Chapter V

CHAPTER V

EFFECT OF VARYING MAGNETIC FIELD ON THE LINEAR STABILITY OF PARALLEL STRATIFIED SHEAR FLOWS

5.1. Introduction

The stability analysis of stratified shear flows under the influence of buoyancy forces is of much interest in astrophysical and meteorological phenomena. A detailed account on theoretical and experimental results on the onset of thermal instability of an incompressible viscous fluid under varying assumptions on hydrodynamics and hydomagnetics has been given in the celebrated monograph by Chandrasekar (1961).

The stability of horizontal layer of a perfectly conducting fluid, with continuous density and viscous stratification in the presence of horizontal magnetic field is investigated by Gupta (1963). Kent (1966) has studied the stability of parallel flows varying in the vertical direction with horizontal magnetic field. It was proved that the system becomes unstable when $U_0'' > \left(\frac{A_0'}{U_0'}\right) A_0''$, while in the absence of the magnetic field the system becomes stable. Howard (1961) found eigen values and eigen bounds for unstable waves in a plane parallel flow of an ideal stratified fluid.

The linear stability analysis of parallel shear flows of an inviscid incompressible fluid to infinitesimal two dimensional disturbances is examined by Blumen (1970). Small disturbances of parallel shear flow in an inviscid incompressible fluid of variable density were studied by Miles (1961). Farrell and Ioannou (1993) investigated the transient development of perturbations in inviscid stratified shear flow.

Some general aspects of the stabilizing influence of a parallel magnetic field on plane parallel flow with two-dimensional disturbances are investigated by Drazin (1960). Lock (1955) discussed the stability of hydromagnetic flow between two parallel plates in the presence of uniform magnetic field. Stability of a heterogeneous shear flow in the presence of parallel magnetic field is analyzed by Agarwal and Agarwal (1969). The linear stability of stratified shear flow of a perfectly conducting bounded fluid in the presence of magnetic field under the action of buoyancy forces aligned with the flow is studied by Parthi and Nath (1991).

Gupta (1992) investigated the stability of stratified flow varying in two directions of an incompressible conducting fluid permeated by a uniform magnetic field. It was concluded that a strong magnetic field can stabilize the flow completely with unstable density stratification. The problem on linear stability of inviscid, incompressible non-parallel stratified shear flows to normal mode disturbances was analyzed in detail by Padmini and Subbiah (1995). Stability analysis of simple shear flow of an incompressible fluid with piecewise linear velocity profile in the presence of a magnetic field has been carried out by Ruderman and Brevdo (2006). The stability of incompressible, inviscid density stratified fluid in sea straits of arbitrary cross section is examined by Sridevi and Ganesh (2016). Rajesh Kumar Gupta and Mahinder Singh (2012) considered the Rayleigh- Taylor instability of rotating stratified Rivilin- Ericksen fluids in the presence of variable magnetic field.

The effect of uniform magnetic field on shear flow instabilities was investigated by many authors as mentioned above. In this chapter, the work of Padmini and subbiah (1995) is extended by assuming varying magnetic field. In the presence of varying magnetic field, the linear stability of an inviscid, incompressible parallel stratified shear flow is studied. The analysis is performed using normal mode analysis.

5.2 Mathematical Formulation

An unsteady three dimensional flow of an electrically conducting inviscid, stratified Boussinesq fluid flowing between two infinite horizontal rigid plates at $y = \pm L$ is considered. The fluid is characterized by a shear layer with arbitrary velocity profile. The basic flow is taken as *(U(y), 0, 0)*. Cartesian coordinate system is introduced in such a way that x axis taken in the direction of the flow, y axis is taken in the perpendicular direction. Varying magnetic field is introduced in the x direction given by $\vec{B} = \mu_m(H(y), 0, 0)$.

The assumptions made for the present problem are:

- Flow of an unsteady, inviscid, incompressible Newtonian fluid is considered.
- Stratified shear flow is taken into account
- Fluid is flowing between two horizontal infinite rigid plates at a distance $2L$.

- No slip boundary conditions are imposed at the boundaries.
- Boussinesq approximation is taken into consideration.
- All the fluid properties are assumed constant except the density variation which takes place with vertical coordinate *y* due to stratification.
- The basic velocity profile is assumed as $\vec{q}_e = (U(y),\ 0,\ 0)$.
- Rotation and viscous dissipation effects are neglected.
- The magnetic profile is taken as $\vec{H} = (H(y), 0, 0)$.

Under the above mentioned assumptions the geometry of the flow configuration is shown in Figure.5.1

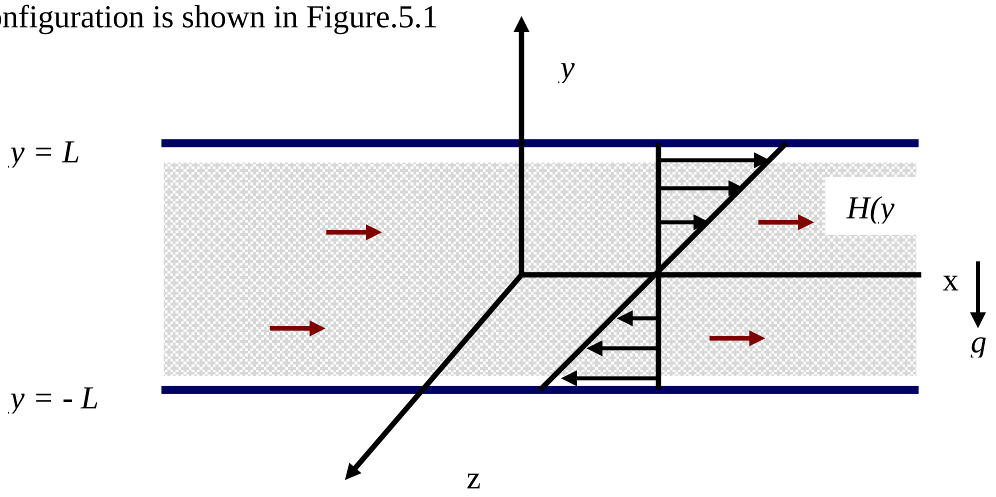

Figure 5.1: Flow configuration

The problem is governed by the following equations

Continuity equation

$$\nabla.\vec{q} \qquad = 0 \tag{5.1}$$

Momentum equation

$$\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = \frac{-\nabla p}{\rho_0} - \frac{\rho g\hat{y}}{\rho_0} + \mu_m(\nabla\times\vec{H})\times\vec{H} \tag{5.2}$$

It is known that the density of the fluid particle moving with the fluid remains unchanged. Hence, the condition for incompressibility is

$$\frac{\partial\rho}{\partial t} + (\vec{q}.\nabla)\rho = 0 \tag{5.3}$$

Magnetic induction equation

$$\frac{\partial\vec{H}}{\partial t} \qquad = \eta\nabla^2\vec{H} + \nabla\times(\vec{q}\times\vec{H}) \tag{5.4}$$

The solenoidal condition for the magnetic field

$$\nabla.\vec{H} \qquad = 0 \tag{5.5}$$

where $\vec{q}$, ρ, p, g, $\vec{H}$, η, μ_m denote respectively the velocity vector, density, pressure, gravitational acceleration, magnetic field vector, magnetic resistivity, magnetic permeability and $\hat{y}$ is the unit vector in the vertical direction. Due to no slip condition velocity vanishes at the plates $y = \pm L$.

At equilibrium, we have

$$p_e' = -\rho_e g - \mu_m H(y) H'(y) \tag{5.6}$$

where prime denotes differentiation with respect to y and $U(y)$, $\rho_e(y)$, $p_e(y)$ and $H(y)$ are continuously differentiable functions of y in the flow domain.

We define the following non-dimensional variables

$$t = \frac{Lt^*}{U_0}, \; p = \rho_0 U_0^2 p^*, \; \rho = \frac{\rho_0 U_0^2 N_0^2}{Lg} \rho^*, \; \vec{H} = H_0 \vec{H}^* \text{ and}$$

$$(x, y, z) = L(x^*, y^*, z^*)$$

where $N^2 = -\frac{g}{\rho_0}\left(\frac{d\rho}{dz}\right)$ is the Brunt-Vaisala frequency which is assumed to be positive for static stability and N_0^2 is the typical value of Brunt-Vaisala frequency in the flow domain, L is the characteristic length and U_0 is the characteristic velocity.

Substitute the above dimensionless quantities in the governing equations, Equationss (5.1) - (5.5) reduce to the form (on removing asterisks)

$$\nabla . \vec{q} = 0 \tag{5.7}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\nabla p - Ri \, g\hat{y} + S(\nabla \times \vec{H}) \times \vec{H} \tag{5.8}$$

$$\frac{\partial \rho}{\partial t} + (\vec{q}.\nabla)\rho = 0 \tag{5.9}$$

$$\frac{\partial \vec{H}}{\partial t} = \frac{1}{Rm} \nabla^2 \vec{H} + \nabla \times (\vec{q} \times \vec{H}) \tag{5.10}$$

$$\nabla . \vec{H} = 0 \tag{5.11}$$

where $S = \frac{\mu_m H_0^2}{\rho U_0^2}$, Magnetic pressure number

$Rm = \frac{LU_0}{\eta}$, Magnetic Reynolds number

$Ri = \frac{g\beta L^2}{\rho_0 U_0^2}$, Richardson number

subject to the boundary condition

$$\vec{q} = 0 \quad \text{on } y = \pm 1 \tag{5.12}$$

To solve equation (5.7) – (5.11), we can take the velocity, density, pressure and magnetic field in a perturbed form

$$(U(y) + u, v, w), \rho_e(y) + \rho, \; p_e(y) + p \text{ and } (H(y) + h_x, \; h_y, \; h_z).$$

Here we incorporate the infinitesimal normal modes of the form $f(z)e^{i(kx+klz-k\sigma t)}$, where *f(z)* is the function of *z* only, *k* and *l* are wave numbers in the *x* and *z* direction respectively and σ is the growth rate of the disturbance which is in general a complex quantity.

Hence, the linearized perturbation equations can be obtained as

$$ik(u+lw)+\frac{\partial v}{\partial y} \quad = 0$$

$$ik(U-\sigma)u+v.\frac{\partial U(y)}{\partial y} = -ik(p+Sh_y)$$

$$ik(U-\sigma)v \quad = -\frac{\partial p}{\partial y} - Ri\,\rho \; + S\left(H(y)\left(ikh_y-\frac{\partial h_x}{\partial y}\right)\right)-h_x$$

$$ik(U-\sigma)w \quad = -ikl(p+SH(y)h_x)$$

$$ik(U-\sigma)\rho-\frac{N^2}{N_0^2}v \quad = 0$$

$$ik(h_x+\; lh_z)+\frac{\partial h_y}{\partial y} \quad = 0$$

$$\left(-ik\sigma-\frac{1}{Rm}\left(-k^2(1+l^2)+\frac{\partial^2}{\partial y^2}\right)\right)h_x = \frac{1}{Rm}\frac{\partial^2(H(y))}{\partial y^2}+\frac{\partial}{\partial y}(Uh_y)-\frac{\partial(v\,H(y))}{\partial y}$$
$$-ikl(H(y)w-Uh_z)$$

$$\left(-ik\sigma-\frac{1}{Rm}\left(-k^2(1+l^2)+\frac{\partial^2}{\partial y^2}\right)\right)h_y = \; ik(H(y)v-Uh_y)$$

$$\left(-ik\sigma-\frac{1}{Rm}\left(-k^2(1+l^2)+\frac{\partial^2}{\partial y^2}\right)\right)h_z = \; ik(H(y)w-\; Uh_z) \qquad (5.13)$$

The corresponding boundary conditions are

$$u = v = w = 0 \qquad \text{on } y = \pm 1 \qquad (5.14)$$

5.3 Eigen Values and Eigen Functions for Long Waves

If $k \ll 1$, it is convenient to solve the problem formulated as in equation (5.13). The flow is assumed to be bounded between two plates $y = \pm 1$. To make the problem mathematically tractable, we consider the linear velocity profile as the basic flow *U(y)* = *y*. From the equilibrium condition, we obtain the magnetic profile as *H(y)* = *y*.

Hence equation (5.13) reduces to the form

$$iku+iklw+\frac{\partial v}{\partial y} \quad = 0$$

$$ik(y-\sigma)u+v.\frac{\partial U(y)}{\partial y} \; = -ik(p+Sh_y)$$

$$ik(y-\sigma)v \quad = -\frac{\partial p}{\partial y} - \; Ri\,\rho \; + \; S\left((1+y)\left(ikh_y-\frac{\partial h_x}{\partial y}\right)\right)-h_x$$

$$ik(y-\sigma)w \quad = -ikl(p+S(1+y)h_x)$$

$$ik(y-\sigma)\rho - \frac{N^2}{N_0^2}v \quad = 0$$

$$ik(h_x + \ lh_z) + \frac{\partial h_y}{\partial y} \quad = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_x = \frac{\partial}{\partial y}\left(y\, h_y\right) - \frac{\partial\left(v\,(1+y)\right)}{\partial y}$$

$$-ikl(Hw - yh_z)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_y = \ ik\left(Hv - yh_y\right)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left(-k^2(1+l^2) + \frac{\partial^2}{\partial y^2}\right)\right)h_z = \ ik(Hw - yh_z) \qquad (5.15)$$

Inorder to solve equation (5.15), we assume the series expansions with respect to the wave number *k* in the form

$$f = f_0 + kf_1 + k^2 f_2 + \cdots \qquad (5.16)$$

where f represents the disturbances $u, v, w, \sigma, \rho, h_x, h_y \ or \ h_z$

Using the equation (5.16) into equation (5.15) and equating the coefficients of like powers of *k*, we get the following set of zeroth order differential equations

$$iu_0 + ilw_0 + \frac{\partial v_0}{\partial y} \quad = 0$$

$$iR(y)u_0 + v_0 \quad = -ip_0$$

$$-\frac{\partial p_0}{\partial y} - Ri\,\rho_0 \quad = 0$$

$$iR(y)w_0 \quad = -ilp_0$$

$$iR(y)\rho_0 - \frac{N^2}{N_0^2}v_0 \quad = 0 \qquad (5.17)$$

$$ih_{x0} + ilh_{z0} + \frac{\partial h_{y0}}{\partial y} \quad = 0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{x0}}{\partial y^2}\right) \ = -v_0 - (1+y)\frac{\partial v_0}{\partial y} - il\left((1+y)w_0\right)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y0}}{\partial y^2}\right) \ = \ i(1+y)v_0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{z0}}{\partial y^2}\right) \ = \ i(1+y)w_0 \qquad (5.18)$$

where $R(y) = y - \sigma_0$

The first order equations are

$$iu_1 + ilw_1 + \frac{\partial v_1}{\partial y} \quad = 0$$

$$-i\sigma_1 u_0 + iR(y)u_1 + v_1 \quad = -ip_1 + Sh_{y0}$$

$$\frac{\partial p_1}{\partial y} + Ri\,\rho_1 = S\left((1+y)\frac{\partial h_{x0}}{\partial y} + Bh_{x0}\right)$$

$$-i\sigma_1 w_0 + iR(y)w_1 = -ilp_1 - Slh_{x0}(1+y)$$

$$iR(y)\rho_1 - i\sigma_1\rho_0 - \frac{N^2}{N_0^2}v_1 = 0 \tag{5.19}$$

$$ih_{x1} + ilh_{z1} + \frac{\partial h_{y1}}{\partial y} = 0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{x1}}{\partial y^2}\right) = i\sigma_0 h_{x0} + h_{y0} + y\frac{\partial h_{y0}}{\partial y} - v_1 - (1+y)\frac{\partial v_1}{\partial y}$$
$$-l\big(w_1 + y(w_1 - h_{z0})\big)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y1}}{\partial y^2}\right) = i(1+y)v_1 - i(R(y))h_{y0}$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{z1}}{\partial y^2}\right) = i(1+y)w_1 - iR(y)h_{z0} \tag{5.20}$$

The relevant boundary conditions reduces to

$$u_0 = u_1 = 0,\;\; v_0 = v_1 = 0,\;\; w_0 = w_1 = 0 \tag{5.21}$$

Eliminating $\rho_0,\ p_0, u_0, w_0$ in favour of v_0 from equation (5.17) we obtain

$$R(y)^2\frac{\partial^2 v_0}{\partial y^2} + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)v_0 = 0 \tag{5.22}$$

In the same way, equation (5.19) is simplified to find v_1.

$$R(y)^2\frac{\partial^2 v_1}{\partial y^2} + \frac{Ri\,N^2}{{N_0}^2}(1+l^2)v_1 = \sigma_1\left(R(y)^2\frac{\partial^2 v_0}{\partial y^2} - Ri\,(1+l^2)\,i\,\rho_0\right)$$
$$-S\,R(y)\left(\frac{\partial\, h_{y0}}{\partial y} - \big(l^2 - i\,(1+l^2)\big)\left(\,(1+y)\frac{\partial h_{xo}}{\partial y} + h_{x0}\right)\right) \tag{5.23}$$

The solution of equation (5.22) is given as

$$v_0 = \begin{cases} C\,R(y)^{m_1} + D\,R(y)^{m_2}, & \lambda > 0 \\ R(y)^{\frac{1}{2}}(E + F\,R(y)), & \lambda = 0 \\ R(y)^{\frac{1}{2}}\begin{pmatrix} G\cos\big(k\log(R(y))\big) \\ +H\sin\big(k\log(R(y))\big)\end{pmatrix}, & \lambda < 0 \end{cases}$$

where $m_{1,2} = \frac{1\pm\sqrt{\lambda}}{2}$, $\lambda = 1 - 4\,Ri\frac{N^2}{N_0^2}(1+l^2)$, k = - λ, C, D, E, F, G and H are arbitrary constants.

By applying the boundary condition that $v_0 = 0\ \ at$ y = ± 1, we obtain the value of σ_0 as

$$\sigma_0 = \begin{cases} \dfrac{1+e^{\frac{2n\pi i}{m_1-m_2}}}{1-e^{\frac{2n\pi i}{m_1-m_2}}}, & \lambda \geq 0 \\ \dfrac{1+e^{\frac{n\pi}{k}}}{1-e^{\frac{n\pi}{k}}}, & \lambda < 0 \end{cases} \tag{5.24}$$

The solution of equations (5.17) and (5.18) are obtained by

$$u_0 = \begin{cases} C_5 R(y)^{m_1-1} + C_6 R(y)^{m_2-1}, & \lambda \geq 0 \\ iR(y)^{-\frac{1}{2}} \begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)\left(1 - \frac{kH-\frac{1}{2}}{1+l^2}\right) \\ -\sin\left(k\log(R(y))\right)\left(\frac{k+\frac{H}{2}}{1+l^2} + H\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$v_0 = \begin{cases} R(y)^{m_1} + DR(y)^{m_2}, & \lambda \geq 0 \\ R(y)^{\frac{1}{2}}\left(\cos\left(k\log\left(R(y)\right)\right) + F\sin\left(k\log(R(y))\right)\right), & \lambda < 0 \end{cases}$$

$$w_0 = \begin{cases} C_7 R(y)^{m_1-1} + C_8 R(y)^{m_2-1}, & \lambda \geq 0 \\ \frac{-lR(y)^{-\frac{1}{2}}}{i(1+l^2)} \begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)\left(kH - \frac{1}{2}\right) \\ +\sin\left(k\log(R(y))\right)\left(k + \frac{H}{2}\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$p_0 = \begin{cases} C_3 R(y)^{m_1} + C_4 R(y)^{m_2}, & \lambda \geq 0 \\ \frac{R(y)^{\frac{1}{2}}}{i(1+l^2)} \begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)\left(kH - \frac{1}{2}\right) \\ +\sin\left(k\log(R(y))\right)\left(k + \frac{H}{2}\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$\rho_0 = \begin{cases} C_1 R(y)^{m_1-1} + C_2 R(y)^{m_2-1}, & \lambda \geq 0 \\ R(y)^{-\frac{1}{2}} \frac{N^2}{iN_0^2} \begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right) \\ +H\sin\left(k\log(R(y))\right) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$h_{x0} = \begin{cases} Rm\begin{pmatrix} (A+By)(C_9\, R(y)^{m_1+1} + C_{10}\, R(y)^{m_2+1}) \\ +C_{11}\, R(y)^{m_1+2} + C_{12}\, R(y)^{m_2+2} \end{pmatrix}, & \lambda \geq 0 \\ Rm\begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)\left(R(y)^{\frac{5}{2}}c_{56} + R(y)^{\frac{3}{2}}c_{57}\right) \\ +\sin\left(k\log(R(y))\right)\left(R(y)^{\frac{5}{2}}c_{58} + R(y)^{\frac{3}{2}}c_{59}\right) \end{pmatrix} & \lambda < 0 \end{cases}$$

$$h_{y0} = \begin{cases} Rm\begin{pmatrix} (A+By)(C_{13}\, R(y)^{m_1+2} + C_{14}\, R(y)^{m_2+2}) \\ +C_{15}\, R(y)^{m_1+3} + C_{16}\, R(y)^{m_2+3}) \end{pmatrix}, & \lambda \geq 0 \\ Rm\, R(y)^{\frac{5}{2}}\begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)(c_{64}y + c_{65}) \\ +\sin\left(k\log\left(R(y)\right)\right)(c_{66}y + c_{67}) \end{pmatrix}, & \lambda < 0 \end{cases}$$

$$h_{z0} = \begin{cases} Rm\begin{pmatrix} (A+By)(C_{17}\, R(y)^{m_1+1} + C_{18}\, R(y)^{m_2+1}) \\ +C_{19}\, R(y)^{m_1+2} + C_{20}\, R(y)^{m_2+2} \end{pmatrix}, & \lambda \geq 0 \\ Rm\, R(y)^{\frac{3}{2}}\begin{pmatrix} \cos\left(k\log\left(R(y)\right)\right)(c_{72}y + c_{73}) \\ +\sin\left(k\log(R(y))\right)(c_{74}y + c_{75}) \end{pmatrix}, & \lambda < 0 \end{cases} \quad (5.25)$$

By imposing the boundary condition that $v_1(\pm 1) = 0$ we get the first order approximation of σ as

$$\sigma_1 = \begin{cases} \frac{S\,Rm\,C_{51}}{Ri\,C_{80} - C_{49}}, & \lambda \geq 0 \\ \frac{-S\,Rm\,C_{109}}{C_{108}}, & \lambda < 0 \end{cases} \tag{5.26}$$

For the sake of brevity the constants are given in *Appendix III*.

5.4 Results and Discussion

The purpose of this study is to analyze the linear stability of parallel stratified shear fluid due to the presence of varying magnetic field. To obtain the physical insight of the problem, a comprehensive numerical computation is performed for various values of dimensionless parameters that describe the stability characteristics, and the results are reported in terms of Figures. (5.2) – (5.14). Imaginary part of growth rate *(σ)* as a function of wave number *(k)* is plotted in Figures. (5.2) - (5. 5) when $\lambda > 0$.

Figure (5.2) represents the growth rate (*σ*) against wave number (*k)* for various values of Magnetic Reynolds number (*Rm*). It is noticed that with the increase in magnetic Reynolds number, the growth rate increases with increasing wave number (*k)*. Figure (5.3) depicts the growth rate (*σ)* against wave number (*k*) for various Magnetic pressure number (*S*). It is observed that, the growth rate (*σ*) increases with increasing values of Magnetic pressure number (*S*).

The effect of longitudinal wave number (*l*) on the growth rate (*σ*) is given in Figure (5.4). It is seen from the figure that, increasing longitudinal wave number (*l*) results in unstable growth rate. From the above figures we conclude that, the frequency of disturbances are unstable with increasing magnetic Reynolds number (*Rm*), Magnetic pressure number (*S*) and longitudinal wave number (*l*). From the analytical expression derived for σ_0, it can be concluded that infinite number of modes exists, both stable and unstable corresponding to the values of n. Few modes are depicted in Figure (5.5).

Figure (5.6) shows the relationship between growth rate (*σ*) and Brunt-Vaisala frequency (N^2). It is understood that with the increase in Brunt-Vaisala frequency the frequency of disturbance increases upto certain level and then decreases thereby the system becomes unstable when $\lambda > 0$. The relation between growth rate (*σ*) and wave number (k) for various parameters are shown through Figures (5.7) – (5.11) when $\lambda < 0$. Figure (5.7) is graphed to see the influence of Magnetic Reynolds number (Rm) on the growth rate (σ). It is observed that, increase in Magnetic Reynolds number (Rm) increases the growth rate with the increase in wave number k. Therefore, the flow becomes unstable.

Figure (5.8) depicts the behavior of growth rate (σ) for different values of Magnetic Pressure Number (S). It is noted that, growth rate (σ) increases with the increase in Magnetic Pressure Number (S) thereby enhances the region of instability. Figure (5.9) depicts the growth rate (σ) as a function of the wave number (k) with various n.

Figure (5.10) presents the relation between growth rate (σ) and longitudinal wave number (l). It is observed that, increase in longitudinal wave number (l) decreases the growth rate (σ). The flow becomes stable. Figure (5.11) shows the growth rate (σ) as a function of wave number (k) for various Richardson number (Ri). It is known that increasing Richardson number (Ri) destabilize the nature of the flow.

Growth rate (σ) as a function of Magnetic Reynolds number (Rm) is exhibited in Figure (5.12) when $\lambda < 0$. From this figure, we observe that, with increasing Richardson number (Ri) growth rate increases and hence the flow becomes unstable. The velocity profile for various nondimensional parameters is shown in Figures (5.13) and (5.14). It is noted that the velocity decreases with the increase of Magnetic Reynolds number (Rm) and Magnetic pressure number (S).

5.5 Conclusion

The linear stability analysis of an inviscid, parallel stratified shear fluid in the presence of varying magnetic field is analyzed in this chapter. The governing equations of the flow coincide with those obtained by Padmini and Subbiah (1995) when $Rm = 0$. The stability of the flow is analyzed using the normal mode approach and the analysis is restricted to long wave approximation. The effect of various nondimensional numbers like Magnetic pressure number (S), Magnetic Reynolds number (Rm), longitudinal wave number (l), transverse wave number (k), Brunt Vaisala frequency (N^2) and Richardson number (Ri) on the stability of parallel shear flow confined between the plates $y = \pm 1$ is discussed.

The salient features of the present study are listed below

- Richardson number (*Ri*) plays a crucial role in the stability of parallel stratified shear flows.
- Increase in wave number (*k*) increases the growth rate (σ) for varying Magnetic pressure number (*S*) and Magnetic Reynolds number (*Rm*) when $\lambda > 0$ thereby destabilizes the flow.
- Increase in transverse wave number (*l*) destabilizes the fluid flow.
- The flow become unstable with the increase in Brunt Vaisala frequency (N^2) when $\lambda > 0$.
- Growth rate (σ) increases for varying Magnetic pressure number (*S*) and Magnetic Reynolds number (*Rm*) thereby destabilizes the flow. In the case of an increase in transverse wave number (*l*) the growth rate (σ) decreases and the flow becomes stable when $\lambda < 0$.
- Increase in Richardson number (*Ri*) destabilizes the flow as Magnetic Reynolds number (*Rm*) increases.

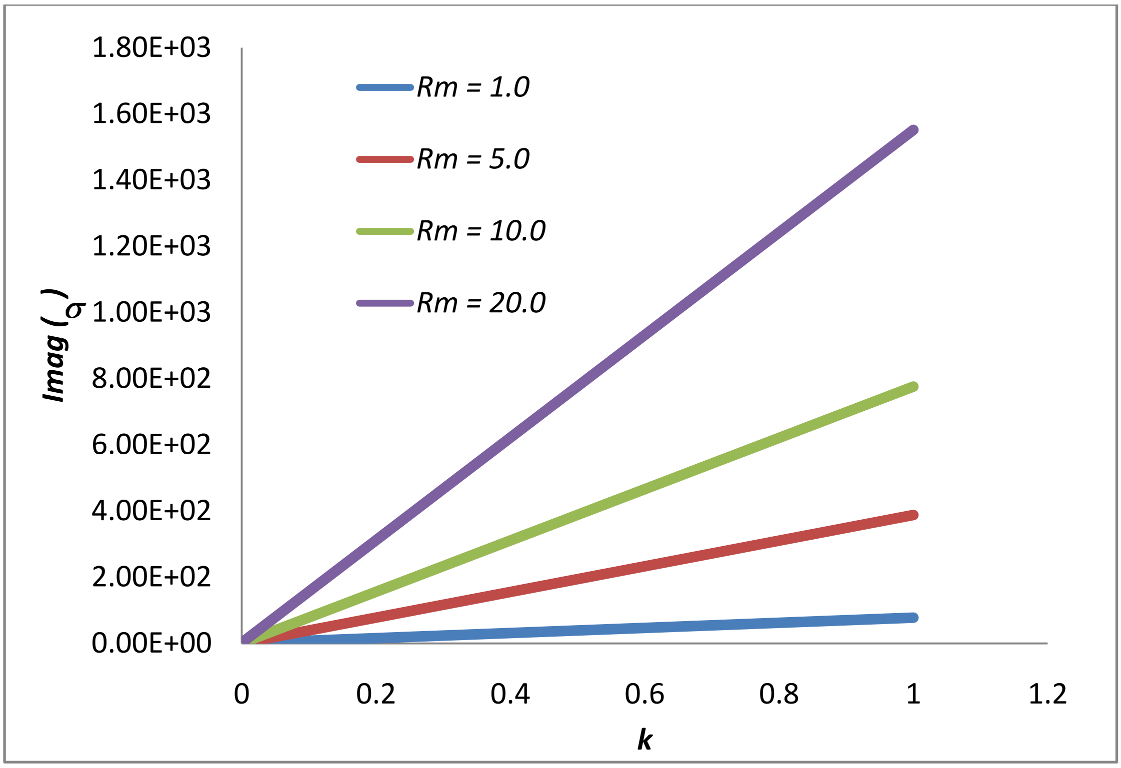

Figure 5.2 Growth rate as a function of wave number for various Rm

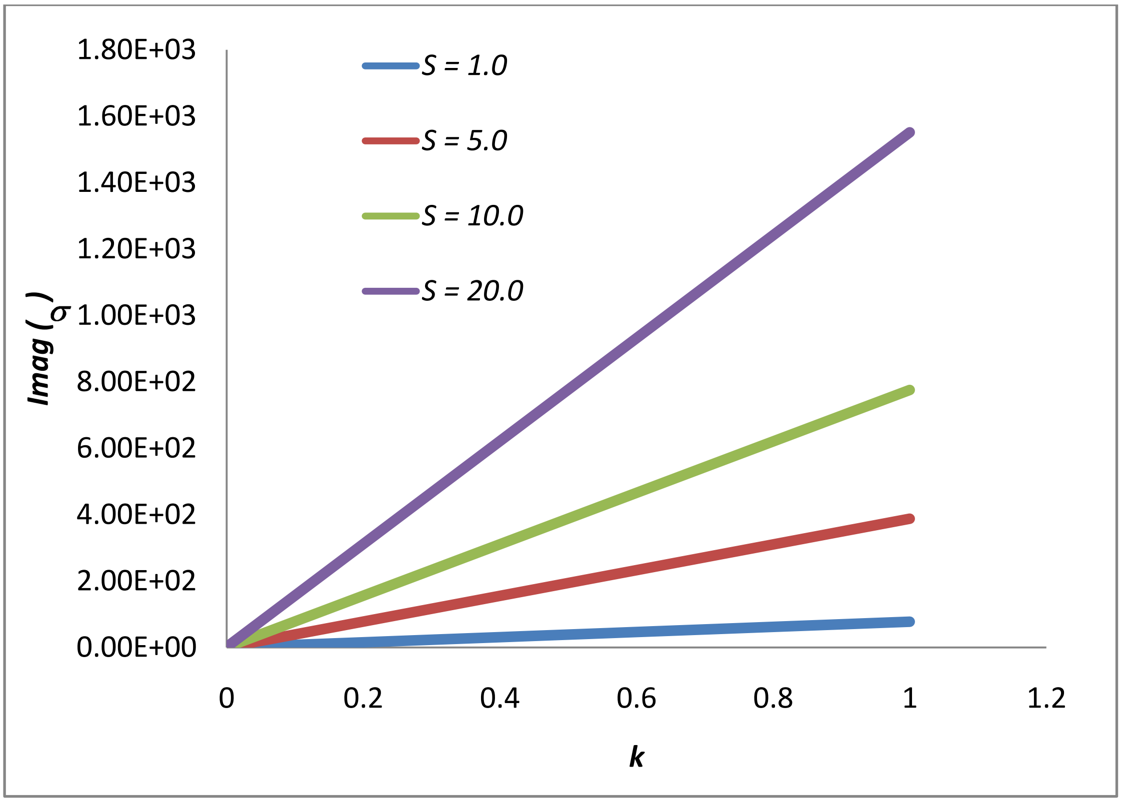

Figure 5. 3. Growth rate as a function of wave number for various S

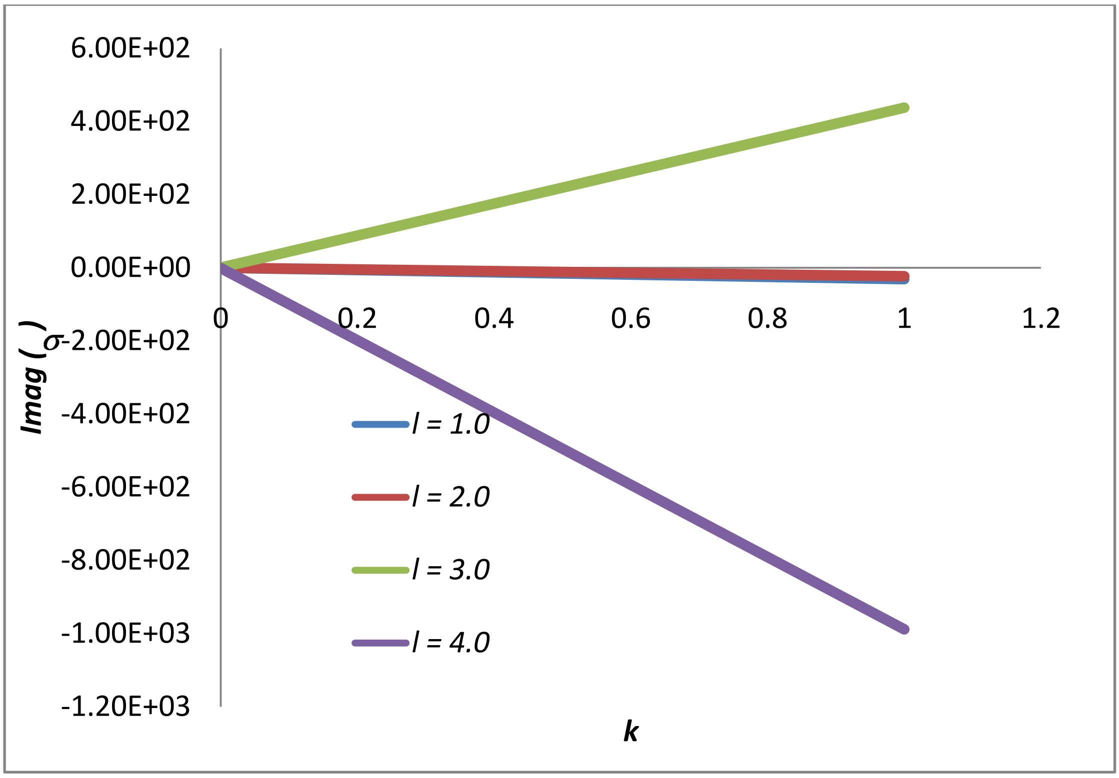

Figure 5. 4. Growth rate as a function of wave number for various l

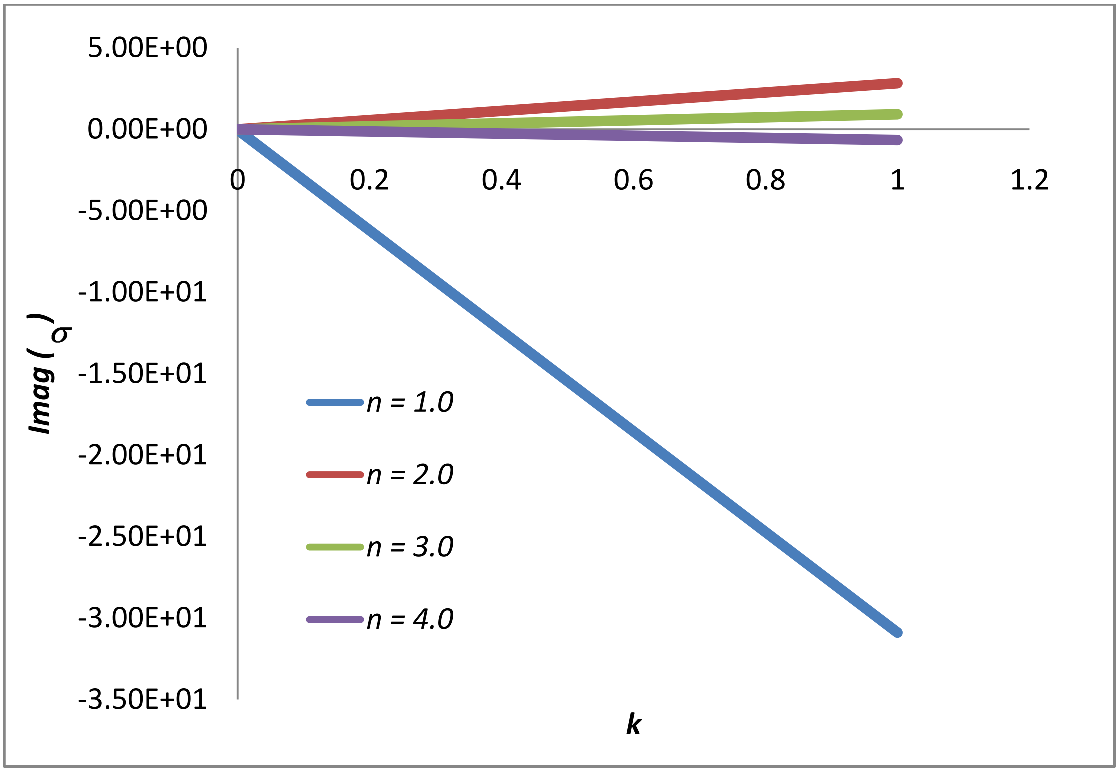

Figure 5. 5. Growth rate as a function of wave number for various n

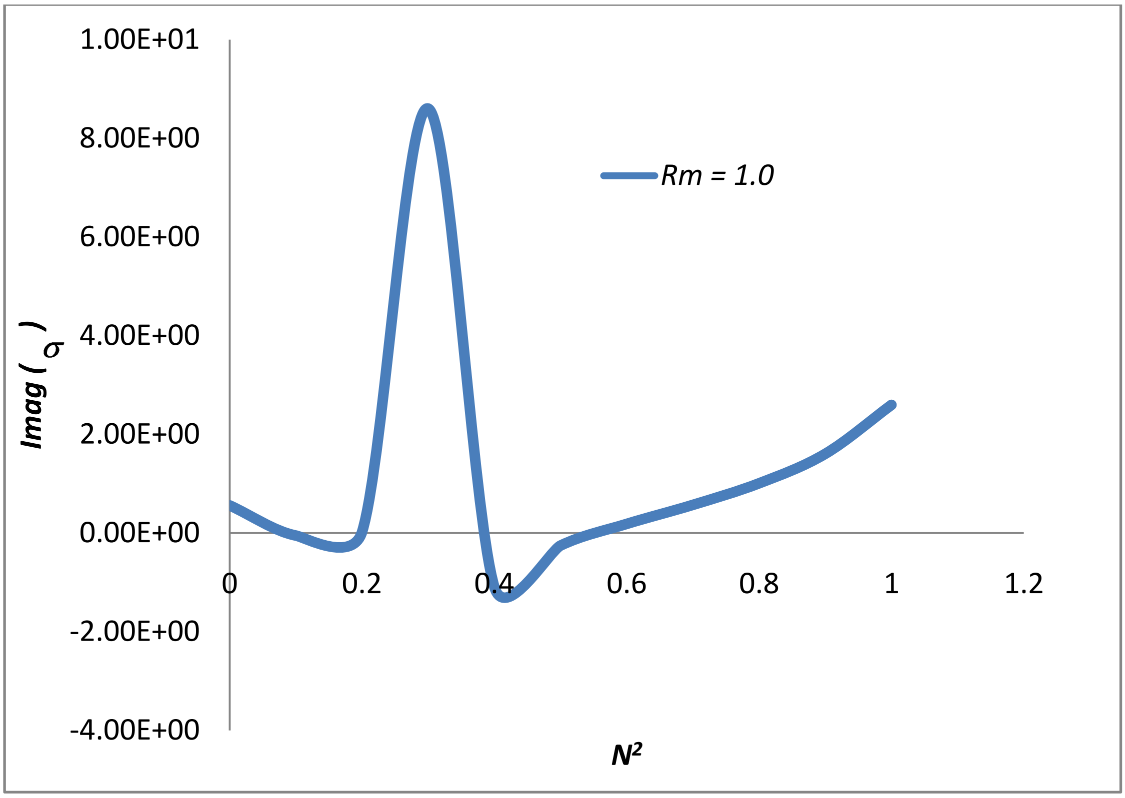

Figure 5. 6. Growth rate as a function of Brunt-Vaisala frequency N^2 ($\lambda < 0$)

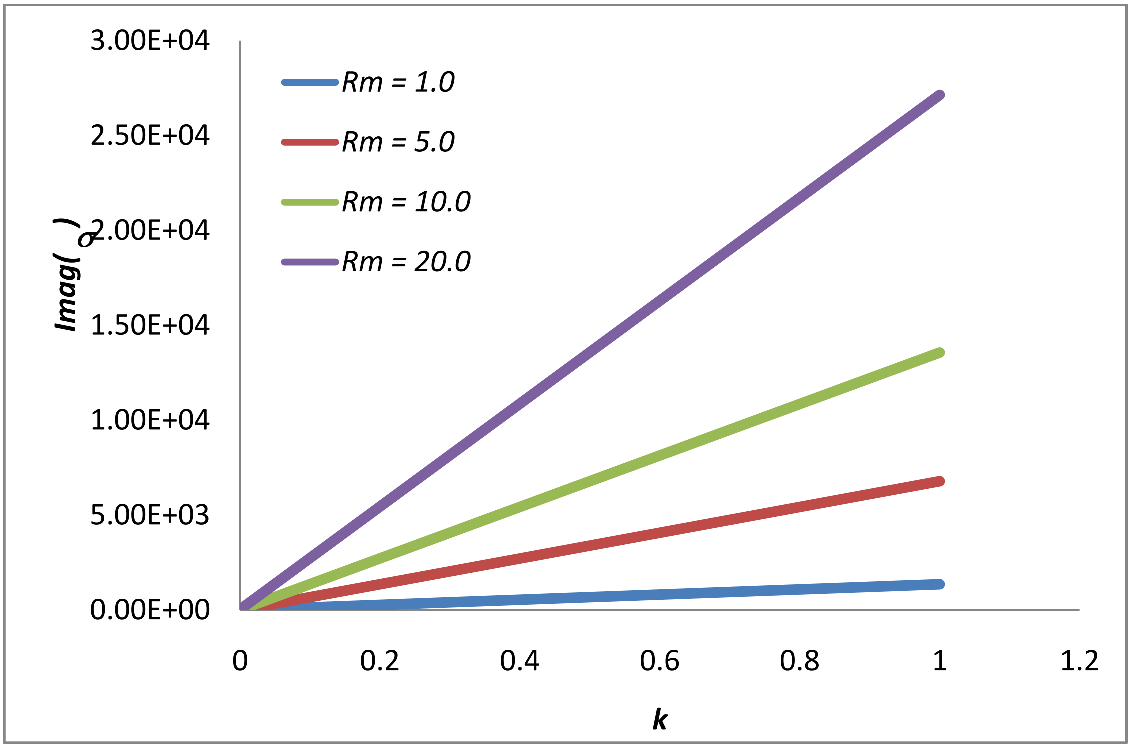

Figure 5. 7. Growth rate as a function of wave number for various Rm ($\lambda < 0$)

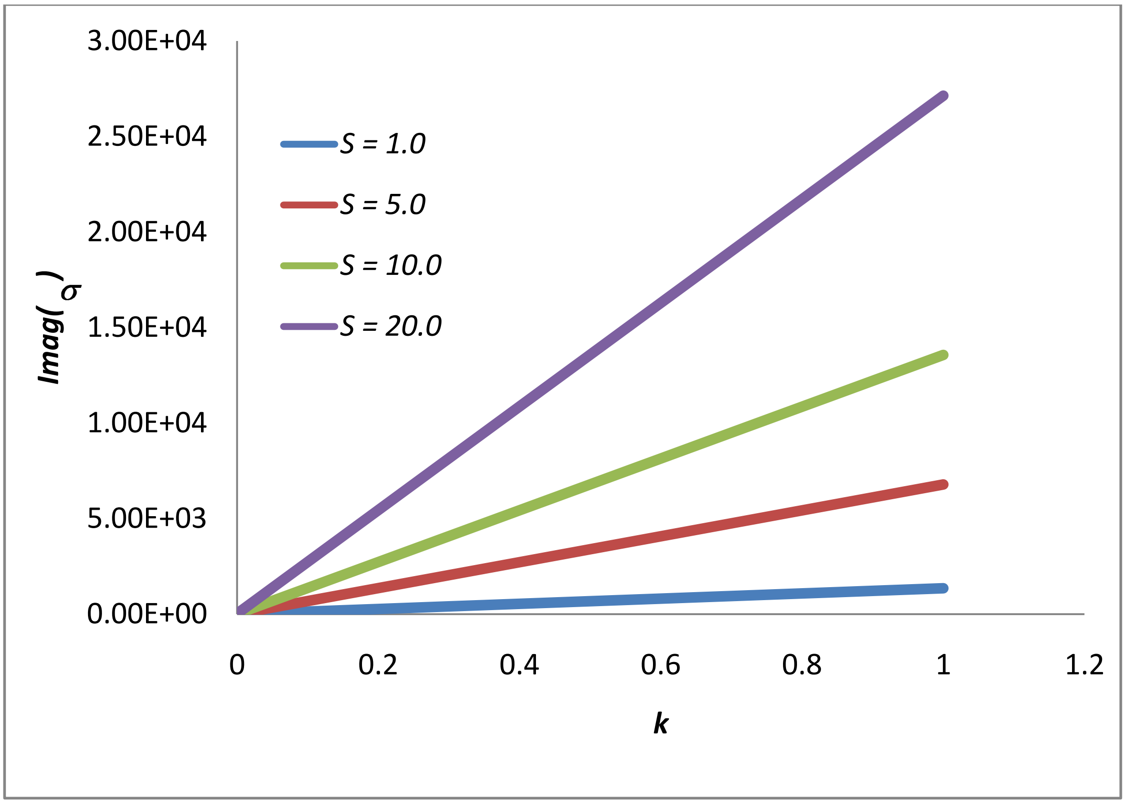

Figure 5. 8. Growth rate as a function of wave number for various S ($\lambda < 0$)

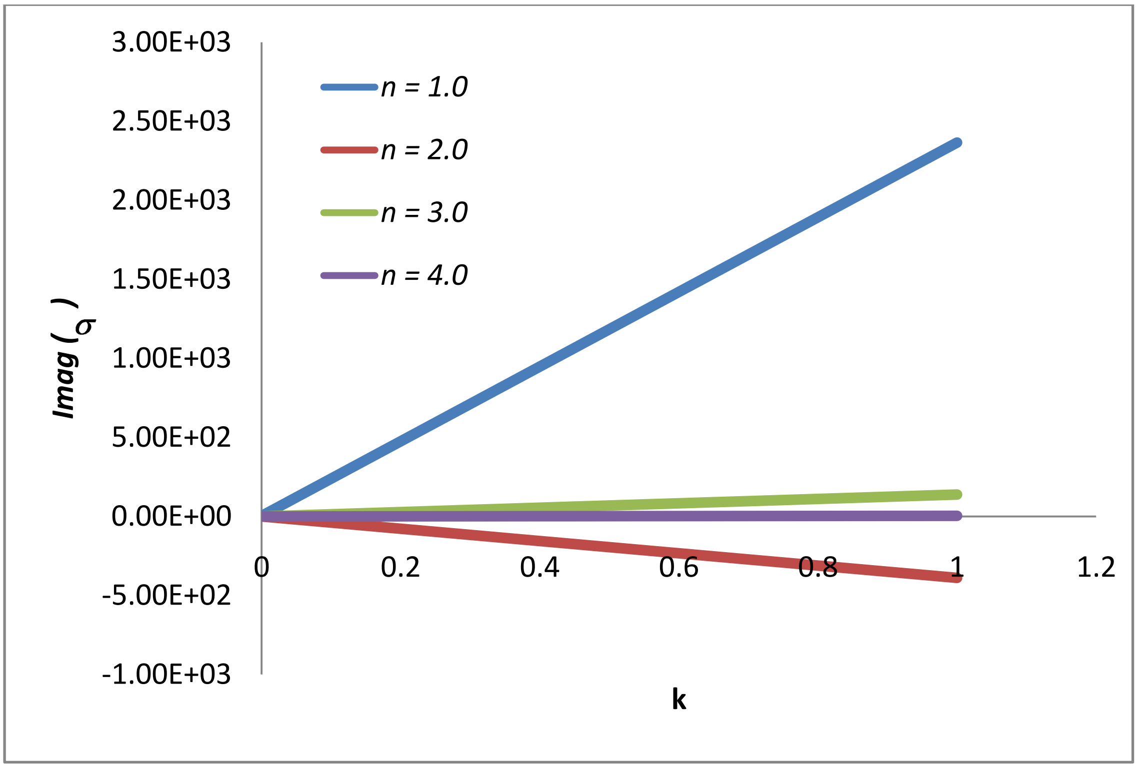

Figure 5. 9. Growth rate as a function of wave number for various n ($\lambda < 0$)

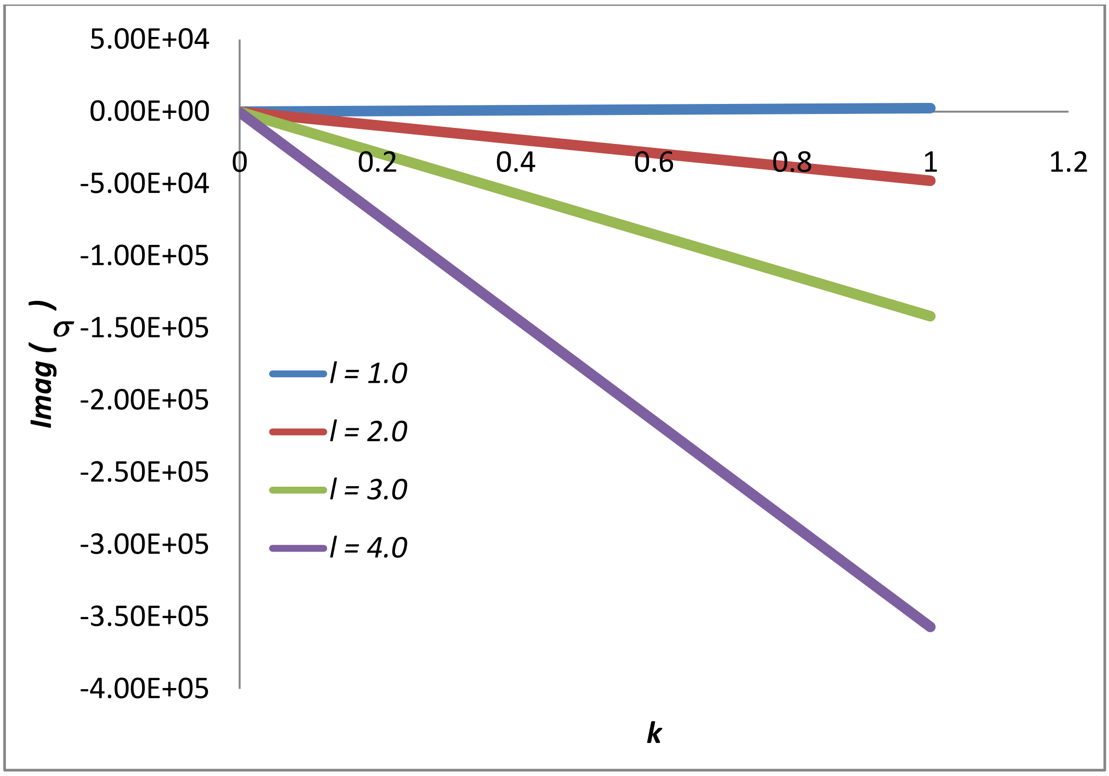

Figure 5. 10. Growth rate as a function wave number for various l ($\lambda < 0$)

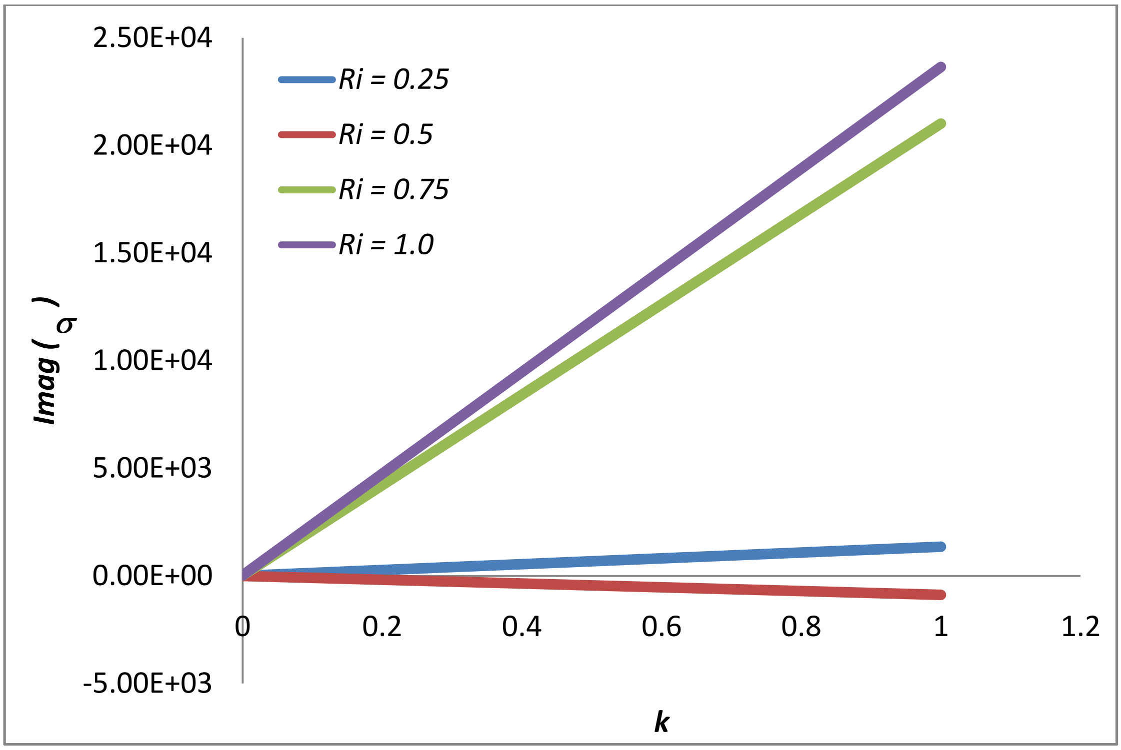

Figure 5. 11. Growth rate vs wave number for various Ri ($\lambda < 0$)

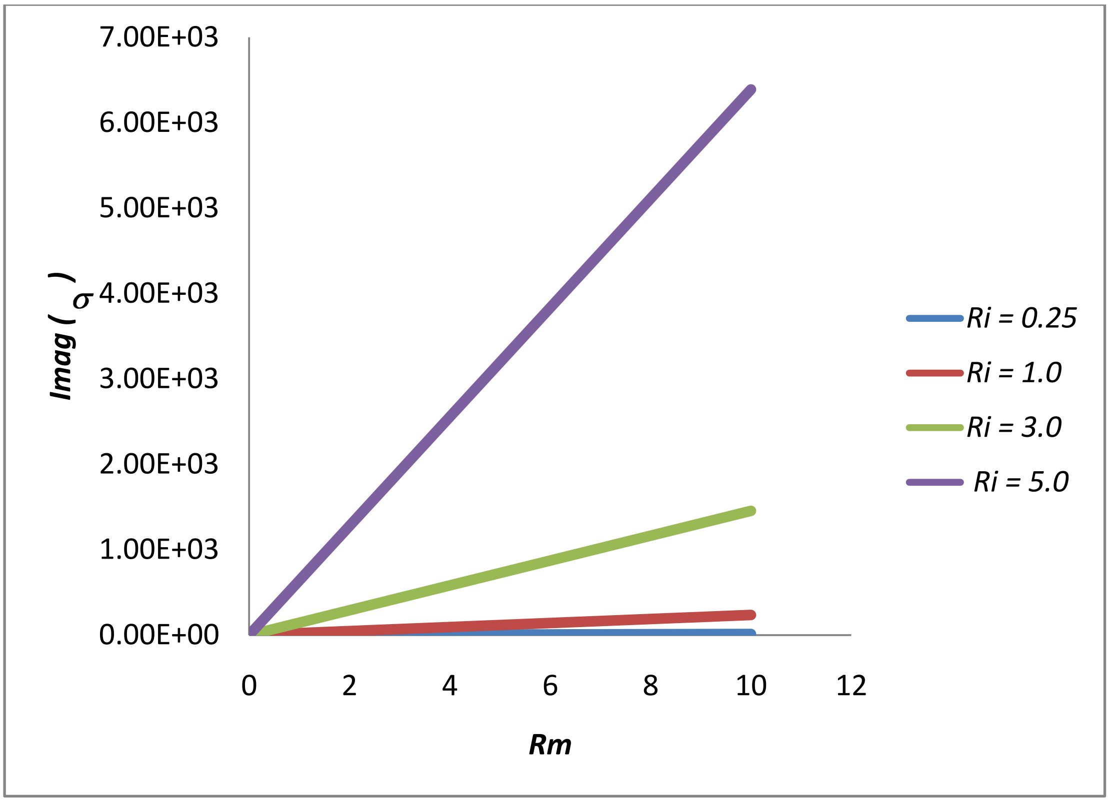

Figure 5. 12. Growth rate as a function of Magnetic Reynolds number for various Ri ($\lambda < 0$)

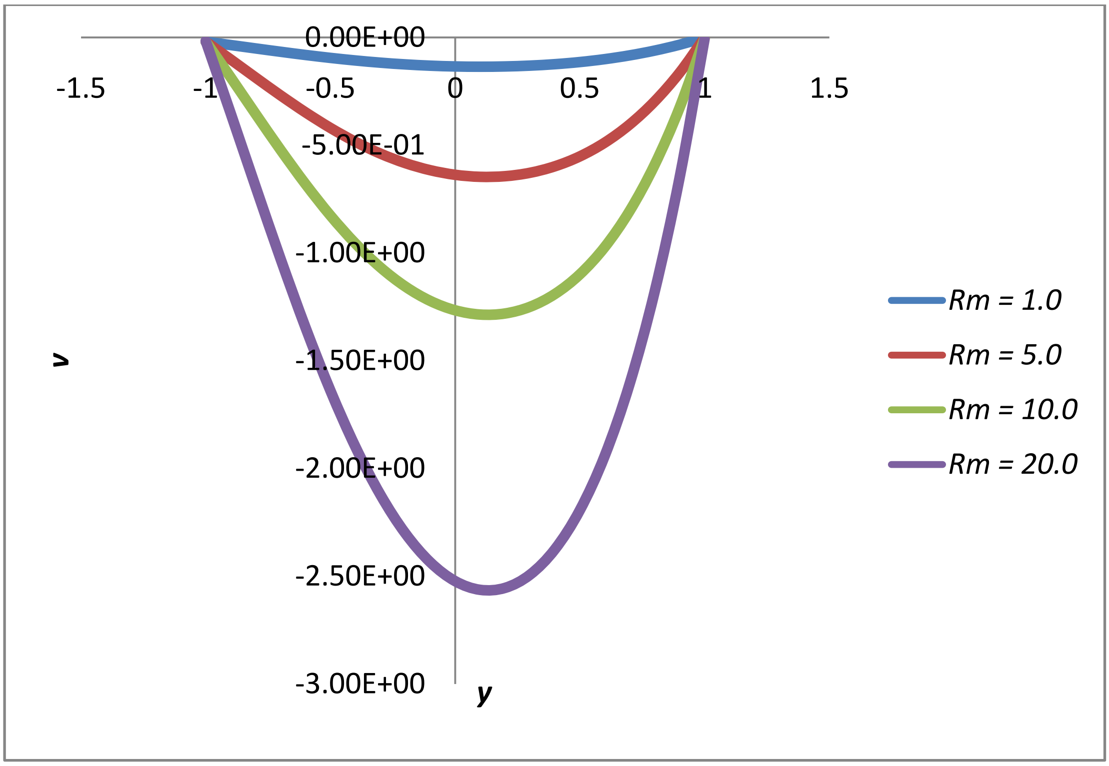

Figure 5. 13. Effect of Magnetic Reynolds number(Rm) on velocity profile ($\lambda > 0$)

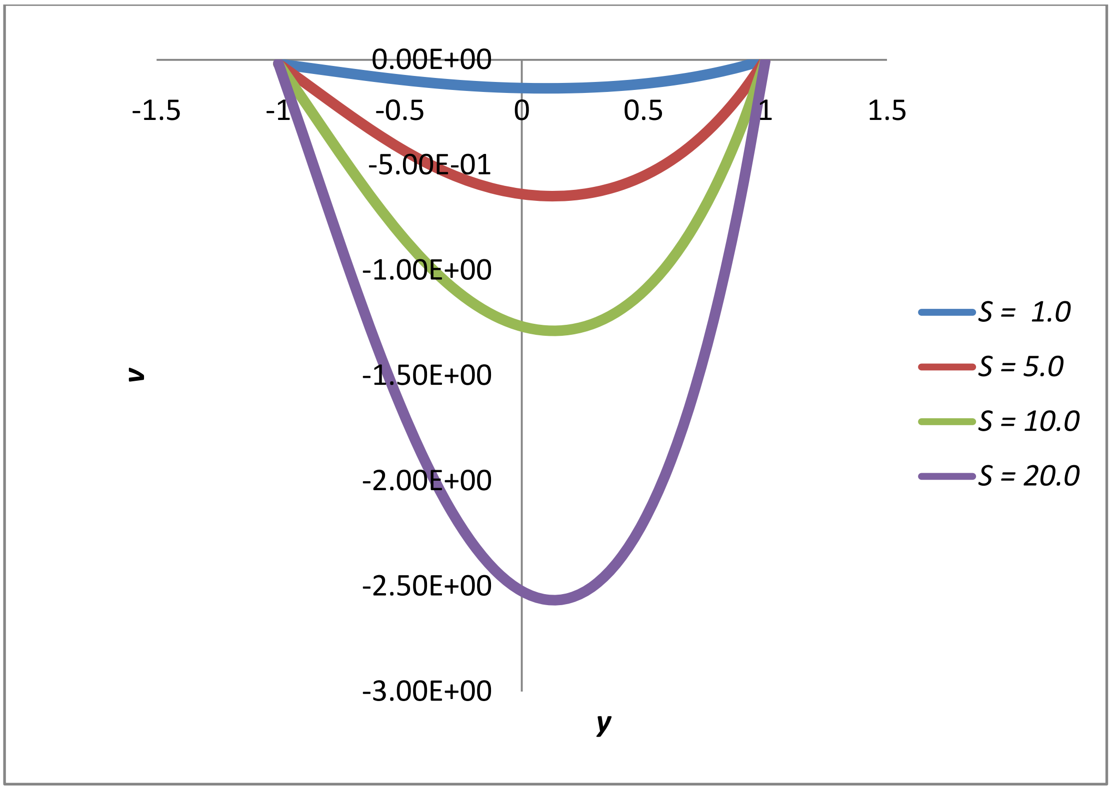

Figure 5. 14. Effect of Magnetic pressure number (S) on velocity profile ($\lambda > 0$)