partial t} + U(y) \frac{\partial v}{\partial x} + W(y) \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} - Ri \rho \quad (3.23)$$

$$\frac{\partial w}{\partial t} + U(y) \frac{\partial w}{\partial x} + v \frac{\partial W(y)}{\partial y} + W(y) \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} - Ha' w \quad (3.24)$$

$$\frac{\partial \rho}{\partial t} + U(y) \frac{\partial \rho}{\partial x} + W(y) \frac{\partial \rho}{\partial z} - \frac{N^2}{N_0^2} v = 0 \quad (3.25)$$

the disturbances are taken to be periodic in the streamwise, spanwise directions and time, which allows us to assume solutions of the form $f(x, y, z, t) = \hat{f}(y)e^{i(kx+klz-k\sigma t)}$, where f represents the disturbances u, v, w, p, ρ and \hat{f} is the amplitude function, k is the longitudinal wave number, l is the transverse wave number and $\sigma = \sigma_r + i\sigma_i$ is the wave velocity which is taken to be complex.

Hence, equations (3.21) – (3.25) becomes,

$$ik(u + lw) + \frac{\partial v}{\partial y} = 0 \quad (3.26)$$

$$ik(-\sigma + U(y) + lW(y))u + v \cdot \frac{\partial U(y)}{\partial y} = -ikp - Ha'u \quad (3.27)$$

$$ik(-\sigma + U(y) + lW(y))v = -\frac{\partial p}{\partial y} - Ri \rho \quad (3.28)$$

$$ik(-\sigma + U(y) + lW(y))w + v \cdot \frac{\partial W(y)}{\partial y} = -iklp - Ha' w \quad (3.29)$$

$$ik(-\sigma + U(y) + lW(y))\rho - \frac{N^2}{N_0^2} v = 0 \quad (3.30)$$

The motion is stable or unstable according as $c_i < 0$ or $c_i > 0$. The corresponding boundary conditions are

$$u = v = w = 0 \quad \text{on } y = \pm 1 \quad (3.31)$$

3.4 Eigen values and eigen functions for long waves

The analysis is restricted to long waves (i.e) k is assumed to be small and the flow is assumed to be bounded between two plates $y = \pm 1$. $U(y)$ and $W(y)$ are assumed to be linear. Hence equation (3.26) – (3.30) takes the form

$$\begin{aligned}
ik(u + lw) + \frac{\partial v}{\partial y} &= 0 \\
ik((1 + l)y - \sigma)u + v &= -ikp - Ha'u \\
ik((1 + l)y - \sigma)v &= -\frac{\partial p}{\partial y} - Ri \rho \\
ik((1 + l)y - \sigma)w + v &= -iklp - Ha' w \\
ik((1 + l)y - \sigma)\rho - \frac{N^2}{N_0^2}v &= 0
\end{aligned} \tag{3.32}$$

By assuming the series expansions with respect to k in the form $f = f_0 + kf_1 + k^2f_2 + \dots$ where $f = (u, v, w, \sigma, \rho)$, $Ha' = k^2Ha$ and substituting into equation (3.32) and collecting the coefficients of like powers of k , we get

O(1)

$$\begin{aligned}
iu_0 + ilw_0 + \frac{\partial v_0}{\partial y} &= 0 \\
iT(y)u_0 + v_0 &= -ip_0 \\
-\frac{\partial p_0}{\partial y} - Ri \rho_0 &= 0 \\
iT(y)w_0 + v_0 &= -ilp_0 \\
iT(y)\rho_0 - \frac{N^2}{N_0^2}v_0 &= 0
\end{aligned} \tag{3.33}$$

where $T(y) = (1 + l)y - \sigma_0$

with the corresponding boundary condition

$$u_0(\pm 1) = v_0(\pm 1) = w_0(\pm 1) = 0 \tag{3.34}$$

Eliminating the above equation interms of v_0 , we get

$$T(y)^2 \frac{\partial^2 v_0}{\partial y^2} + \frac{Ri N^2}{N_0^2} (1 + l^2)v_0 = 0 \tag{3.35}$$

The solution of equation (3.35) is given as

$$v_0 = A T(y)^{m_1} + B T(y)^{m_2} \tag{3.36}$$

where $m_{1,2} = \frac{1 \pm \sqrt{\lambda}}{2}$, $\lambda = 1 - 4 Ri \frac{N^2}{N_0^2} (1 + l^2)$, A, B are arbitrary constants.

By imposing the boundary condition that the velocity must vanish at the boundaries

(i.e) $v_0 = 0$ at $y = \pm 1$, we obtain the value of σ_0 as

$$\sigma_0 = (1 + l) \frac{1 + e^{\frac{2n\pi i}{m_1 - m_2}}}{1 - e^{\frac{2n\pi i}{m_1 - m_2}}} \tag{3.37}$$

The first order approximation is given by

$$\begin{aligned}
iu_1 + ilw_1 + \frac{\partial v_1}{\partial y} &= 0 \\
iT(y)u_1 + v_1 - i\sigma_1 u_0 &= -ip_1 - Ha u_0 \\
-\frac{\partial p_1}{\partial z} - Ri \rho_1 &= 0 \\
iT(y)w_1 + v_1 - i\sigma_1 w_0 &= -ilp_1 - Ha w_0 \\
iT(z)\rho_1 - i\sigma_1 \rho_0 - \frac{N^2}{N_0^2} v_1 &= 0
\end{aligned} \tag{3.38}$$

with the boundary condition

$$u_1(\pm 1) = v_1(\pm 1) = w_1(\pm 1) = 0 \tag{3.39}$$

By simplifying equation (3.38) in terms of v_1 , we get

$$T(y)^2 \frac{\partial^2 v_1}{\partial y^2} + \frac{Ri N^2}{N_0^2} (1 + l^2) v_1 = (\sigma_1 + iHa) \left(T(y) \frac{\partial^2 v_0}{\partial z^2} \right) + iRi \rho_0 \sigma_1 (1 + l^2) \tag{3.40}$$

The solution of equation (3.40) is given as

$$v_1 = C(T(y))^{m_1} + D(T(y))^{m_2} + P_1 A_1 (T(y))^{m_1-1} + P_2 A_2 (T(y))^{m_2-1} \tag{3.41}$$

The value of σ_1 can be obtained from the above equation by applying the boundary condition that $v_1(\pm 1) = 0$

$$\sigma_1 = \frac{-Ha A_9}{A_7 - Ri A_8} \tag{3.42}$$

The second order approximation is given by

$$\begin{aligned}
iu_2 + ilw_2 + \frac{\partial v_2}{\partial y} &= 0 \\
iT(y)u_2 + v_2 - i\sigma_1 u_1 - i\sigma_2 u_0 &= -ip_2 - Ha u_1 \\
-\frac{\partial p_2}{\partial z} - Ri \rho_2 &= iT(y)v_0 \\
iT(y)w_2 + v_2 - i\sigma_1 w_1 - i\sigma_2 w_0 &= -ilp_2 - Ha w_1 \\
iT(y)\rho_2 - i\sigma_1 \rho_1 - i\sigma_2 \rho_0 - \frac{N^2}{N_0^2} w_2 &= 0
\end{aligned} \tag{3.43}$$

with the boundary conditions

$$u_2(\pm 1) = v_2(\pm 1) = w_2(\pm 1) = 0 \tag{3.44}$$

By simplifying equation (3.43) in terms of v_2 , we get

$$\begin{aligned}
T(y)^2 \frac{\partial^2 v_2}{\partial y^2} - \frac{Ri N^2}{N_0^2} (1 + l^2) v_2 &= -(\sigma_1 + iHa) \left(T(y) \frac{\partial^2 v_1}{\partial y^2} \right) - \sigma_2 \left(T(y) \frac{\partial^2 v_0}{\partial y^2} \right) \\
&\quad - Ri \rho_1 \sigma_1 (1 + l^2) - iT(y)v_0
\end{aligned} \tag{3.45}$$

The solution of equation (3.45) is given as

$$\begin{aligned}
v_2 &= E (T(y))^{m_1} + F (T(y))^{m_2} \\
&\quad + \left((\sigma_1 + iHa) A_{24} + Ri \sigma_1 A_{25} + \sigma_2 (A_{26} + Ri A_{27}) \right) (T(y))^{m_1-1}
\end{aligned}$$

$$\begin{aligned}
& +((\sigma_1 + iHa)A_{28} + Ri \sigma_1 A_{29} + \sigma_2(A_{30} + Ri A_{31}))(T(y))^{m_2-1} \\
& +((\sigma_1 + iHa)A_{32} + Ri \sigma_1 A_{33})(T(y))^{m_1-2} \\
& +((\sigma_1 + iHa)A_{34} + Ri \sigma_1 A_{35})(T(y))^{m_2-2} \\
& + A_{36}(T(y))^{m_1+2} + A_{37}(T(y))^{m_2+2}
\end{aligned} \tag{3.46}$$

The value of σ_2 can be obtained from the above equation by applying the boundary condition that $v_2(\pm 1) = 0$

$$\sigma_2 = \frac{-A_{54} - \sigma_1(A_{50} + Ri A_{51}) - iHa A_{50}}{A_{52} + Ri A_{53}} \tag{3.47}$$

For the sake of brevity, constants are given in *Appendix I*.

3.5 Discussion of the results

In this section, the influence of wave number (k), Brunt - vaisala frequency (N^2), Richardson number (Ri) and transverse wave number (l) on the stability of non parallel stratified shear flow confined between the plates $y = \pm 1$ is examined.

Figures (3.2) - (3.6) show the dependence of growth rate (σ) on the longitudinal wave number (k). From Figure (3.2) we can conclude that increase in longitudinal wave number (k) decreases the growth rate (σ). Growth rate decreases with the increase in Hartmann number (Ha) and the system becomes stable.

Figure (3.3) shows that growth rate (σ) decays when k is increasing thereby increasing the stability or instability of the fluid flow accordingly. As transverse wave number (l) increases growth rate is decreasing. Figure (3.4) exhibits that the growth rate (σ) increases as the Richardson number (Ri) increases. The system becomes stable with the increase in wave number (k).

Figure (3.5) presents the nature of growth rate for various Brunt-Vaisala frequency (N^2). It is understood that, growth rate increases with the increase in N^2 and makes the system unstable. From the analytical expression derived for σ_0 , it can be noted that there exist infinite number of modes both stable and unstable corresponding to the values of n . This nature of n is shown through Figure (3.6).

The effect of Ri on growth rate is exhibited using Figures (3.7) - (3.10). From Figure (3.7), it can be observed that increase in Hartmann number (Ha) increases the growth rate (σ) as Richardson number (Ri) increases and the system becomes stable. From Figure (3.8), it is noted that growth rate (σ) decreases with the increase in transverse wave number (l). The system becomes stable with the increase in Richardson number (Ri).

Figure (3.9) shows the nature of growth rate (σ) for increasing wave number k . It can be concluded from this figure that increase in k decreases the growth rate rapidly with the increase in Ri and the system becomes stable. From Figure (3.10) we can observe that growth rate (σ) decreases with the increase in Brunt-Vaisala frequency N^2 . The system is stable with the increase in Richardson number Ri .

Figures (3.11) – (3.13) exhibit the nature of growth rate (σ) as a function of transverse wave number (l). From Figures (3.11) and (3.12), it is observed that growth rate (σ) decreases with an increase in l for $Ha = 1, 2$ and the system becomes unstable. Figure (3.13) presents the nature of growth rate for various n . We can conclude that variation exists with the increase in l and the system is initially stable and becomes unstable as transverse wave number (l) increases.

Figures (3.14) and (3.15) show the nature of growth rate with respect to longitudinal wave number (k) for parallel flows. From Figure (3.14), it is noted that growth rate (σ) decreases with the increase in Hartmann number (Ha) and the system becomes stable. Figure (3.15) exhibits the nature of growth rate (σ) for various Richardson number (Ri). It is observed that growth rate decreases with the increase in longitudinal wave number (k) for various Ri . It is known that the system is unstable.

The nature of growth rate (σ) in the absence of magnetic effect is shown through Figures (3.16) – (3.18). From figure (3.16), it is noted that, growth rate (σ) decreases with the increase in l and the system is stable. From Figures (3.17) and (3.18), it is observed that growth rate (σ) increases with the increase in Brunt Vaisala frequency (N^2) and Richardson number (Ri). The system is stable for various Ri and N^2 .

Figures (3.19) - (3.20) exhibit the velocity profile for various dimensionless parameters Ha and k . It is observed that velocity increases with the increase in Ha and k .

3.6 Conclusion

The linear stability analysis of an inviscid, nonparallel stratified shear fluid to normal mode disturbances with applied magnetic effect is analyzed. The governing equations of the flow coincides with those obtained by Padmini and Subbiah (1995) when $Ha = 0$. The stability of the flow is analysed using normal mode approach and the analysis is restricted to long wave approximation. The behavior of various

nondimensional numbers on the stability of non parallel and parallel shear flow confined between the plates $y = \pm l$ is discussed.

From the results obtained, it is concluded that

- ◆ Increase in longitudinal wave number (k) decreases the growth rate rapidly for various Ha and l . While an increase in the longitudinal and transverse wave number destabilizes the system, increase in the magnetic field stabilizes the system.
- ◆ The system is stable for various Richardson number (Ri) and becomes unstable when Brunt - Vaisala frequency (N^2) increases.
- ◆ Growth rate amplifies rapidly with increasing Ha , weakens with increasing longitudinal and transverse wave number (k and l), Brunt Vaisala frequency (N^2), shows the stable nature of the system with the increasing Richardson number (Ri).
- ◆ Growth rate decreases with the increase in k for various Ha thereby increases the region of instability when l increases.
- ◆ The flow becomes stable when Hartmann number (Ha) and Richardson number (Ri) increases for parallel flow.
- ◆ In the absence of applied magnetic effect, the system becomes stable with the increase in transverse wave number (l), Brunt-Vaisala frequency (N^2) and Richardson number (Ri) when longitudinal wave number (k) increases.

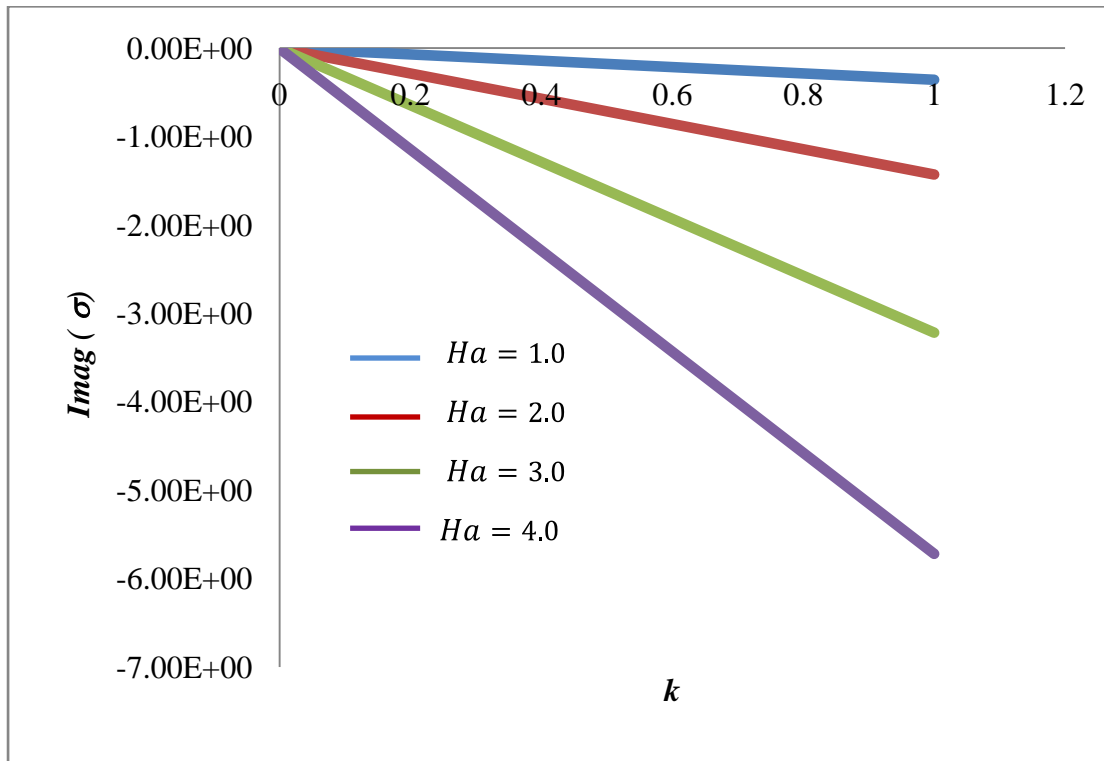


Figure 3. 2. Growth rate as a function of longitudinal wave number for various Ha

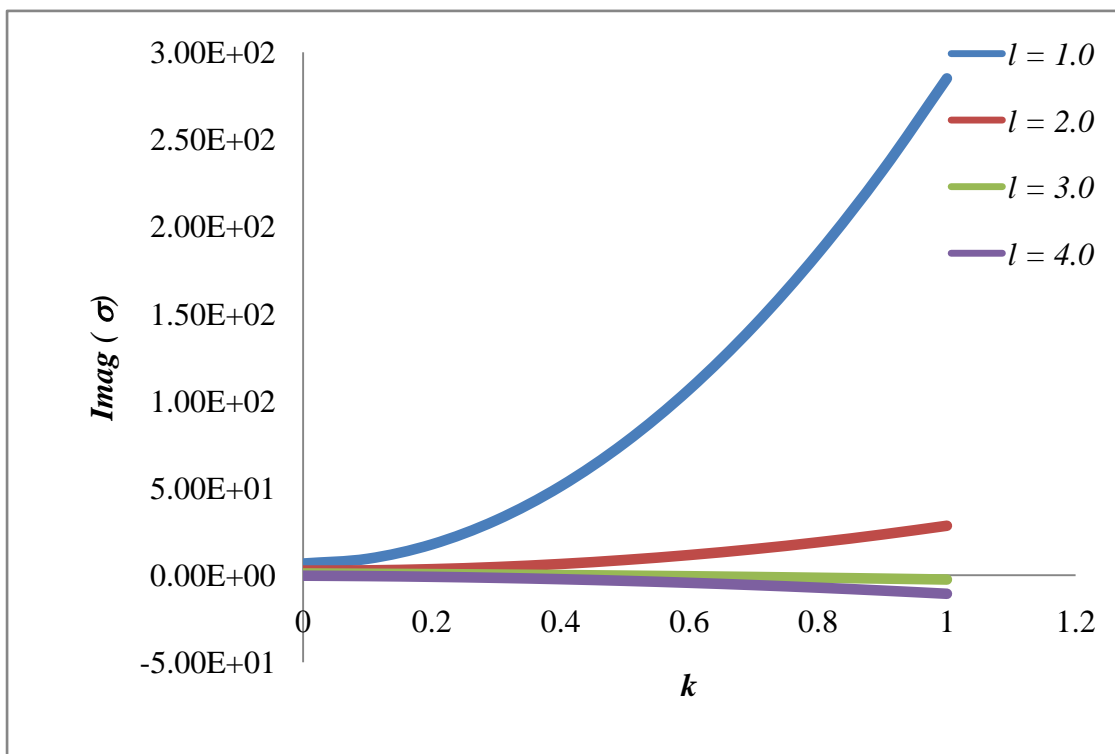


Figure 3. 3. Growth rate as a function of longitudinal wave number for various l

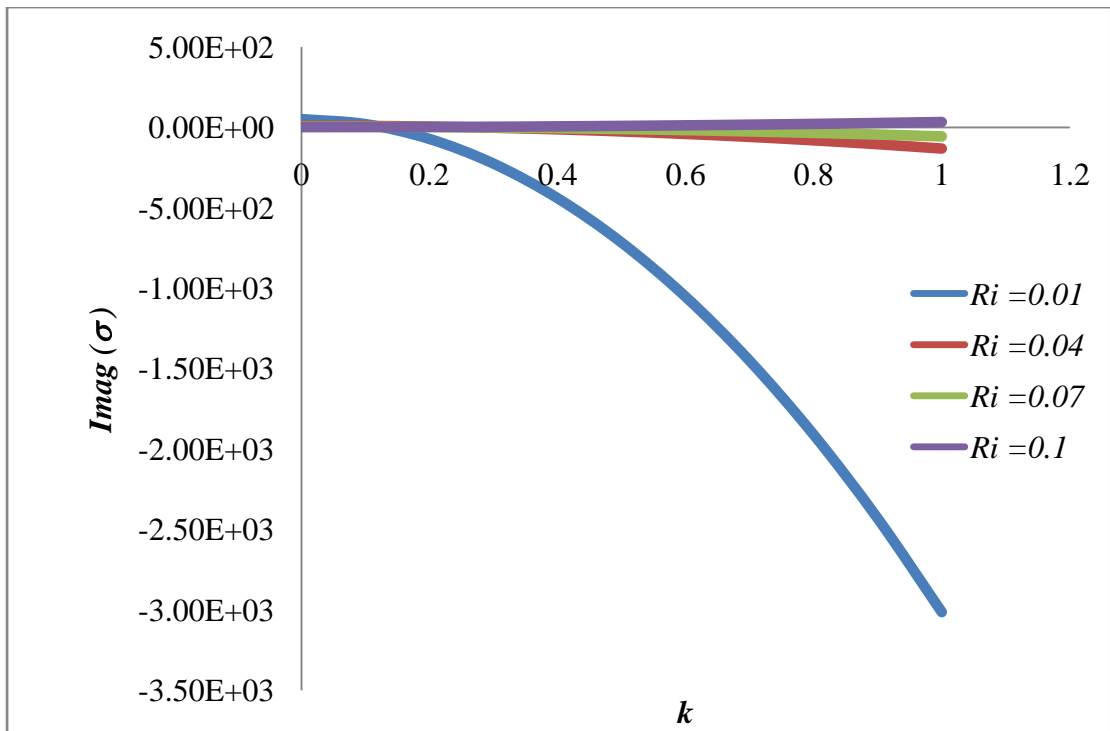


Figure 3. 4. Growth rate as a function of longitudinal wave number for various Ri

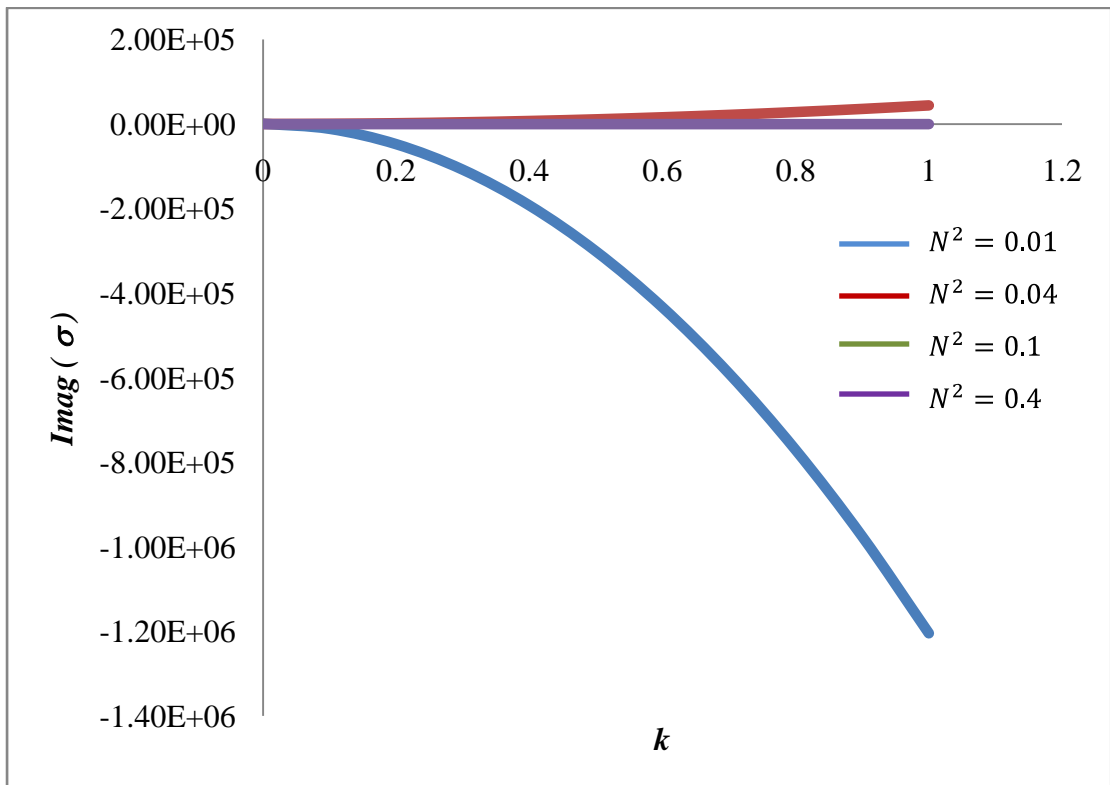


Figure 3. 5 Growth rate as a function of longitudinal wave number for various N^2

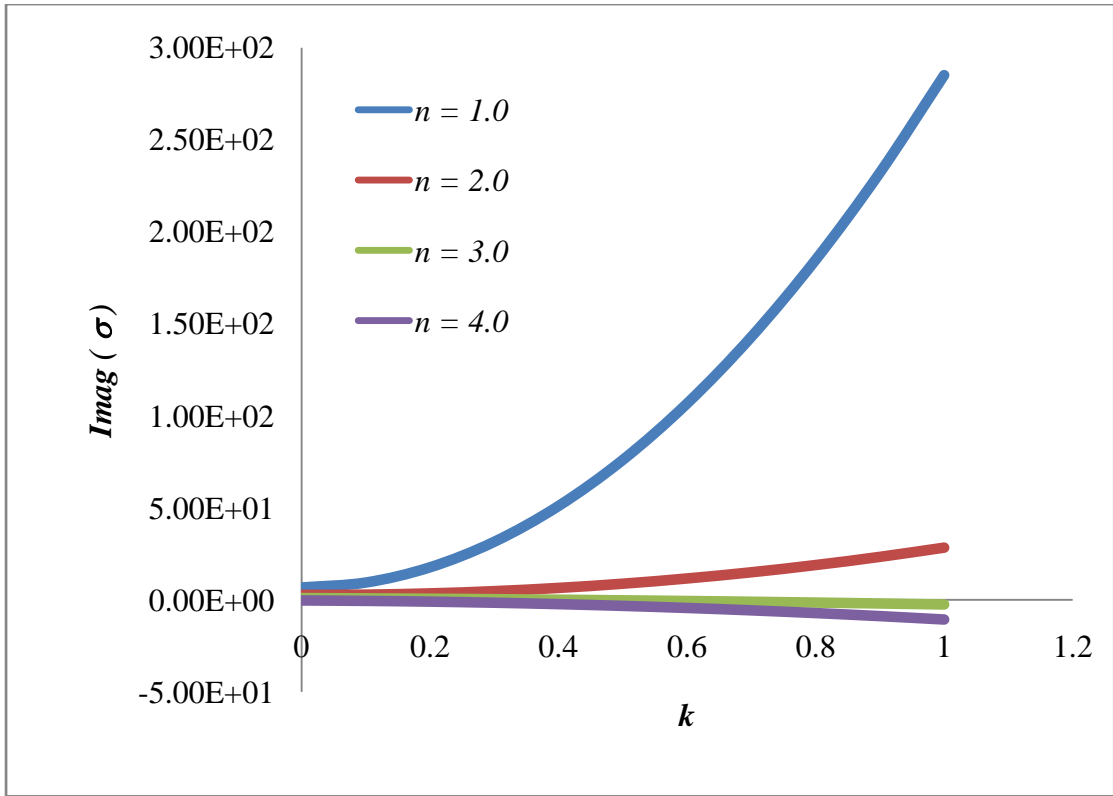


Figure 3. 6. Growth rate as a function of longitudinal wave number for various n

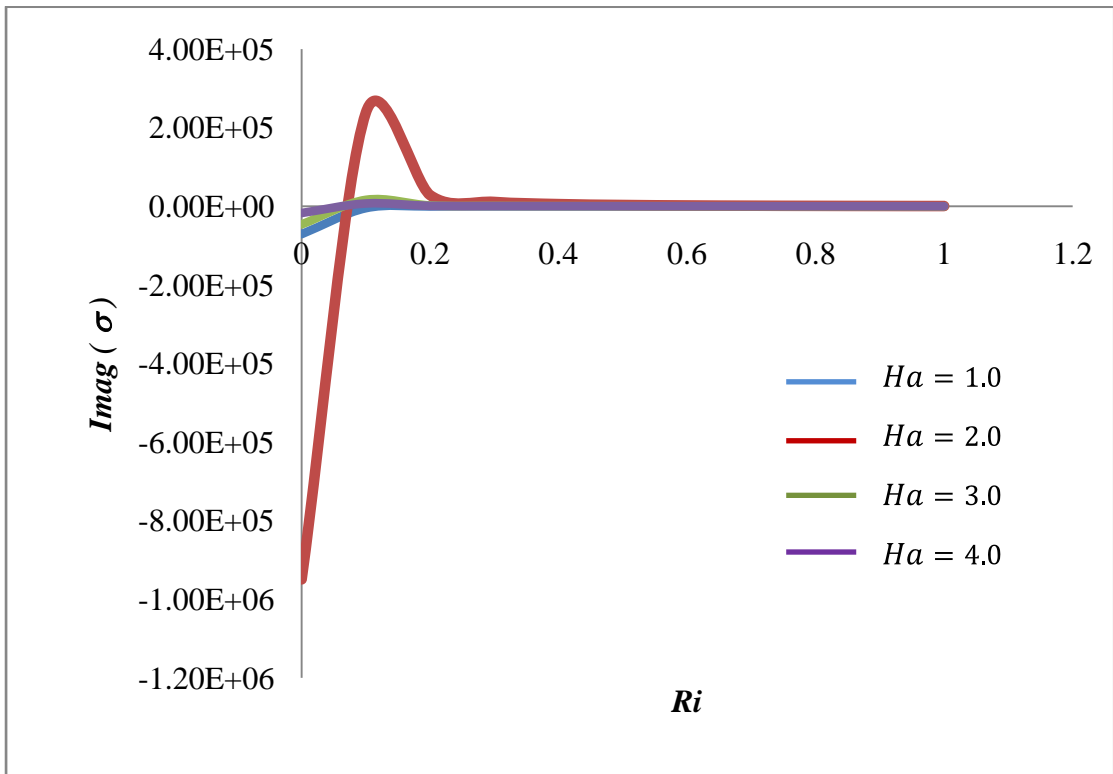


Figure 3. 7. Growth rate as a function of Richardson number for various Ha

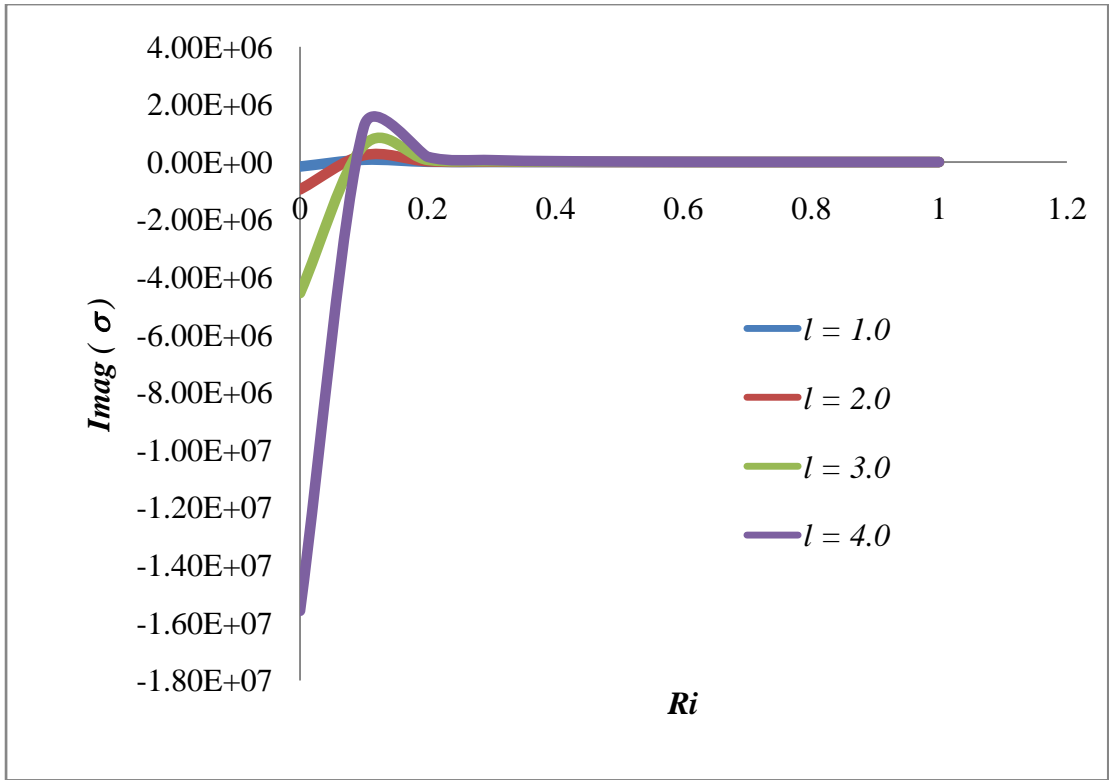


Figure 3. 8. Growth rate as a function of Richardson number for various l

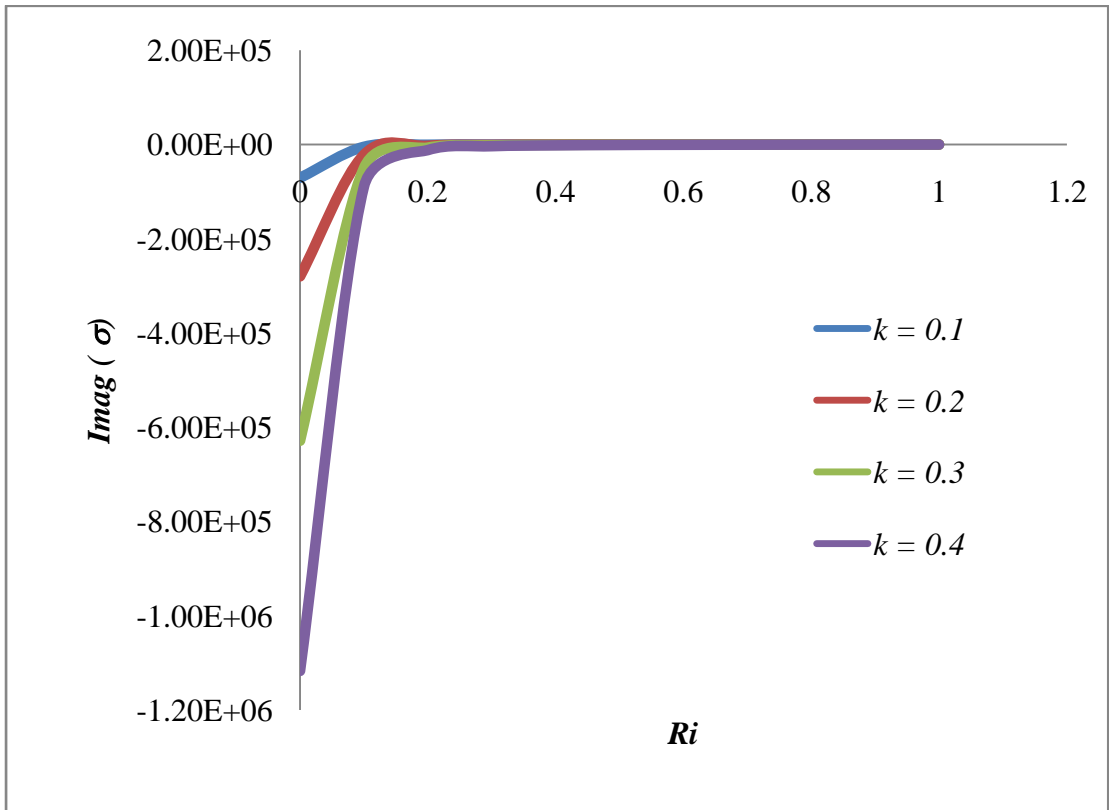


Figure 3. 9. Growth rate as a function of Richardson number for various k

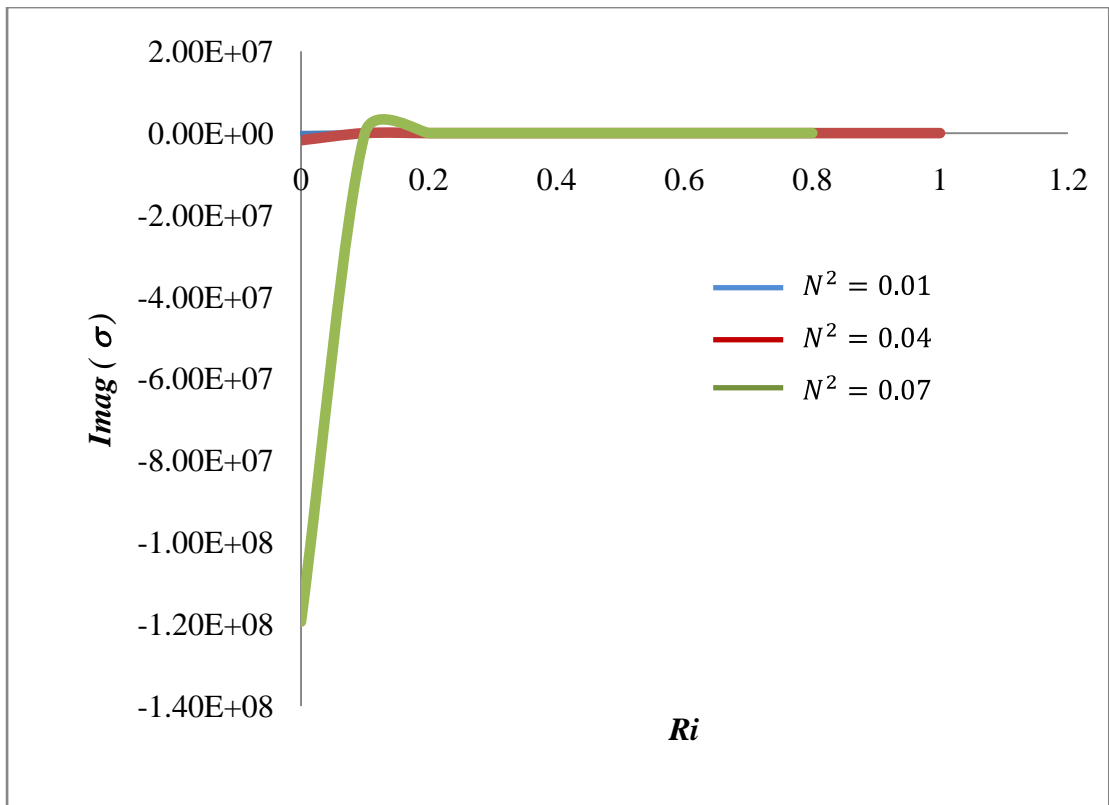


Figure 3. 10. Growth rate as a function of Richardson number for various N^2

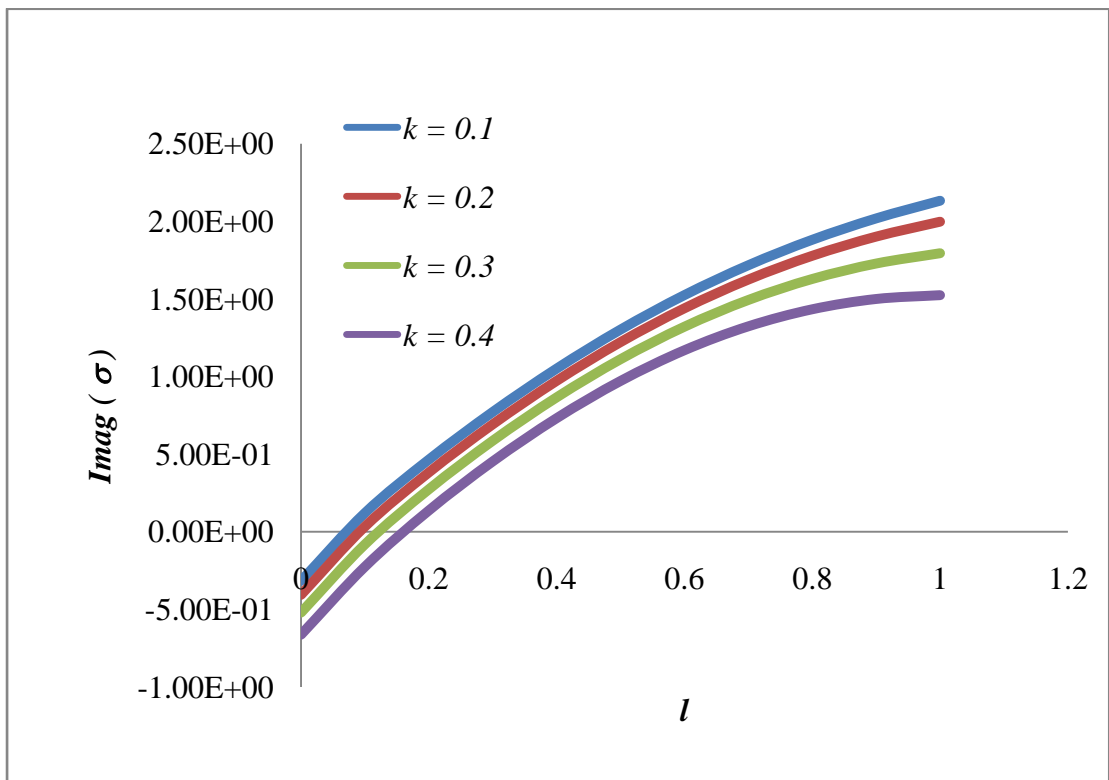


Figure 3. 11. Growth rate as a function of transverse wave number for various k ($Ha = 1.0$)

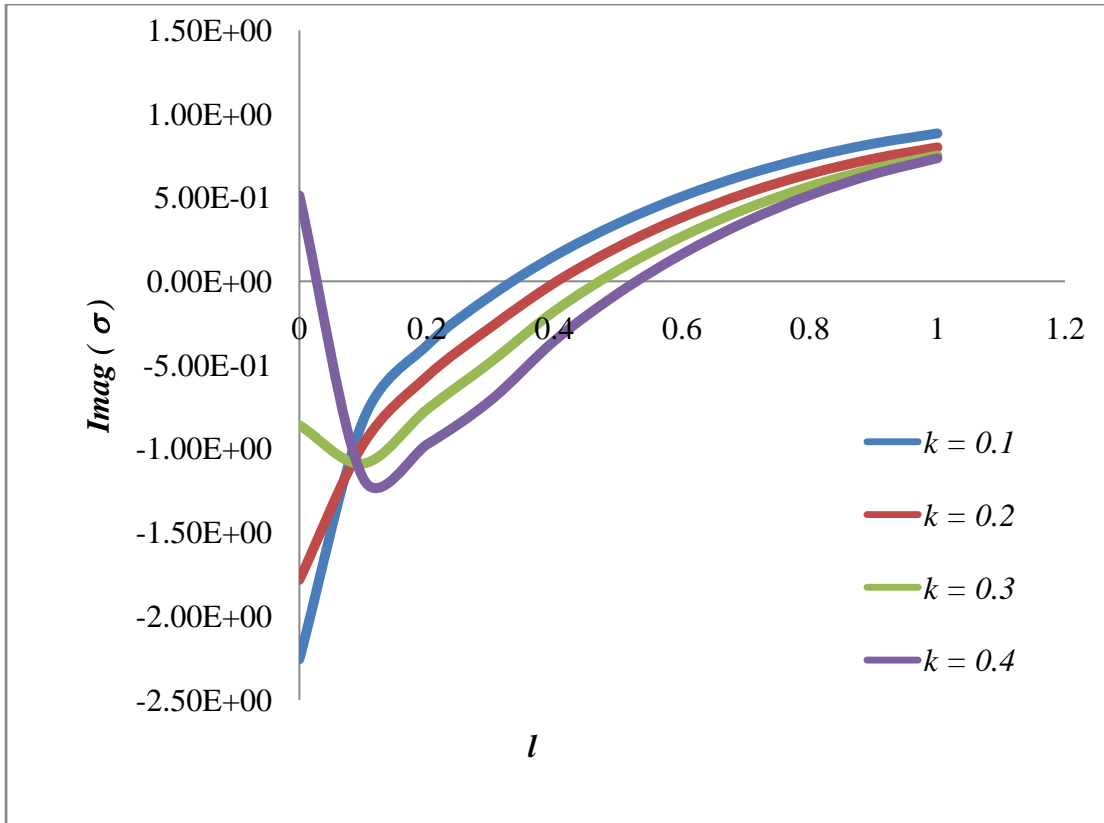


Figure 3.12. Growth rate as a function of transverse wave number for various k ($Ha = 2.0$)

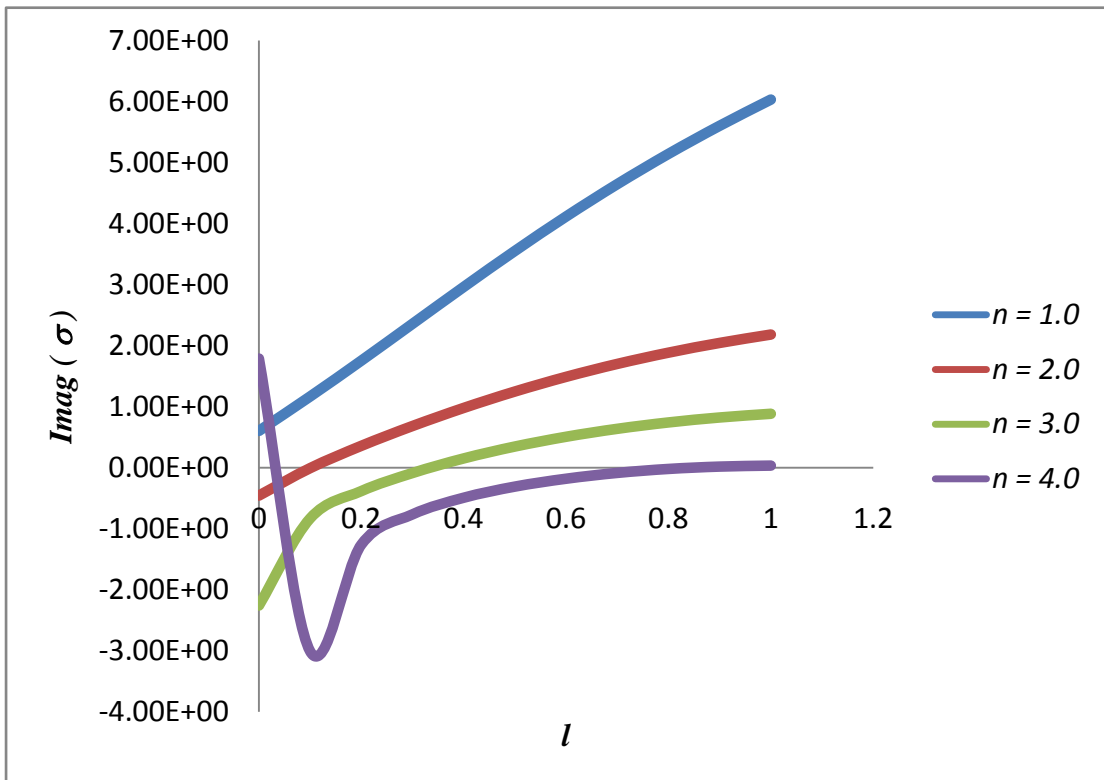


Figure 3.13. Growth rate as a function of transverse wave number for various n

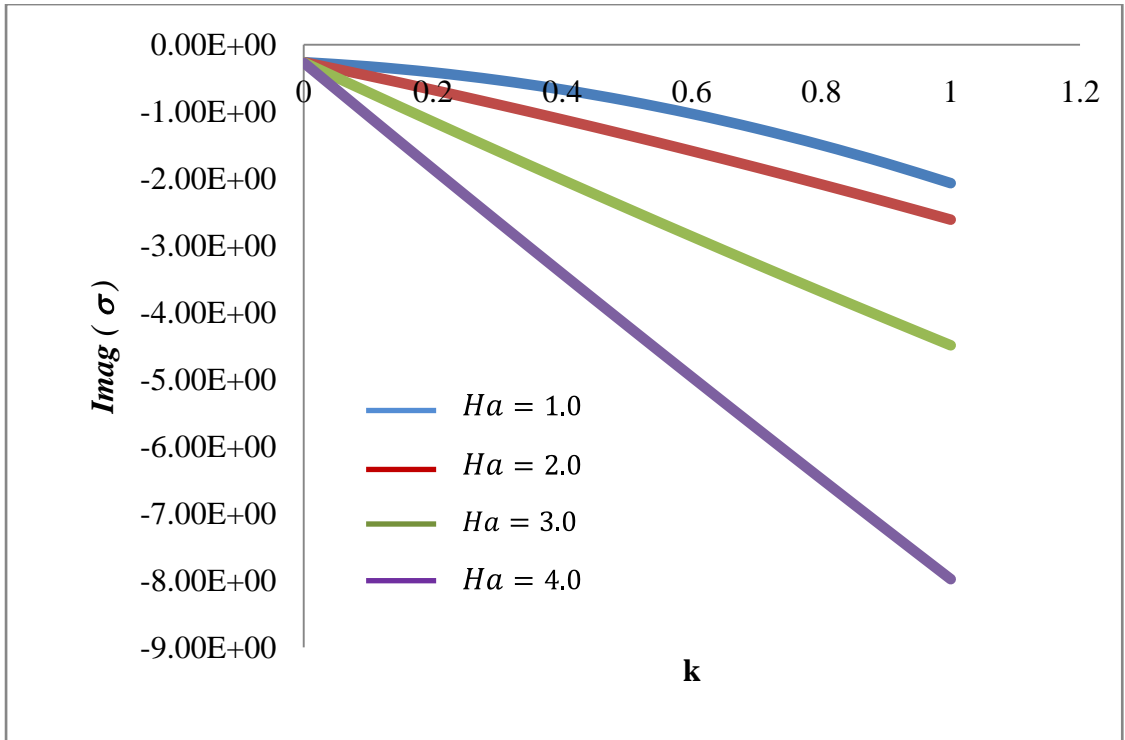


Figure 3.14. Growth rate as a function of longitudinal wave number for various Ha ($l = 0.0$)

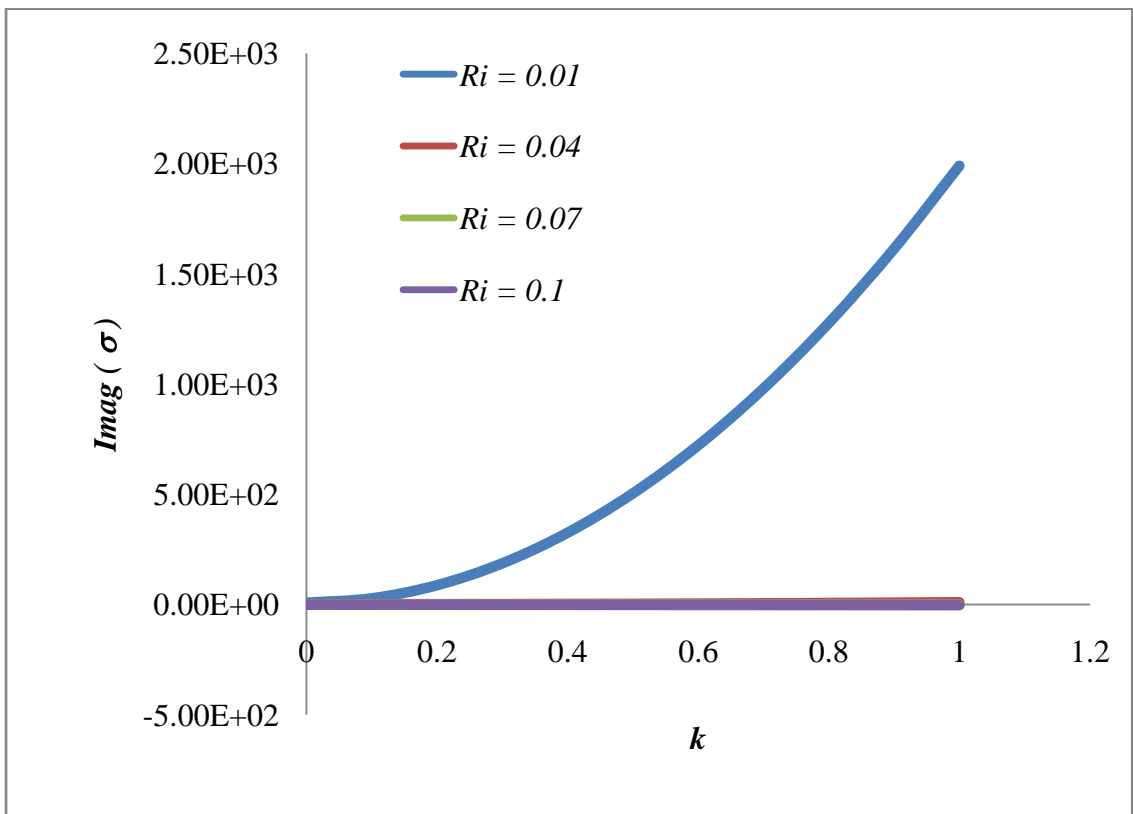


Figure 3.15. Growth rate as a function of longitudinal wave number for various Ri ($l = 0.0$)

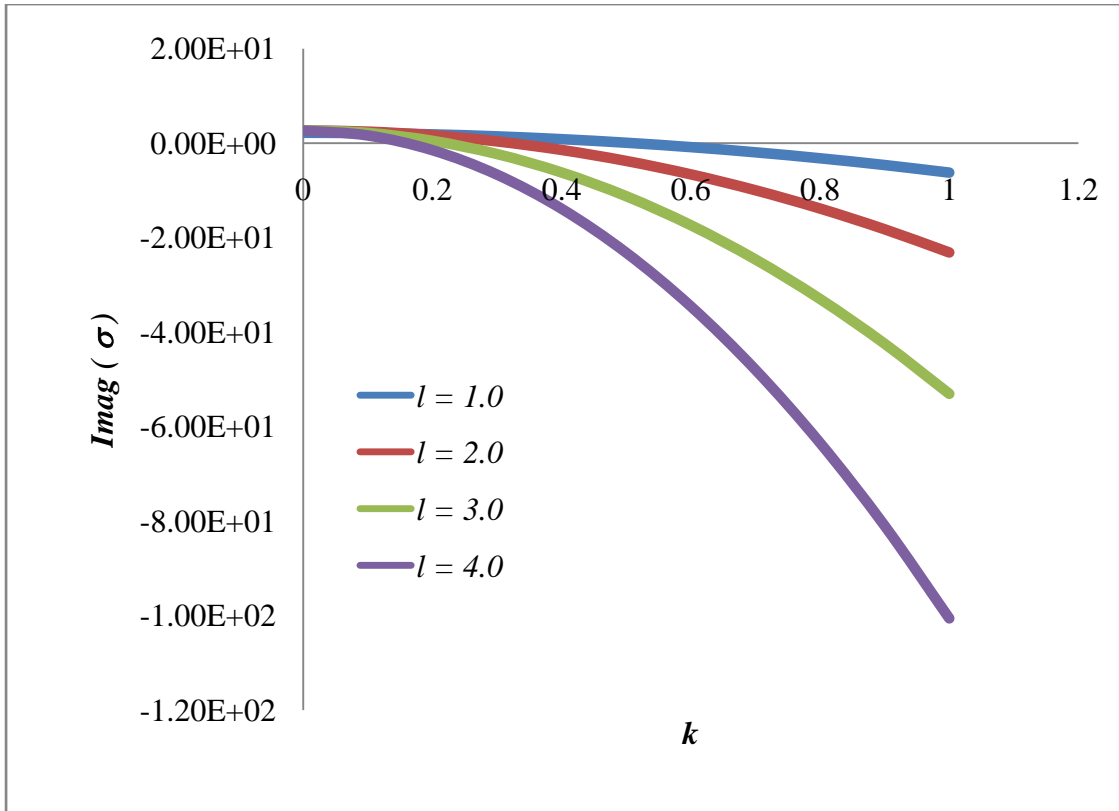


Figure 3.16. Growth rate as a function of longitudinal wave number for various l ($Ha = 0$)

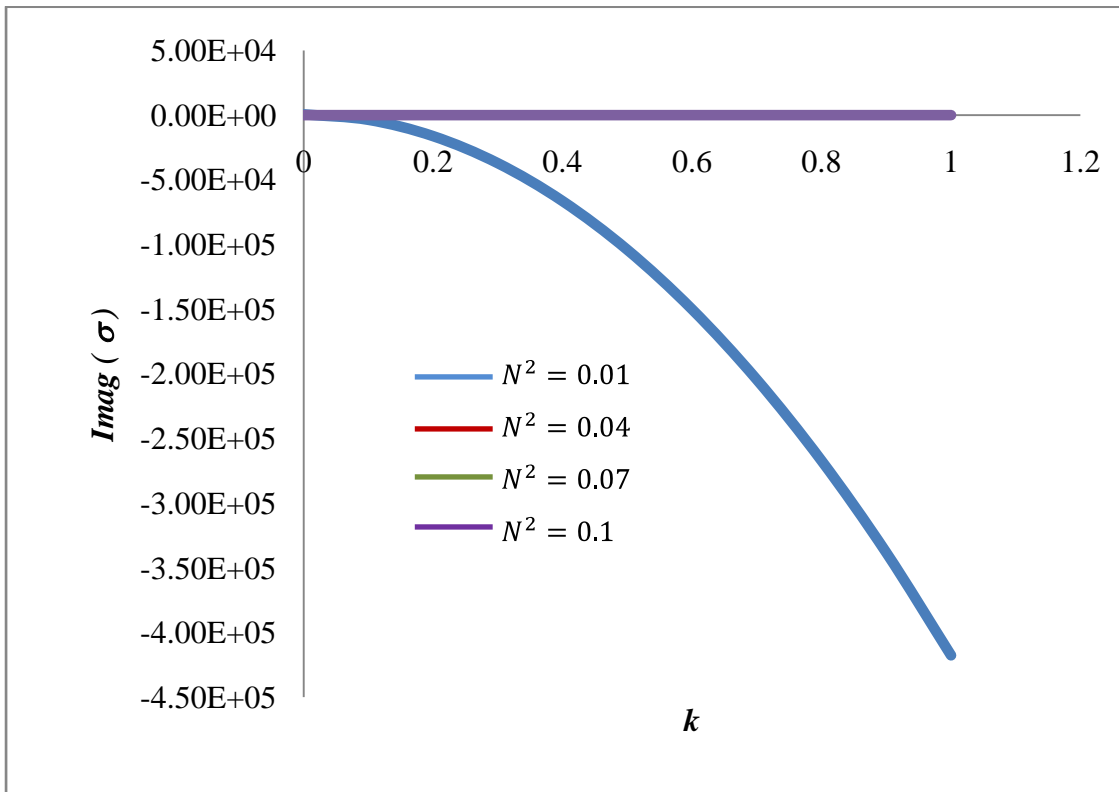


Figure 3.17. Growth rate as a function of longitudinal wave number for various N^2 ($Ha = 0$)

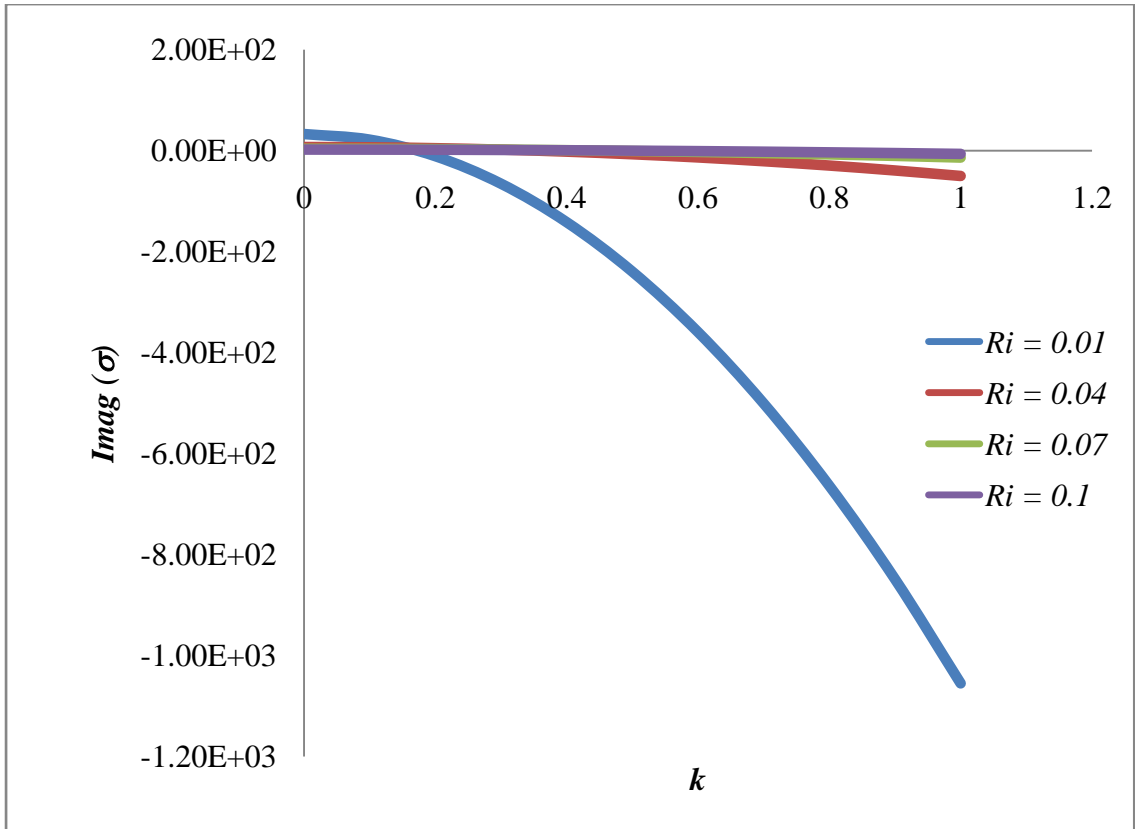


Figure 3.18 Growth rate as a function of longitudinal wave number for various Ri ($Ha = 0$)

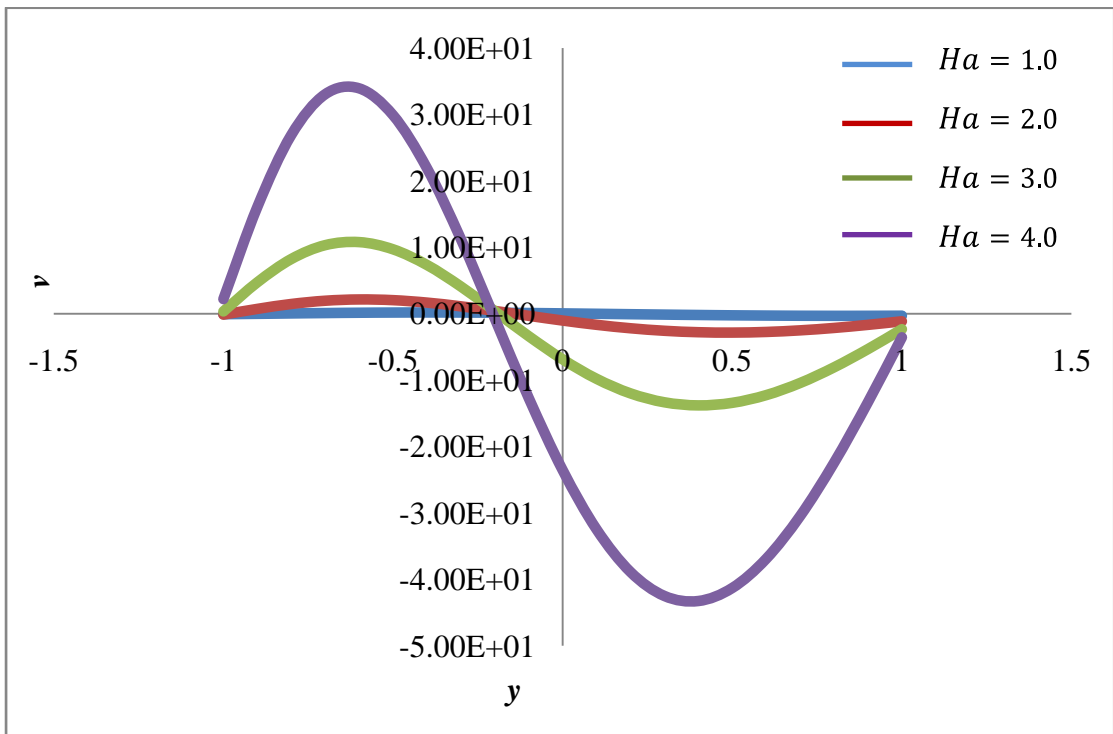


Figure 3.19. Effect of Hartmann number (Ha) on Velocity profile

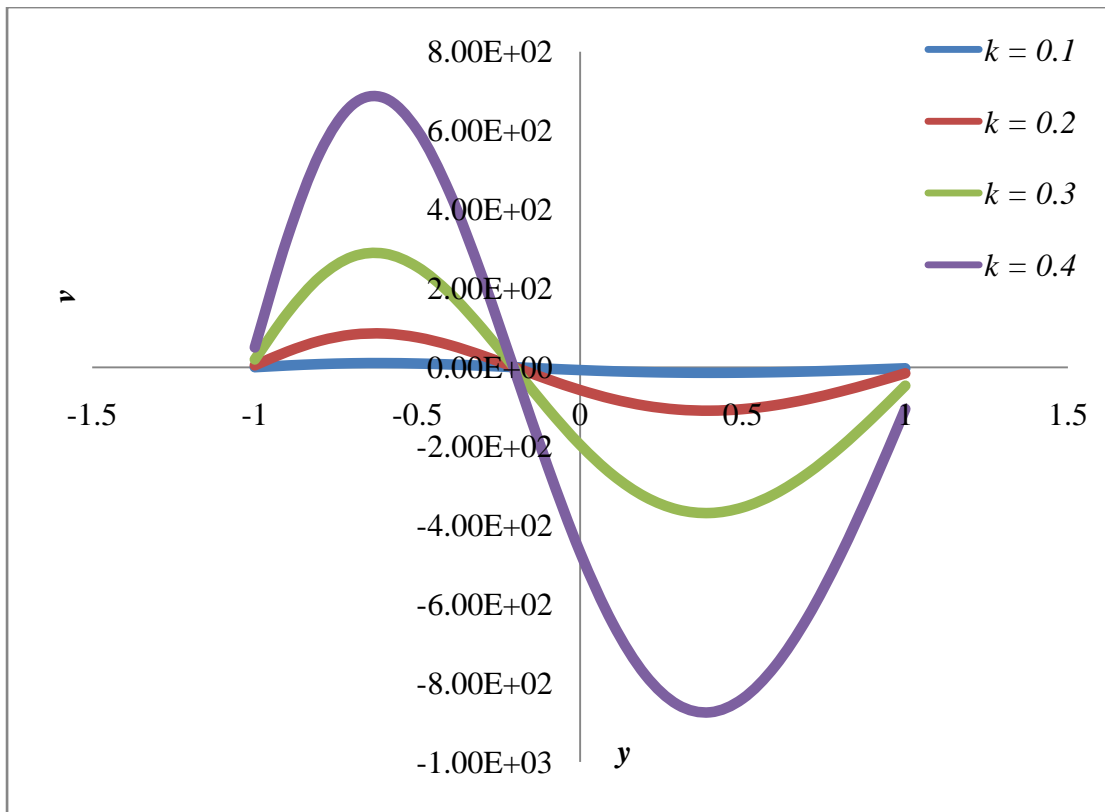


Figure 3. 20. Effect of small wave number (k) on Velocity profile