CHAPTER I

Introduction

1.1 Motivation

A monsoon is a seasonal change in the wind direction. This occurs because the temperature differences between the land and sea across all of the tropics and deflection of wind due to rotation of earth. The shift of wind brings about a remarkable change in local weather. Monsoons are often associated with rainy seasons in the tropics and the subtropics. In these areas, life is critically dependent on the monsoon rains.

India being a tropical country, the south-west monsoon winds have great importance in Indian agriculture. The regions which receive the largest rainfall are along the west coast of India and the states of Assam and West Bengal in northeast India. In these regions orographic features play an important role, because the moisture laden monsoon winds strike against physical barriers by way of mountains. Apart from orographic features, atmospheric convection plays an important role during the monsoon. Indian farmer largely depends on the monsoon rains. The Indian economy is heavily dependent on agriculture.

On the other hand, weak monsoon rains result in crop failure which affects the economy in a negative manner due to lower production. Later on, this translates into price-rise, low industrial output and other issues. A weak monsoon rainy season may cause drought, crop failures and hardship for people and wildlife. However, heavy monsoon rains have caused massive floods that have killed thousands of people.

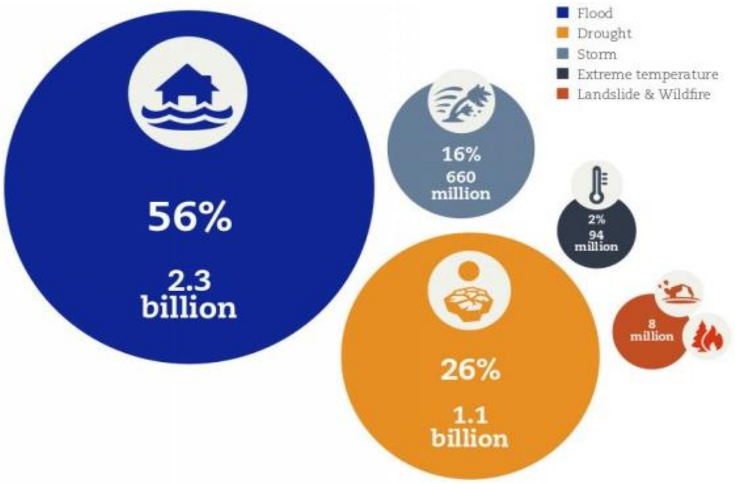


Fig 1.1 Number of people affected by weather-related disasters (1995-2015)

During 2015 South Indian floods resulted from heavy rainfall generated by the annual northeast monsoon in November 29 – December 2, 2015. They affected the Coromandel Coast region of the South Indian states of Tamil Nadu, Andhra Pradesh, and the union territory of Puducherry. In Tamil Nadu, city of Chennai was particularly hard-hit. More than 500 people were dead, over 18 lakh people were displaced and Tamilnadu faced enormous economic damages, with estimates of damages and losses ranging from nearly ₹200 billion (US$3 billion) to over ₹1 trillion (US$15 billion).

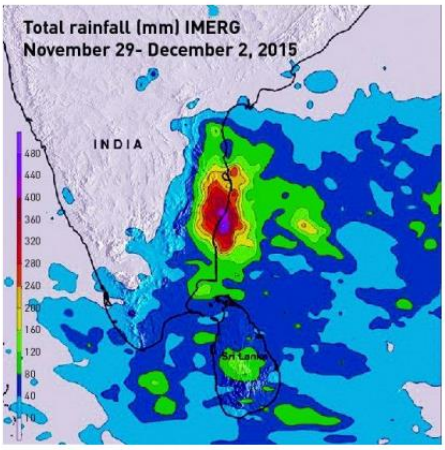


Fig 1.2 Accumulated rainfall between November 29 - December 2 over Chennai

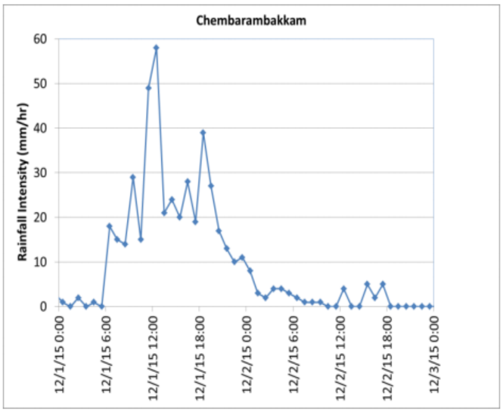


Fig 1.3 Rainfall Intensity at Chembarambakkam rain gage station

Fig 1.4 South Indian floods resulted from heavy rainfall –Affected people and properties

Hence it is important to study the detailed analysis of convective flow which will throw light on various atmospheric phenomena.

1.2 Porous Medium

The study of flow through porous medium plays a vital role in recent years because of its applications in industrial, bio-physical and hydrological problems, especially in petroleum, chemical and nuclear industries. This has received much attention among researchers and motivated them to do research through porous medium. The role played by porous medium in the study of the flow of blood and other fluids and filters in bio-chemical engineering is more important.

A porous medium or a porous material containing pores is filled with a fluid (liquid or gas). Porous material is a solid substance permeated by an interconnected channel of pores. The flow of one or more fluids is allowed through the interconnectedness of the solid material. The small holes distributed throughout the solid porous material may be effective or ineffective. By effective holes, we mean those holes through which the fluid can actually pass. By ineffective holes, we mean those holes through which fluid cannot pass. There are many varieties of artificial and natural porous materials such as spunches, a pack of sand, a piece of dolomite rock, sandstone, limestone, wood, foamed plastics, human lung, etc.

Many natural substances such as rocks and soil (e.g., aquifers, petroleum reservoirs), biological tissues (e.g. bones, wood, cork) and manmade materials such as cements and ceramics can be considered as porous medium.

Porous medium plays an important role in the study of the flow of blood and other fluids, electro-osmosis, biological membranes and filters in bio-chemical engineering. We understand the mechanism of transfer of heat from the deep interior of the earth to a shallow depth in the geothermal region. The study of the behavior of fluid saturated porous medium is known as Poromechanics.

The concept of porous medium is used in many areas like filtration, mechanics (acoustics, soil mechanics, rock mechanics), engineering (petroleum engineering, bio-remediation, construction engineering), geosciences (hydrogeology, petroleum geology, geophysics), biology and biophysics, material science etc. A fluid flow through porous medium is a subject of most common interest and has emerged as a separate field of study.

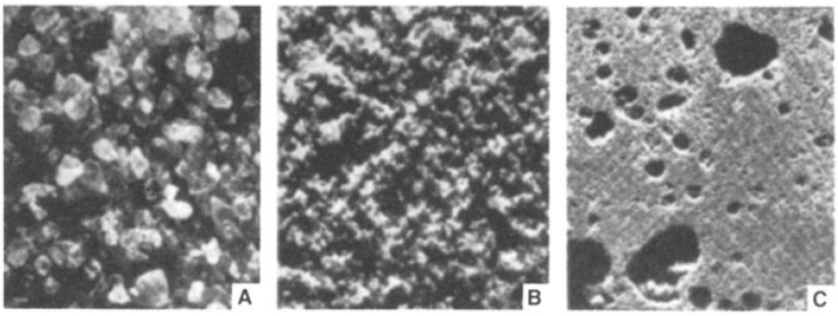


Fig 1.5 Examples of natural porous materials: a) beach sand, b) sandstone, c) limestone

Fig 1.6 Granular porous materials used in the construction industry, Liapo® spheres and crushed limestone

1.3 Properties of Porous Medium

Two important properties of porous medium used to describe the fluid flow are,

- porosity (ε)
- permeability (k_1)

1.3.1 Porosity

The porosity of a porous medium is its most important geometrical property. The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by the void space. Mathematically, it is the ratio between the unit volume of void space to the total volume containing both fluid and solid which may be either sphere or cube.

$$\varepsilon = \text{void volume/total volume} = V_v/V$$

where $0 < \varepsilon < 1$. If $V_v = V$ then it is the case for free fluid.

The porosity of a porous medium describes how densely the material is packed. Thus 1-ε is the fraction that is occupied by solid. Porosity ranges between the fraction 0 and 1, typically ranging from less than 0.01 for solid granite to more than 0.5 for peat and clay. Manmade materials such as metallic frames can be more porous; ε can approach to the value 1. For natural media, ε does not normally exceed 0.6.

Materials with low porosity are less permeable and have small pores, making it more difficult for gas or liquid to pass through them, while materials with high porosity have large pores and are easily permeable. Porosity is an important consideration in filtering, since if particles must be removed from a porous medium, the pores must be small enough to effectively trap them. Geologists also consider the porosity of the surrounding stone and soil when conducting observations of oil and natural gas reservoirs.

Apart from porosity, other properties for any porous medium of interest are:

- Viscous resistance - the obstruction of a fluid flow through a porous medium. It is inversely proportional to the permeability of the medium.
- Inertial resistance – additional resistance expected for fluid flow in a porous medium, beyond those predicted by Darcy’s law

The viscous resistance and inertial resistance are independent of the nature of the fluid but dependent only on the nature of the porous medium.

S. No.	**Substance**	**Porosity(in fraction)**
1	Silica powder	0.37 – 0.49
2	Loose sand	0.37 – 0.50
3	Soils	0.43 – 0.54
4	Sand stone	0.08 – 0.38
5	Lime stone	0.04 – 0.10
6	Brick	0.12 – 0.34
7	Leather	0.56 – 0.59
8	Fiber glass	0.88 – 0.93
9	Coal	0.02 – 0.12
10	Black State Powder	0.57 – 0.66

Table 1.1 Various representative values of Porosity

A number of methods such as the Direct method, Optical method or Visual method, Density method, Gas expansion method, Mercury injection method, Absorption method, Methods based on flows etc. are available for measuring the porosity of a medium all aiming at measuring somehow the void volume and bulk volume of the material.

1.3.2 Permeability

Permeability is the fluid conductivity of the porous material. It is the property of a porous material that characterizes the case with which a fluid is made to flow through the material by an applied pressure gradient. The permeability of a porous medium is defined as the measure of its ability to transmit fluids. The permeability and porosity are related since if the porosity is zero then permeability is also zero.

1.3.2.1 Factors affecting Permeability

Numerous factors affect the magnitude and/or direction of permeability.

- Textural properties
    - Pore size/ grain size
    - Grain size distribution
    - Shape of grains
    - Packing of grains
- Gas slippage
- Amount, distribution and type of clays
- Type and amount of secondary porosity
- Overburden pressure
- Reactive fluids
- High velocity flow effects

1.4 Applications of Porous Medium

The study of flow through porous medium plays an important role in recent years because of its application in industrial, bio-physical and hydrological problems, especially in petroleum, chemical and nuclear industries. Flow through porous medium is also used in the field of geophysics. Geologists used pores in the extraction of oil, where the oil is surrounded by porous rock. Multi-layered linings of porous materials have been engineered that can absorb a wider range of sounds than traditional linings, and have the advantage of being much lighter. The heat retention of a building is also determined by the porosity of the building material. It is also applied for dechlorination of concrete structures such as bridges contaminated and corroded by sea water.

Common application of porous medium in science and industry is filtration. Example for filtration in industry is water treatment and petroleum refining. Porous medium is also used as the aquaculture. It absorbs undesirable organic compounds and metals from the water.

Fig 1.7 Porous medium (Aerosols) in filters

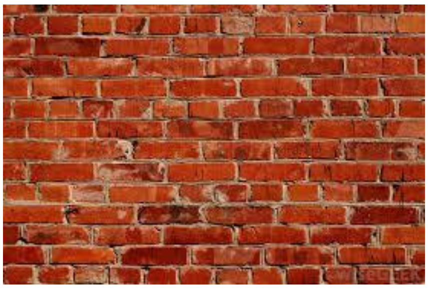


Fig 1.8 Porous medium in building materials

An important application of porous medium concept in engineering is the optimization of internal spacing of heat exchangers subjected to overall volume constraints. A potentially revolutionary application of the formalism of forced convection in porous medium is in the field of heat exchanger simulation and design. There are many applications in biomedicine e.g. the delivery of drugs by iontophoresis which is a physical process in which ions flow diffusively in a medium driven by an applied electric field.

The study of the effect of rotation on a porous layer is of particular interest in a practical setting with applications in the food processing industry, the chemical engineering industry and in the process of centrifugal casting of metals and rotating machinery.

1.5 Heat transfer

The study of heat transfer becomes more important because of its application in science and engineering such as the design of cooling system of motors, generators, transformers, geophysics, meteorology, oceanography and in organic metabolism. Heat transfer is the transition of thermal energy from a higher temperature region to a lower temperature region. This process continues until the object and the surroundings reach the state of thermal equilibrium. We cannot measure the transfer of heat flow or energy transfer by heat flow directly. But the concept has physical meaning because it is related to the measurable quantity called temperature. Heat flux is defined as the amount of heat transfer per unit time. Heat flux is determined easily, once the temperature difference is known. All are exposing the heat transfer problems in our normal life. Some examples are adjusting cooking power and looking for breezes in summer.

Heat exchangers, condensers, boilers, solar collectors are few typical heat-transfer devices. Theory of heat transfer is based on thermodynamics, physical transport phenomena, space-time modeling and additional mathematical modeling.

1.6 Heat transfer mechanisms

Heat transfer is energy transfer due to a temperature difference between two or more medium. Heat can be transferred from one place to another by three methods:

- Conduction
- Convection
- Radiation

1.6.1 Conduction

The transfer of heat between two bodies in direct contact is called conduction. Conduction is the transfer of heat between two bodies or two parts of the same body through molecules, which are more or less stationary. It occurs without bulk motion, and is caused by the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones. Through solids, this is the only possible mode of heat transfer.

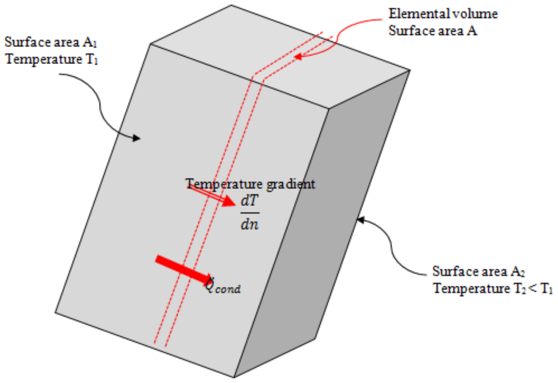


Fig 1.9 Schematic diagram showing heat conduction through a material

Convection

Some examples are,

- When ironing the cloth, the iron is hot and the heat is transferred to the cloth
- Chocolate in the hand will eventually melt as heat is conducted from the hand to the chocolate
- After a car is turned on, the engine becomes hot. The hood will become warm as heat is conducted from the engine to the hood

1.6.2 Convection

Convection is the transfer of heat between a wall and a fluid system in motion. It is not fully a heat transfer mechanism because it depends on its progress or operation on the motion of fluid action as a carrier of energy. Convection is a process involving the mass movement of fluids.

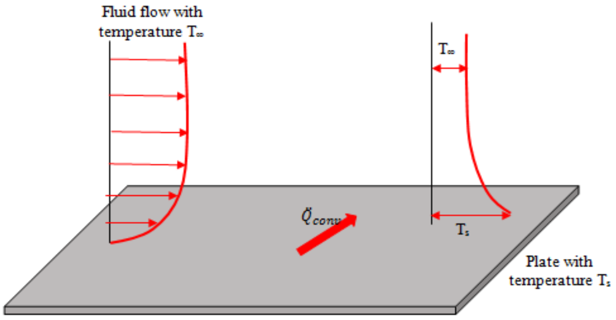


Fig 1.10 Schematic diagram showing forced convective heat conduction from a heated plate to the fluid

There are two types of convective heat transfer. They are

- Free or Natural Convection
- Forced Convection

Free Convection

When a temperature difference produces a density difference that results in mass movement, that process is called free or natural convection. The free convection is induced by the heat transfer itself, i.e., due to fluid motion caused by the buoyancy forces. The phenomenon of free convection has many important technological applications such as cooling of nuclear reactors, providing heat sinks in turbine blades etc. Further, the structure of stars and planets are greatly influenced by thermal convection in their interior. Examples of free convective flows include the following.

- The formation of breeze on land or sea. We observe that the land near the sea is warmer in the afternoon than in the evening. This warm air rises by the principle of convection, and is replaced by cooler air. Similarly, during the night, the air near the sea is warmer than that at the shore. That is because the warm air rises and is replaced by cooler air.
- Warm water from the oceans rises up in the air and turns into saturated water drops that form clouds. When this process continues, the smaller clouds collide with each other and bigger clouds are formed. Upon reaching the final growth stage, cumulonimbus clouds or thunderstorms are formed.
- The simplest example of convection is a steaming beverage. We observe steam coming out of a cup of hot tea or coffee. Due to the heat of the fluid, the warm air rises up. This warm air is the steam.

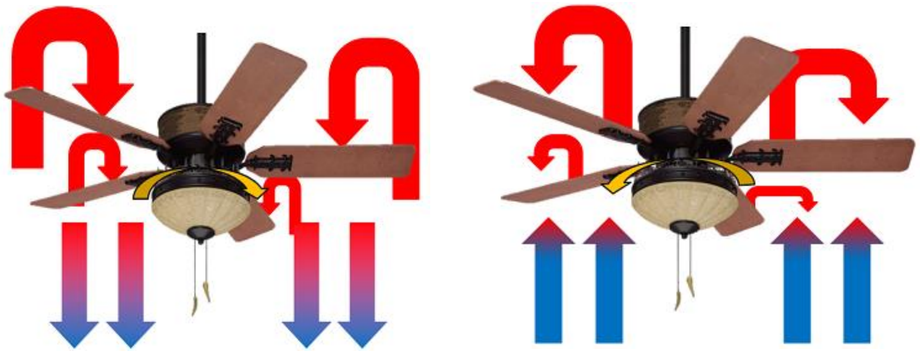
a) Land and Sea Breeze *b) Thunderstorm* *c) Steaming Beverage*

Figs 1.11 Examples of free convective flows

Forced Convection

The motion of the fluid is caused by an external agent i.e., externally imposed flow of a fluid over a heated object is called forced convection. When an external device like a pump, compressor, fan etc., causes the motion of the fluid, it is termed as forced convection. Here, the fluid is made to flow along the hot surface and heat is transferred from the wall to the fluid. Some examples of forced convection are given below.

- In our home or office, we are using ceiling fans. It's a good example of forced convection. Ceiling fans are used in both the seasons i.e., winter and summer. But the setting of ceiling fan is different in order to act in the desired task. Normally in summer seasons, outside air is so hot. In order to cool the air, we put the fan in a higher speed. The angle of blades forces the air down through the room. This corresponds to a counter-clockwise rotation when looking at the fan from below. This process mix warm air and force a cool breeze downwards, creating downdraft.



a) In summer, ceiling fans should rotate counter clockwise to mix warm air and force a cool air downwards

b) In winter, ceiling fans should rotate clock-wise to pull cool air up and force warm air downwards

Figs 1.12 Examples of forced convection flow

- In winter seasons, outside air is cooler. In order to warm it, we put fan on a slower speed. The blades also spin in a different direction, generally clockwise direction when looking from below the fan. This pulls the cooler air up from the room and forces warm air downwards, creates an updraft. This cooler air mixes with the warmer air that has risen and mixes the two, distributing warmer air throughout the room.

1.6.3 Radiation

All solid bodies, as well as liquids and gases have tendency of radiating thermal energy in the form of electromagnetic waves and of absorbing similar energy emerging from the neighboring bodies. This type of heat transfer is known as radiation. Heat may be transferred between two bodies separated by empty space or gases by the mechanism of radiation through electromagnetic waves. The conductive and convective heat transfers require a material medium for transfer, whereas the radioactive transfer does not require a medium and can occur in a vacuum. In addition the energy transfer takes place at the speed of light.

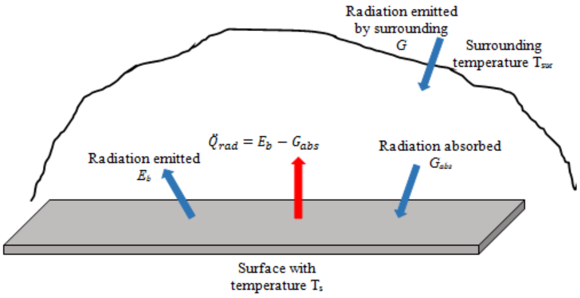


Fig 1.13 Schematic diagram showing net radioactive heat transfer from a plate.

Some examples for radiation are,

- Keep our hands or face near to the freezer door in refrigerator. Our hands or face will feel cooler. The reason is that we radiate heat directly into the cold region and it radiates very little heat to us. Consequently, our hands or face cools perceptibly.
- X-Rays are one of the most common uses of radiation in medicine, which provides valuable information to doctors and other medical professionals on patient injuries.
- All objects radiate heat, but some radiate much more heat than others. The biggest source of radiation is the Sun. Earth is heated due to solar radiation. Cooling of earth at night is due to radioactive heat loss.
- If we keep our hands a few inches away from the fire, without touching the fire, we can feel heat, because heat passes through the empty space until it reaches our hands.

a) Refrigerator *b) X-Rays*

c) Earth is heating due to solar radiation *d) Heat passes through the empty space (Hands near the fire)*

Figs 1.14 Examples of Radiation

In industrial processes, all three mechanisms often participate simultaneously in the transmission of heat. For a study of heat transfer process, however, it is necessary to distinguish clearly among the various modes, since they are subject to different laws.

Conduction travels faster through solids, but liquids and gases can also conduct heat. Convection is how heat travels through fluids – liquids and gases. Radiation is how heat travels through empty space.

Fig 1.15 Example for conduction, convection and radiation – Heating the liquid in a pan

1.7 Applications of Heat Transfer

Heat transfer mechanism is the heart of industries based on energy conversion. Principle of heat transfer is used in many fields like, geophysics, oceanography, science and engineering such as automotive engineering, thermal engineering, power station engineering, insulation, materials processing etc., Heat transfer is used in polymers, medical cream manufacturing, formation and dispersion of fog, molten plastics, distribution of temperature and moisture over agricultural fields, drying process of paper, glues, ink, pulps and foodstuffs. Heat transfer principle is used in spacecrafts to lose excess heat. For this process, all space crafts have thermal radiators.

Heat transfer principle is very important in the field of chemical engineering. Chemical engineers apply their knowledge of heat transfer for the evaporation, condensation, heating and cooling of fluids. Most of the chemical unit processes and unit operations are accompanied by absorption or evolution of heat.

The mechanical engineers deal with problems of heat transfer in the field of internal combustion engines, steam generation, refrigeration and heating and ventilation. The heat transferred by free convection from the pipes carrying the steam, from the walls of the surfaces, from the walls of air conditioned houses are popular examples.

Civil engineers are using heat transfer principles in the construction of dams, structures and to the architect in the designing of buildings. The heating of houses in cold weather with the use of radiators is a practical example of heat transfer.

Heat transfer is common in heat exchangers, which are tools built for efficient heat transfer from one fluid to another, whether the fluids are separated by a solid wall so that they never mix, or the fluids are in direct contact. Heat exchangers are widely used in refrigeration, air conditioning, space heating, power generation and chemical processing. One common example of a heat exchanger is a car radiator, in which the hot coolant fluid is cooled by the flow of air over the radiator surface.

Further, the mode of heat transfer is recently used in nuclear power plants to cool the core of the reactor and carry out the heat generated in the reactor by nuclear fission.

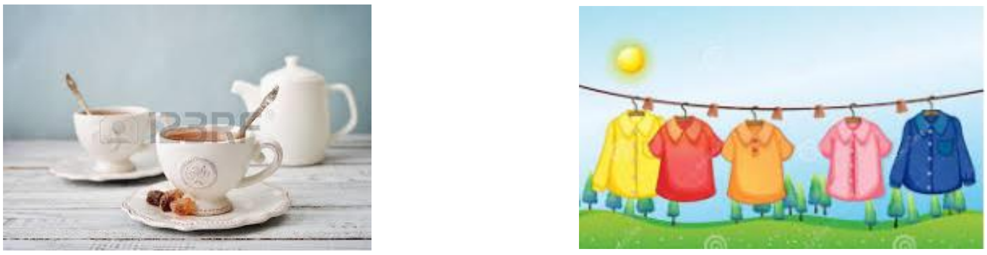
1.8 Introduction to Mass Transfer

Phenomena of mass transfer is seen everywhere in nature. The transport of one component in a mixture from a higher concentration region to that of lower concentration region is called mass transfer. Principle of mass transfer is used in many industries and chemical engineering processes. There is a natural tendency for mass to be transformed from a region of higher concentration to the region of lower concentration. Some examples of mass transfer flow are evaporation of water from pond, lake, water reservoir to the atmosphere, separation of chemical species in distillation columns, the diffusion of impurities in rivers, oceans, etc., Mass is transferred from one region to another using the following two modes,

- Convection
- Diffusion

The common examples of mass transfer are,

- A lump of sugar added to a cup of tea. The sugar dissolves in the liquid and diffuses throw out uniformly.
- Drying of clothes under the sun.



A lump of sugar added to a cup of tea *Drying out the clothes under the sun*

Figs 1.16 Examples of mass transfer

1.8.1 Convection

The mass transfer due to the movement of fluid is known as convection mass transfer. There are two types of convective mass transfer. They are,

- Free or Natural Mass Convection
- Forced Mass Convection

When a concentration difference produces a density difference that results in mass movement, the process is called free or natural mass convection.

The mass motion of the fluid caused by an external applied force is called forced mass convection.

1.8.2 Diffusion

In diffusion mass transfer, the material is transferred due to the concentration difference. The word diffusion is derived from the Latin word "Diffundere", which means "to spread out". It is a substance or a material that "spreads out" from the region of high concentration to the region of low concentration. The main feature of diffusion is, bulk motion or bulk flow of mass transport is not required. Diffusion occurs as a result of the random thermal movement of molecules. During its movement, one molecule collides with other molecules and changes its speed and direction. So the rate of diffusion depends on the number of collisions between molecules. The process of diffusion continues until uniformity of concentration is reached, or in other words, a state of equilibrium has been achieved. Here equilibrium is defined as a state of uniform concentration in which no diffusion occurs through the molecules, which are in continuous thermal motion.

1.9 Applications of Mass Transfer

The applications of mass transfer are significant in chemical industry, petrochemical industry, separation and purification processes of fluid mixtures and in refineries. It is also used in biotechnology and in the processes of environmental protection. Almost all chemical processes require a preliminary purification step of the raw materials, and also separations of the products from byproducts. For example, extracting perfumes, dyes from plants, especially from flower, getting salt from sea water by evaporation and so on. For all these processes principle of mass transfer is used very much. Preparation of reactants for chemical syntheses, separation of products from the reaction mixture and purification of gases and liquids are all associated with mass transfer.

Mass transfer plays the dominant role in absorption, rectification, extraction and adsorption. Mass transfer has tremendous application in meteorology, solar physics, cosmic fluid dynamics, astrophysics and geophysics.

1.10 Governing Equations

The motion of a fluid is governed by conservation of laws – the conservation of mass, the conservation of momentum and the conservation of energy. These conservation laws can be written in the form of partial differential equations as well as in the form of integral equations.

Conservation of Mass

Conservation of mass states that, mass cannot be created in a fluid nor can be destroyed from it. This means, the amount of the fluid within the region is conserved. The general conservation principle is defined as,

Rate at which mass enters = Rate at which mass leaves from the region + Rate of accumulation of mass in the region.

or

In – out = Accumulation

In general, the amount of fluid entering is more than the fluid leaving. Hence there is an accumulation of fluid inside the control volume.

The law can be defined as, the variation of the mass in the given volume is change in the mass of system and unit vector of the system.

$$\frac{\partial \rho}{\partial t} + \rho(\nabla.\vec{q}) = 0 \tag{1.1}$$

where $\vec{q}$, t and ρ are respectively flow velocity of flowing fluid in the system, time and density. For an incompressible fluid in which ρ is constant, the equation reduces to

$$\nabla.\vec{q} = 0 \tag{1.2}$$

Conservation of Momentum

Conservation of momentum states that, the rate of momentum accumulation is equal to difference of rate of momentum in and rate of momentum out and sum of forces acting on the system. In other words, the total momentum of an isolated system of interacting bodies remains constant.

The equation of momentum is,

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho\vec{F} + \mu\nabla^2\vec{q} + \vec{j} \times \vec{B} \tag{1.3}$$

where $\rho,\ \vec{q},\ p, t,\ \mu, \vec{F},\ \vec{j},\ \vec{B}$ are respectively fluid density, flow velocity vector field, pressure of the fluid, time, dynamic viscosity, body force per unit mass, total electric current density and magnetic field.

Conservation of Energy

This law states that the energy can neither be created nor be destroyed but the energy can change its one form to another form. The equation of energy states that in a fixed volume in space within a given surface the difference in the rate of supply of energy going through the surface must be equal to the net rate of increase of energy in that volume. Energy equation is,

$$\rho C_p\left(\frac{dT}{dt}\right) = \kappa\nabla^2 T + \mu\varphi + \frac{j^2}{\sigma} \tag{1.4}$$

$$\varphi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 \tag{1.5}$$

where $\rho, C_p, T, \kappa, \mu, j, \sigma$ and φ are respectively density, specific heat of the fluid at constant pressure, temperature, thermal conductivity, dynamic viscosity, Joule heating, electrical conductivity and viscous dissipation.

Equation of Species Concentration

The equation of the species concentration shows the relation between various species and their concentration. The following is the equation of species concentration within the given volume.

$$\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = D\nabla^2 C + D_m\nabla^2 T \tag{1.6}$$

where $\vec{q}, C, t, D, D_m$ and T are respectively flow velocity vector field, concentration of the species, time, molecular diffusivity, thermal diffusivity and temperature.

1.11 Methodology

Ordinary differential equations arise in the study of the physical problems frequently. It has many applications in the real-world problems but solving these differential equations analytically is difficult. It is very important to solve these equations numerically and get the exact solutions. Hence, we have used the Fourth order Runge-Kutta method with shooting technique to get the numerical solution of the governing equations of the problems considered in this thesis.

1.12 Shooting Method

The shooting method is a well-known method for solving a boundary value problem by reducing it to an initial value problem.

Let us consider a non-linear second order differential equation,

$$y'' + p(x)yy' + r(x)y^4 = f(x) \tag{1.7}$$

$$y(a) = A, \qquad y(b) = B \tag{1.8}$$

Let us recast the problem as a sequence of two first order equations

$$y' = z$$

$$z' = f(x) - p(x)yz - r(x)y^4 \tag{1.9}$$

Now, if $y(a)$ & $z(a) = y'(a)$ were known, equation (1.9) would define an IVP, and could use a fourth order Runge-Kutta scheme to construct the solution in a step-by-step manner for values of $x > a$. Let assume some value for $z(a) = \alpha_1$. The system of two equations (1.9) may be integrated forward in x as an initial value problem. But, when we reach $x = b$, $y(b) = B$ will not be satisfied. Other value $y(b) = \gamma_1$.

Problem is to find an intelligent way to go back and adjust the guess for $y'(a)$ so that the condition at $x = b$ will be satisfied. Select another value of $y'(a)$, let $z(a) = \alpha_2$ and integrate again and produce another value $y(b) = \gamma_2$.

Our aim is to determine where this numerical function intersects the true boundary condition.

Having guessed two values α_1 and α_2 for $z(a) = y'(a)$,

$$z(a) = \alpha_1 \rightarrow y(b) = \gamma_1$$

$$z(a) = \alpha_2 \rightarrow y(b) = \gamma_2$$

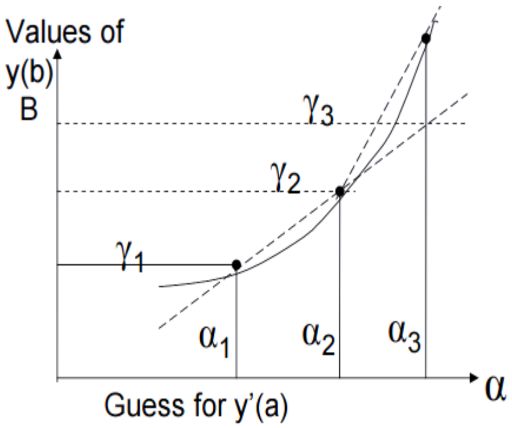


Fig 1.17 Schematic diagram for Shooting method

Equation of the line passing through (α_1, γ_1) and (α_2, γ_2)

Linear interpolation $\frac{\gamma - \gamma_1}{\gamma_1 - \gamma_2} = \frac{\alpha - \alpha_1}{\alpha_1 - \alpha_2}$ (1.10)

But we want $y = B$, so we guess to try for α_3

$$\alpha_3 = \alpha_1 + \frac{(\alpha_1 - \alpha_2)(B - \gamma_1)}{(\gamma_1 - \gamma_2)}$$

Use α_3 to start another integration of equation (1.9)

$$y'(a) = z(a) = \alpha_3 \rightarrow y(b) = \gamma_3$$

$(\alpha_1, \gamma_1), (\alpha_2, \gamma_2), (\alpha_{3,}\gamma_3)$ take a line between whichever of the three points have values of y closes to B, and use this line to obtain a new estimate of $z(a) = \alpha_4$

$$|\alpha_{i+1} - \alpha_i| \leq \varepsilon \quad \text{tolerance} \tag{1.11}$$

$$|\gamma_{i+1} - B| \leq \varepsilon$$

This technique is called shooting method.

Since the ODE associated with the IVP is of second order, it must normally be rewritten as a system of first order equations before it can be solved by Fourth order Runge-Kutta method.

1.13 Runge-Kutta Method

Runge-Kutta fourth order method is a well-tested and validated numerical method for initial value problems. To avoid the disadvantage of the Taylor series method, we can use Runge-Kutta method. In Runge-Kutta method with shooting technique, the boundary value problem (BVP) is first transformed in to an initial value problem (IVP). Then the IVP is solved by systematic guessing of missing initial values using shooting technique until the boundary conditions at ∞ asymptotically decay to zero value for several sets of parameters.

Runge-Kutta method was developed by the German mathematicians C.Runge and M.W. Kutta in 1900.

Let an initial value problem be,

$$\dot{y} = f(t, y), \qquad y(t_0) = y_0 \tag{1.12}$$

The general form of fourth-order Runge-Kutta method is given by,

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1.13}$$

$$t_{n+1} = t_n + h \text{ , for } n = 0,1,2,3,\dots.. \tag{1.14}$$

$$k_1 = f(t_n, y_n), \tag{1.15}$$

$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right), \tag{1.16}$$

$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right), \tag{1.17}$$

$$k_4 = f(t_n + h, y_n + hk_3). \tag{1.18}$$

These formulae yield an approximation of the fourth degree in h, that is, the expansion in powers of h of y_{n+1} defined by these formulae agree through terms of the fourth degree with the expansion of y_{n+1} obtained directly from the differential equation. It may be observed that RK4 uses four functional evaluations in the interval $[t_0, t_1]$. These points are shown as P_0, P_1, P_2 and P_3 in figure 1.18.

Runge-Kutta method is used in the field of engineering and science. This method is step by step method because the values of y are calculated by short steps. Comparing to all methods like Taylor's method, Euler's method, improved Euler's method, the better method is Runge-Kutta method.

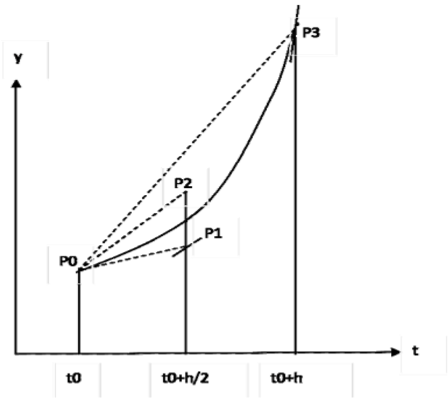


Fig 1.18 Schematic diagram for RK4 method

Merits

- The approach is faster than analytical methods.
- Easily adaptable by solving system of equations of any order.
- The results are straight forward and always give exact solution.

- Accurate, stable and easy to program.
- Special devices are not required for starting the computation.
- The length of the step can be modified at any time in the course of the computation without additional labor.
- One step method – global error is of the same order as local error.
- Easy for *Automatic Error Control*.

Demerits

- Requires more computer time.
- The cost of computer run is also high.
- Difficult to simulate realistic conditions.
- Change of the interval h in the course of a computation is somewhat problematic.

1.14 Basic definitions

One dimensional flow

The fluid flow constrained to single degree of freedom is called one-dimensional fluid flow. In one dimensional flow characteristics like cross sectional area, velocity and pressure vary in one direction and this direction is the direction of flow. In this world, there are some flows which are called one dimensional flow like ideal fluid passing through nozzle.

Two dimensional flow

The fluid flow constrained to two degrees of freedom is called two-dimensional fluid flow. In two dimensional flow characteristics like cross sectional area, velocity and pressure vary in two directions. Real fluid passing through a nozzle is an example of this type of flow.

Three dimensional flow

The most common physical fluid flow has all three degrees of freedom and the flow characteristics like cross sectional area, velocity and pressure vary in three dimensions like flow of air over vehicle and plane.

Thermal conductivity

Thermal conductivity is the inherent property of a material which is defined as the capability of the material to conduct heat from high temperature end to low temperature end. Materials like copper and aluminum are good conductors of heat and hence their thermal conductivity is higher than other materials like plastic and wood.

Using the Fourier's law we can define the thermal conductivity as the rate of heat transfer through a unit thickness of a material per unit area and per unit temperature difference. Thermal conductivity of most materials vary with temperature.

Thermal conductivity of most liquids decrease with increasing temperature. Water is, however, an exception to this rule. According to the kinetic theory of gases, the thermal conductivity of gases is proportional to the square root of the absolute temperature and inversely proportional to the square root of the molar mass. It is obvious that the thermal conductivity of a gas increases with the increasing temperature.

Material **Thermal conductivity**	**(W/m K)**
Copper (pure)	399
Gold (pure)	317
Aluminum (pure)	237
Iron (pure)	80.2
Carbon steel (1 %)	43
Stainless Steel (18/8)	15.1
Glass	0.81
Plastics	0.2 – 0.3
Wood (shredded/cemented)	0.087
Cork	0.039
Water (liquid)	0.6
Ethylene glycol (liquid)	0.26
Hydrogen (gas)	0.18
Benzene (liquid)	0.159
Air	0.026

Table 1.2 Thermal conductivity values for various materials at 300 K

Specific heat

Specific heat is defined as the amount of heat required to raise the temperature of unit mass of the substance by one degree. It is the inherent property of a material. Specific heat is measured in Joule/Kg Kelvin.

Typical values of C_p for various materials (at 300 K) are shown below:

Material	$\boldsymbol{C_p}$**(J/Kg.K)**
Aluminum (pure)	903
Copper (pure)	385
Gold	129
Silicon	712
Water	4180
Air	1005

Table 1.3 Specific heat for various materials

Thermal Diffusivity

When a temperature gradient is applied to a material, the heat travels from the high temperature region to the low temperature region. A measure of how heat propagates through a medium may be defined by the ratio of the heat conducted through the material to the heat stored in the material.

Heat capacity is defined as the product of density and specific heat ρC_p. The thermal diffusivity is defined as:

$$\alpha = \frac{k}{\rho C_p} \tag{1.19}$$

The thermal diffusivity is, therefore, the ratio of heat conducted through the material to the heat stored per unit volume. The larger the thermal diffusivity the faster the propagation of heat into the material. If the thermal diffusivity is small it means that a big part of the heat is absorbed by the material and only a small portion is conducted through.

Some typical values of thermal diffusivity are:

Materials	$\boldsymbol{\alpha = \frac{k}{\rho C_p}}$ **(m^2/s)**
Copper (pure)	113e-6
Glass	0.34e-6
Air	22.1e-6
Aluminum (pure)	97.5e-6
Gold	127e-6
Water	0.14e-6

Table 1.4 Thermal Diffusivity for various materials

1.15 Physical Parameters

In the study of hydrodynamics fluid flow, several physical parameters, which characterize the flow pattern of the fluid, are called as dimensionless numbers. Now we have discussed a few physical parameters, which we have used in our problem.

Hartmann Number

Hartmann number is measured as the ratio of magnetic force to the viscous force. The hydromagnetic effects are important when the Hartmann number is significant and is given by

$$M = \sqrt{\frac{\sigma B_0^2 L^2}{\mu}}$$

where B_0, σ, L and μ are respectively the magnetic field, electrical conductivity, length and coefficient of viscosity. Magnetic Reynolds numbers R_m for some typical MHD flows are given by:

Substance	$\boldsymbol{R_m}$
Mercury	0.01
Liquid Sodium	0.1
Laboratory Plasma	10
Earth's Core	10^6
Interstellar Gas	10^{17}

Table 1.5 Typical values of Magnetic Reynolds number

Prandtl Number

When the temperature distribution is present in the flow, then we must study the physical parameter Prandtl number. It is measured as the ratio of kinematic viscosity to the thermal diffusivity. It is written as

$$Pr = \frac{viscous\ diffusion\ rate}{thermal\ diffusion\ rate} = \frac{\mu C_p}{\kappa}$$

where C_p is the specific heat at constant pressure and κ is the thermal conductivity. Prandtl number $Pr \ll 1$ means the thermal diffusivity dominates, whereas $Pr \gg 1$ means the momentum diffusivity dominates the behavior.

Some typical values of Pr are:

Fluid	**Prandtl Number**
Air and many other gases,	Around 0.7-0.8
Earth's mantle	Around 1×10^{25}
Engine oil	Between 100 and 40,000
For R-12 refrigerant	Between 4 and 5
Mercury	Around 0.015
Mixtures of noble gases or noble gases with hydrogen	Around 0.16-0.7
Seawater (At 0 degrees Celsius and 20 degrees Celsius respectively)	13.4 and 7.2
Water (At 20 degrees Celsius)	7.01
Dry air	0.71
Freon(different chlorofluorocarbons)	3
Ethanol Liquid	14.4
Liquid metals	0.004-0.03
Gases	0.7-1.0

Table 1.6 Typical values of Prandtl Number

Rotation Number

It is a dimensional quantity which is the ratio of Coriolis force and inertial force and is defined by

$$\tau = 2\Omega d/u_0$$

where Ω, d and u_0 are respectively the angular velocity of rotation, length scale and velocity of the flow.

Some typical values of rotational temperature θ_R are given by:

Molecules	$\boldsymbol{\theta_R\ (K)}$
CO_2	0.561
F_2	1.27
H_2	87.6
HCl	15.2
HF	30.2
N_2	2.88
O_2	2.08

Table 1.7 Typical values of Rotational Temperature

Suction Parameter

Suction parameter is the ratio of suction velocity to the characteristic velocity. This number is used in hydraulics. It is given by,

$$S = \frac{V_0}{U}$$

where V_0 is the suction/injection velocity and U is characteristic velocity.

Reynolds Number

The Reynolds number is defined as the ratio of inertial forces to viscous forces. It is given by,

$$Re = \frac{inertial\, force}{viscous\, force} = \frac{UL}{\gamma}$$

where U is the mean velocity of the fluid (m/s),

L is the characteristic length (m) and γ is the kinematic viscosity (m^2/s).

Some typical values of Re are:

Fluids	**Reynolds Number**
Spermatozoa	0.0001
Blood flow in the brain	100
Blood flow in the aorta	1000
Typical pitch in the major leagues Baseball	200,000
Person swimming	4,000,000
Blue whale swimming	300,000,000
A Large ship(RMS Queen Elizabeth II)	5,000,000,000
Laminar flow	less than 2300
Transient flow	between 2300 to 4000
Turbulent flow	greater than 4000

Table 1.8 Typical values of Reynolds Number

The value of Reynolds number denotes the quality of flow. If it is less than 2000, the flow is considered to be streamlined; if the value is more than 3000, the flow is turbulent.

Schmidt Number

The Schmidt number is defined as the ratio of momentum diffusivity to mass diffusivity. It is given by

$$Sc = \frac{\gamma}{D_m} = \frac{viscous\, diffusion\, rate}{mass\, diffusion\, rate}$$

where, γ is the kinematic viscosity, D_m is the mass diffusivity.

Some typical values of Sc are:

Fluid	Schmidt Number
Ethyl Benzene	2.01
Hydrogen (H_2)	0.22
Helium (He)	0.30
Water vapour	0.60
Ammonia (NH_3)	0.78
Methane gas in air	1.06
Ethanol Liquid in water	1100

Table 1.9 Typical values of Schmidt Number

Grashof Number

The Grashof number is the dimensionless number which is defined as the ratio of buoyancy to viscous force acting on the fluid. It frequently arises in the situations involving natural convection. It is used in analyzing the velocity distribution in free convection systems. It is defined by

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\gamma^2}$$

where g, β, γ, T_s and T_∞ represent respectively the acceleration due to Earth's gravity, coefficient of thermal expansion, kinematic viscosity, surface temperature and temperature far away from the sheet.

The transition to turbulent flow occurs in the range $10^8 < \mathrm{Gr} < 10^9$ for natural convection from vertical flat plates. At higher Grashof numbers, the boundary layer is turbulent, at lower Grashof numbers, the boundary layer is laminar.

Modified Grashof number

The Modified Grashof number is the ratio of the buoyancy to viscous force acting on a fluid. The Grashof number can be modified by species concentration rather than the temperature. It is defined by

$$Gm = \frac{g\alpha\Delta C L^3}{\gamma^2}$$

where $g, \alpha, \Delta C, L$ and γ are represent respectively gravity, thermal expansion coefficient, concentration difference, length scale and kinematic viscosity (m^2/s).

Soret Effect

Soret effect (also known as thermo diffusion or thermophoresis) is defined as the migration of a colloidal particle of large molecules in a solution in response to a macroscopic temperature gradient. Soret coefficient is defined as the ratio of the thermal diffusion coefficient and the species diffusion coefficient. It is a measure of the degree of separation of the species.

Nusselt Number

Nusselt number is defined as the ratio of convective to conductive heat transfer across the boundary. A Nusselt number of order unity would indicate a sluggish motion little more effective than pure fluid conduction: for example, laminar flow in a long pipe. A large Nusselt number means very efficient convection: For example, turbulent pipe flow yields of order 100 to 1000.

$$Nu = \frac{Convective\ heat\ transfer}{Conductive\ heat\ transfer} = \frac{hL}{\kappa}$$

where L - characteristic length

h - convective heat transfer coefficient of the fluid

κ - thermal conductivity of the fluid.

Flow Conditions	**Average Nusselt Number**	**Restrictions**
Laminar	$Nu = 0.664\, Re^{1/2} Pr^{1/3}$	$Pr \geq 0.6$
Turbulent	$Nu = \left(0.037 Re^{\frac{4}{5}} - A\right) Pr^{1/3}$	$0.6 \leq Pr \leq 60$

Table 1.10 Typical values of Nusselt Number

Sherwood Number

The Sherwood number is used in mass-transfer operation. It is defined as the ratio of convective to diffusive mass transport. It is given by

$$Sh = \frac{Convective\ mass\ transfer\ coefficient}{Diffusive\ mass\ transfer\ coefficient} = \frac{K}{D/L}$$

where L - characteristic length

D - mass diffusivity

K - mass transfer coefficient.

Fluids	**Sherwood Number**
Gas	$5 \times 10^{-5} – 5 \times 10^{-4}$
Gas-Liquid interfacial surface area	10^{2} - 10^{3}
Liquid	$10^{-9} – 10^{-8}$

Table 1.11 Typical values of Sherwood Number

1.16 Thesis Outline

Given the wide range of applications of porous medium, the study of transport in porous medium has become quite important and numerical study of these flows is the objective of this dissertation.

The broad outline of the thesis is as follows,

- Introduction
- Literature Review
- Effect of magnetic field on steady boundary layer slip flow along with heat and mass transfer over a flat porous plate embedded in a porous medium
- Effect of Hall current on heat and mass transfer of free convective flow over a flat porous plate embedded in a porous medium
- Combined effects of Hall and ion-slip currents on steady free-convective MHD flow of an incompressible viscous and electrically conducting fluid with heat and mass transfer over a porous flat plate embedded in a porous medium
- Thermal diffusion and magneto hydrodynamic effects on heat and mass transfer of steady, viscous incompressible, electrically conducting fluid in a rotating disk embedded in a porous medium
- An investigation of heat and mass transfer of three dimensional magneto hydrodynamics free convective flow over a flat porous plate embedded in a porous medium
- Effect of thermal radiation and chemical reaction on three dimensional magneto hydrodynamics fluid flow in a porous medium
- Concluding Remarks

In Chapter I preliminary aspects of the flows through porous medium are presented. In Chapter II, significant earlier contributions related to the problem studied in the thesis are summarized.

Chapter III extends the work of Asim Aziz *et al.* (2014) to understand the effect of magnetic fluid on a steady boundary layer slip flow along with heat and mass transfer over a flat porous plate embedded in a porous medium. Numerical solutions are found for velocity, temperature and concentration profiles.

Chapter IV presents the effect of Hall current on heat and mass transfer of free convective flow over a flat porous plate embedded in a porous medium. The effects of permeability, magnetic parameter, Schmidt number, Soret number and Hall parameter over velocity, temperature and concentration profiles, skin friction, rate of heat and mass transfer at the plate are discussed in detail. Uniform magnetic field $\vec{B}_0$ is imposed along y-axis and the effect of Hall current is taken into account.

Chapter V investigates the combined effects of Hall and ion-slip currents on steady free-convective MHD flow of an incompressible, viscous and electrically conducting fluid with heat and mass transfer over a porous flat plate embedded in a porous medium.

Chapter VI analyses the effect of thermal diffusion and magnetic field on the steady free convective, viscous incompressible flow of an electrically conducting fluid along with heat and mass transfer due to a rotating disk embedded in a porous medium using shooting procedure with fourth order Runge-Kutta Method. The fluid is subjected to an external uniform magnetic field perpendicular to the plane of the disk. The velocity, temperature and concentration distributions are obtained for different governing parameters. It is inferred from the graphs that increase in the Darcy number and magnetic parameter M decrease the axial and tangential skin-friction.

Chapter VII deals with the heat and mass transfer of three dimensional magneto hydrodynamics free convective flows over a flat porous plate embedded in a porous medium. Here, we have assumed that the sheet is stretched in both the directions along the $xy-$plane. Moreover, we have considered the magneto hydrodynamic effect within the fluid and convective condition along the surface.

Chapter VIII investigates the effect of thermal radiation and chemical reaction on three dimensional MHD fluid flow in porous medium. Numerical solutions are found for velocity, temperature and concentration profiles. We observed that increase in magnetic parameter tends to reduce skin-friction and increase the rate of heat and mass transfer. Increase in radiation parameter enhances the rate of heat transfer and decrease mass flux. Increase in chemical reaction decreases rate of mass transfer.

Chapter IX summarizes the significant results obtained from these problems.

In all the above mentioned problems, the effect of MHD fluid flow through porous medium is studied. The similarity transformations are used to transform the governing partial differential equations into nonlinear ordinary differential equations. The resulting system of ordinary differential equations are then reduced to a system of first order differential equations which are solved by shooting procedure using fourth order Runge-Kutta Method. The numerical results of the flow characteristics are presented graphically using the mathematical softwares FORTRAN Powerstation and Mathematica 8.0.