CHAPTER-2
REVIEW OF LITERATURE

The word 'topology' has two meanings: it is both the name of a mathematical subject and the name of a mathematical structure. A topology on a set (as a mathematical structure) is a collection of what are called "open subsets", satisfying certain relations about their intersections, unions and complements. The notion of generalized closed sets in point set topology have been found considerable interest in the field of research work among general topologists during the past few decades to investigate different types of generalized closed sets that has led to several new and interesting concepts.

Earlier, topological spaces were characterized by open sets. Later Stone [103] introduced regular openness which is stronger than openness. The notion of open sets is a powerful tool for defining a topological space. Norman Levine [58] introduced the notion of semi-openness which is weaker than the notion of openness in topological spaces. Since several interesting generalized open sets have been introduced by many topologists. Njastad [75] introduced α-closed sets in a new way and its properties are further investigated and established. Velicko [114] introduced δ-closed sets in topological spaces. Sayed MEL and Mansour FHAL [91] introduced new class of near open sets namely, b^*-open set, and studied some of their properties.

Closed and open sets

Open and closed sets have fundamental importance in topology. The concept is required to define and make sense of topological space and other topological structures that deal with the notions of closeness and convergence for spaces such as metric spaces and uniform spaces. Every subset A of a topological space X contains a (possibly empty set) open set, the largest such open set is called the interior of A. It can be constructed by taking the union of all the open sets contained in A. Many mathematicians contributed several articles on different and interesting new open sets as well as generalized open sets. The notion of g-closed sets and investigated its fundamental properties.

Norman Levine [58] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces by comparing the closure of a subset with its open supersets and showed that compactness, countably compactness, para compactness and normality were all g-closed hereditary. This definition uses both the closure operator and openness of the superset. By considering other generalized closure operators or classes of generalized open sets, various notions analogous to Levines g-closed sets have been constructed. Generalized closed sets are a strong tool in the characterization of topological spaces satisfying weak separation axioms. The productivity and fruitfulness of the notion of generalized closed sets motivated the mathematicians to introduce weaker and stronger forms of generalized closedness for the past four decades. With the aid of g-open sets, they introduced, investigated and modified continuous functions which are the core concept of topology. Detailed study in this regard by many investigators has enriched the field of generalized closed sets to a considerable extent. Palaniappan. N. [79] highlights the definition and properties of rg-closed sets, rg-open sets. Further some of their characterizations are investigated with examples. Maki et al [62] and [63] introduced generalized α-closed sets and α-generalized closed sets and used this notion to consider new weak and stronger forms of continuities associated with these sets. Gnanambal. Y., [42] introduced generalized pre regular closed sets and their properties are studied and the notion of pre regular $T_{1/2}$ space and generalized pre regular continuity are introduced. Veera kumar [110] introduced and studied the concept of g^*-closed and g^*-continuity in topological spaces. g^*-closed set lies between closed set and g-closed set. Sekar S., [94] introduced a new class of generalized α regular-closed sets, generalized α-regular interior and generalized α-regular closure in topological spaces. Some characterizations and several properties concerning generalized α-regular interior and generalized α-regular closure are obtained.

Continuity

Continuity of functions is one of the core concepts of topology. Continuous functions in topology have significant role in the applications of mathematics, as it has applications to engineering fields like digital signal processing and neural networks. Many topologist studied weaker and stronger forms of continuity. Various authors have generalized many types of continuity concepts.

Maheshwari S. N. et. al. [61] introduced α-continuous mappings and presented some properties of such mappings. Some important characterizations were obtained. Cueva [22], defined a new class of functions called generalized continuous (briefly g-continuous) functions, which contains the class of continuous functions and its properties and characteristics were analyzed. Balachandran. K, Sundharam. P and Maki. H., [10] introduced and studied the concepts of a new class of maps, namely g-continuous maps, which includes the class of continuous maps, and a class of gc-irresolute maps as an analogy of irresolute maps. Also introduced the concepts of GO-compactness and GO-connectedness of topological spaces and prove the following product theorems for GO-compact spaces and GO-connected spaces. Devi. R., et. al. [25] introduced and studied α-continuity, α-generalizerd continuity and generalized α-continuity and some of the properties. Arocikiarani et al [6] introduced a significant class of generalized function called g-continuous, rg-continuous functions and r-continuous functions and its properties and characteristics are analyzed. Gnanambal. Y and Balachandran. K., [44] introduced gpr-continuous functions. The concept of GPR-compact and GPR-connected sets have been introduced and some properties are investigated. Mukunthan. C. et. al. [69] introduced the concept of g^*-continuity in topological spaces and obtained relation between sets. Some important characterizations are also obtained. Sekar. S. et. al [95] introduced gαr-continuous and obtained some of their properties.

Contra continuity

A new generalization of contra continuity by Dontchev [32] introduced a stronger form of LC-continuity called contra-continuity. Then variations of contra continuity have been investigated.

Dontchev. J. [32] introduced a stronger form of LC-continuity called contra-continuity. Contra continuous images of strongly S-closed spaces are compact. Every strongly S-closed space satisfies nearly compact. Ekici. E., [36] introduced the concept of contra r-continuity, contra rg-continuity and its properties are discussed. Caldas M. et. al., [16] introduced contra g-continuous and investigated their properties. Alli. K. [2] introduced and investigated contra $g^{\#}$p-continuous functions. This new class properly contains the class of contra continuous, contra α-continuous, contra pre-continuous, contra g*p-continuous functions, contra gα-continuous functions. And contra αg-continuous functions is properly contained in the class of contra gsp-continuous functions and contra gpr continuous functions. Sekar. S and Kumar. G., [95] introduced a new class of functions called contra generalized α-regular continuous function in topological spaces. Some characterizations and several properties concerning contra gαr-continuous functions are obtained. Gomathi [41] introduced a new generalization of contra-continuity called contra g*-continuity and obtained some of their properties.

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Homeomorphism

The notion of homeomorphisms plays a dominant role in topology and so many authors introduced various types of homeomorphisms in topological spaces. A topological property is a property of spaces that is invariant under homeomorphisms. To prove that two spaces are not homeomorphic it is sufficient to find a topological property not shared by them. Examples of such properties include connectedness, compactness, and various separation axioms. A homeomorphism is a bijection that is continuous, whose inverse is also continuous. From the point of topology, homeomorphic spaces are essentially identical. Open maps and closed maps are very useful in topological spaces. For researchers on various closed sets, the study is not complete without extending their definitions to open (closed) maps and homeomorphisms.

Maki. H et. al. [65] defined the notion of generalized homeomorphisms which are generalizations of homeomorphism and they have investigated some of the properties of generalized-homeomorphism. A homeomorphism is a map between spaces that preserves all topological properties. Arokiarani I., [5] introduced a class of rg-homeomorphism, rgc-homeomorphism and obtained the relations among them. Gnanambal.Y [43] introduced gpr-homeomorphism and compared them with some existing weaker forms of homeomorphism. Also studied some properties of gpr-homeomorphism from quotient space to other spaces. Devi. R. et. al. [24], introduced new forms of α-homeomorphism, gα-homeomorphism and αg-homeomorphism in topological spaces and investigated the properties of their mappings from the quotient space to other space. Vadivel. A., [107] introduced gαr-homeomorphism and some of their fundamental properties are investigated, and in addition the connection between these maps and other existing topological maps are also studied. Krishna. P, Antony Rex Rodrigo. J., [55] introduce R-closed maps, R-open maps, R-homeomorphisms, R^*-homeomorphisms, strongly R-continuous, perfectly R-continuous and studied their properties. Using these new types of maps, several characterizations and properties have been obtained. Gomathi. N., [41] introduced g*-homeomorphism and studied some of their topological properties and also investigated the group structure of set of all homeomorphism.

Separation axioms

Separation axioms is one of the most important and interesting concept in topological spaces. One of the most well-known low separation axioms is the T_1 separation axiom in which singleton sets are closed.

Aull and Thron [4] proposed and studied extensively various separation axioms between T_0 and T_1. Several topological spaces that fail to be T_1 are very often of significant importance in the study of the geometric and topological properties of digital images. Several new separation axioms were defined in the course of the investigation of generalized closedness. They, also Introduced separation axiom between T_0 and T_1 and states that by identifying "undistinguishable" points in a topological space every space can be made into a T_0-space. Also introduced a number of new separation axioms, giving equivalent forms analyzed their inclusion relations, and observed that they all can be described in terms of derived sets of points. Levine [59] proved a subset of a topological space to be generalized closed (g-closed) if its closure is contained in each of its neighborhoods. And he shows that g-closed sets possess many of the familiar and important properties of closed sets. Kar. A., et.al. [50] introduced and explore weak separation axioms namely T_i (for i = 0, 1, 2) spaces, properties of some separation axioms between T_0 and T_1 and investigates each of the axioms. Moreover, D_i (for i = 0, 1, 2) spaces are developed and the relations between these spaces are discussed and he initiated the concept of continuous function and discussed the behavior of D_1 space under continuous function. Jafari. S. [46] introduced and studied weak separation axioms on topological spaces. Authors have tried to find a suitable form of separation axioms and described a T_2-axiom in the form of regular frames. T_2-frames coincide for topological spaces with Hausdorff spaces but they are described independently on points. Erdal Ekici, [37], introduced and studied some new separation axioms by using γ-open sets, the notion of weakly γR_0 and γR_1 spaces are introduced and basic properties of them are investigated. Keskin. A and Takashi noiri, [53], introduced γR_0 and γR_1 spaces and obtained some properties of them and investigted further properties of γR_0 and γR_1 spaces. Bishwambhar Roy and M. N. Mukherjee, [14], introduced a unified theory for R_0, R_1 and certain other separation properties and their variant forms. The study of certain separation axioms and their neighbouring forms, introduce unified definitions of R_0, R_1, T_0 and T_1 spaces and derive results concerning them from which many of the existing results follow as special cases. Al-Swidi. A.L., Basim Mohammed M., [3] deals with the relation between the separation axioms T_i-space, i = 0,1,2,3 and R_i space i = 0,1,2 throughout kernel set associated with the closed set and some theorems related to them. Alias B. Khalaf and Suzan N. Dawod, [1] introduced new types of separation axioms in topological spaces by using g*b-open set. Also the concept of g*b-R_0 and g*b-R_1 are introduced. Several properties of these spaces are investigated.

Bitopological spaces

A triplet, (X, τ_1, τ_2) where X is a non-empty set, τ_1 and τ_2 are topologies defined on X is called a bitopological space. Many researchers developed the new area in topology equipped with two topological branches called Bitopology. In particular, different families of subsets of bitopological spaces are introduced and the relationships between the two topologies are analyzed on one and the same set.

Kelly. J. C., [52] introduced the idea of bitopological space as a set equipped with two topologies on a set and initiated a systematic study of bitopological spaces and thereafter the theory has been developed by different Mathematicians from different stand points. It is confined in considering the pairwise properties of the two topologies and their interrelations. Fukutake [40] introduced the concepts of g-closed sets and g-continuous maps in topological spaces to bitopological spaces and obtained some interesting results. After that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Sheik John., et. al. [99] introduced g*-closed sets in bitopological spaces and properties of these sets are investigated. New bitopological spaces, continuity as its applications are also studied. Rajarajeswari P., [84], introduced αg-closed set, sαg-closed set in bitopological spaces and studied some of their properties. Also introduced spaces, continuous maps to bitopological spaces. Janaki, C. [49], introduced α-closed set, gα-closedset, πgα-closed set in bitopological spaces and obtained some of their properties and extended the idea of closure and continuous maps to bitopological spaces. Arockiarani I., [5], introduced r-closed sets in bitopological spaces. And different concepts are also found to develop a good insight into the structure of these spaces. The idea of spaces, pr-closure, pr-continuous maps have been extended to bitopological spaces.

Topological ordered spaces

Nachbin [70] defined a topological ordered space by adding a partial order relation to the structure of a topological space. It can be considered that the topological ordered spaces are one of the generalizations of the topological spaces.

Veera Kumar. M.K.R.S., [111], introduced I-homeomorphisms, D-homeomorphisms and B-homeomorphisms for topological ordered spaces after introducing I-continuous maps, D-continuous maps, B-continuous maps, I-open maps, D-open maps, B-open maps, I-closed maps, D-closed maps and B-closed maps for topological ordered spaces together with their characterizations. Rao K. K., [86], introduced I-α-homeomorphisms, D-α-homeomorphisms and B-α-homeomorphisms for topological ordered spaces after introducing I-α-continuous maps, D-α-continuous maps and B-α-continuous maps, I-α-open maps, D-α-open maps, B-α-open maps, I-α-closed maps, D-α-closed maps, B-α-closed maps for topological ordered spaces together with their characterizations. Krishna rao K., et. al. [56] introduced some concepts in Topological Ordered Spaces using semi-open sets, pre-open sets, α-open sets and β-open sets also introduced different mappings between topological ordered spaces and established relation between these mappings. The major aspect of the study is the properties relating to the equivalent conditions of different homeomorphisms in topological ordered spaces. Srinivasarao G., [101], introduced ig-closed sets, dg-closed sets, bg-closed sets and studied relationship between them. Also introduced ig*-closed sets, dg*-closed sets, bg*-closed sets and discussed the relations between the newly introducing sets. Amarendra babu.V., [7] introduced a new class of sets called g*-closed sets in topological ordered spaces, also discussed some of their properties and investigated the relationship along with some counter examples. Dhanapakyam. C., [31], introduced a new class of closed sets in topological ordered spaces called increasing βg*-closed sets, decreasing βg*-closed sets and balanced βg*-closed sets and obtained some of their characteristics.