CHAPTER II

Literature Review

In recent years the theory of boundary layer has received the attention of many researchers.

Prandtl (1904) introduced boundary layer theory to understand the flow structure of a viscous fluid near a solid boundary. The early contribution in this field is due to Blasius (1908). Blasius solved the famous boundary layer equation for a flat moving plate problem and found a power series solution of the model.

Rotating disk flows of conducting fluids have practical applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, viscometry and crystal growth processes. In most cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the applications of magneto hydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable. Stuart (1954) studied the effect of uniform suction on the steady flow due to a rotating disk and obtained an analytic series solution for the strong suction case.

The effect of simultaneous action of buoyancy and induced magnetic forces on free convective heat transfer was studied by Sparrow and Cess (1961). The analysis was carried out for laminar boundary layer flow about an isothermal vertical plate. It was observed that the free convective heat transfer to liquid metals is affected significantly by magnetic field, but other fluids experienced very small effects.

Crane (1970) found an exact similarity solution in closed analytical form for steady boundary layer flow of an incompressible viscous fluid past a stretching plate.

Gupta and Gupta (1974) studied the effect of radiation on combined free and forced convection of an electrically conducting fluid flow through an open end vertical channel in the presence of a uniform magnetic field. Closed form solutions for the velocity, temperature and the induced magnetic field were obtained in the optically thin limit case when the wall temperatures are varying linearly with the vertical distance. He found that radiation tends to increase the rate of heat transport to the fluid thereby reducing the effect of natural convection.

Three dimensional laminar flow of a viscoelastic fluid past a stretching sheet was studied by Ariel (1994). He found an analytical solution when the fluid was

extracted from the sheet at a uniform rate. He also observed that when there is a suction of the fluid, the solutions are possible only up to a critical value of the viscoelastic parameter. Also, for values less than this critical value, dual solutions exist.

Attia (2002) studied the flow of a dusty fluid in the presence of Hall and ion slip current using analytical procedure. He considered the transient Hartmann flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plate. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to a constant pressure gradient.

Steady three-dimensional flow of an incompressible, viscous fluid past a stretching sheet using the homotopy perturbation method was studied by Donald Ariel (2007). Three-dimensional flow of a viscous incompressible fluid through a porous medium bounded by two vertical walls, one wall being impermeable and the other is permeable was discussed by Guria *et al*. (2009). He found an approximate solution of the velocity and temperature fields using a perturbation technique. It was found that the cross flow increases near the impermeable wall and decreases near the permeable wall with increase in Darcy number, but it decreases with an increase in the source parameter.

The effect of Hall and ion-slip currents on fully developed electrically conducting fluid flow between vertical parallel plates in the presence of a temperature dependent heat source was investigated using homotopy analysis method by Srinivasacharya and Kaladhar (2012). Later, Darbhasayanam and Kaladhar (2012) considered the effect of Hall and ion-slip currents on electrically conducting couple stress fluid flow between two circular cylinders in the presence of a temperature dependent heat source. He found an analytical solution using homotopy analysis method.

Heat and mass transfer of a MHD free convective viscous incompressible fluid flow through a highly porous medium with periodic permeability in presence of chemical reaction was studied by Pravat Kumar Rath *et al.* (2013). In the presence of periodic permeability, the solutions of the coupled non-linear equations of the main flow are assumed as the superimposition of the perturbed solutions with a small amplitude of variation on the two-dimensional flow with a small amplitude of variation are considered.

Anjalidevi and Kayalvizhi (2013) employed nonlinear hydromagnetic flow with radiation and heat source over a stretching surface with prescribed heat and mass flux embedded in a porous medium. The system of equations was solved using confluent hypergeometric functions by the usage of suitable transformations.

Unsteady three-dimensional MHD flow due to impulsive motion with heat and mass transfer in a saturated porous medium was studied by Rajagopal *et al.* (2013). He also studied the development of velocity, temperature and concentration fields of an incompressible viscous electrically conducting fluid, caused by impulsive stretching of the surface in two lateral directions and by suddenly increasing the surface temperature from that of the surrounding fluid in a saturated porous medium. Governing equations are solved analytically. For some particular cases, closed form solutions are obtained, and for large values of the independent variable asymptotic solutions are found.

Sarma and Mahanta (2015) studied the effect of chemical reaction on three-dimensional mass transfer flow in the presence of magnetic field using perturbation technique.

The above mentioned studies dealt with heat and mass transfer of free convective flows using parametric approach. In this literature, we also found many work regarding the study of heat and mass transfer of free convective flows using various numerical methods. This type of solutions dates back to 1921 wherein Von Karman (1921) studied the hydrodynamic steady flow problem due to an infinite rotating disk. He introduced a tractable method to transform the system of partial differential equations into nonlinear ordinary differential equations which simplified the mathematical handling. Solutions are found using momentum integral method.

Cochran (1934) obtained the asymptotic solutions for the steady flows due to a rotating disk. He resolved Karman's problem and obtained a more accurate numerical solution. Compressible laminar flow and heat transfer about a rotating disk was studied by Ostrach and Thornton (1958). If viscous dissipation is neglected, the compressible problem is correlated to the incompressible problem by assuming linear variations of viscosity and thermal conductivity with temperature.

Sparrow and Gregg (1960) studied the effect of Prandtl number on steady state heat transfer due to a rotating disk at a constant temperature. He considered the gaseous systems which are composed of either one or two component gases. He also

studied the effect of mass injection or removal at the surface on heat and mass transfer about the rotating disk.

Sakiadis (1961) was the first to initiate the study of boundary layer flow on a continuous moving surface. Flow of this type gives a new class of boundary layer problems, with solutions substantially different from those for boundary layer flow on surfaces of finite length. He derived the basic differential and integral momentum equations for boundary layer theory for continuous surfaces.

Later, Sakiadis (1961) investigated the laminar boundary layer behavior on moving continuous flat surface by two methods, integral method and numerical solution. The turbulent boundary layer behavior on a moving continuous flat surface was investigated by the integral method only. Equations for the boundary layer thickness, displacement thickness, momentum thickness and skin friction were presented for both laminar and turbulent boundary layers.

The effects of Hall current are very important if the strong magnetic field is applied and electromagnetic force is noticeable due to strong magnetic field. Steady laminar incompressible flow of an electrically conducting fluid through a straight channel of arbitrary cross section with conducting or non-conducting walls in the presence of a uniform transverse magnetic field and Hall current was studied by Tani (1962). He carried out the numerical calculation in the case of a rectangular channel.

The steady flow of an incompressible, viscous and electrically conducting fluid due to a rotating disk of infinite radius was investigated in the presence of a transverse magnetic field by Katukani (1962). This was an extension of the well-known Karman's problem to the hydomagnetic case. He assumed that the magnetic Prandtl number of the fluid is so small, that magnetic field is not affected by the electric currents flowing in the fluid.

A high accurate solution of the Von Karman (1921) problem was obtained by Benton (1966). Benton (1966) developed Cochran's (1934) solutions to solve the unsteady problems. The Von Karman (1921) rotating disk problem was extended to the case of flow started impulsively from rest and also the steady state problem was solved to a higher degree of accuracy than Cochran's (1934) well known solution. Benton (1966) solved the first four equations for velocity components exactly in closed form, and the next six by numerical integration. This gives four terms in the series for the primary flow and three terms in each series for the secondary flow.

Katagiri (1969) investigated the effect of Hall current on the steady boundary layer flow in the presence of a transverse magnetic field on the assumption of small magnetic Reynolds number. The numerical investigation of boundary layer equations were obtained by the difference-differential method.

When applying the Ohm's law Hall current was ignored, because it has no remarkable effect for small and moderate values of magnetic field. Hall current are very important if the strong magnetic field is applied because for strong magnetic field electromagnetic force is noticeable was studied by Cramer and Pai (1973).

Hall effects on hydromagnetic flow were studied by Pop and Soundalgekar (1974). Gupta and Gupta (1977) studied heat and mass transfer on a stretching sheet with suction or blowing. The heat transfer of MHD Couette flow in the presence of Hall and Ion-slip currents was studied by Soundelgekar *et al.* (1979). It was observed that the flow may become unstable when magnetic field is small and Hall and Ion-slip parameters are large.

Three-dimensional flows provide more physical insights of the real world problems when compared with two-dimensional flows. Three-dimensional flow due to a stretching flat surface was studied by Wang (1984) and the author found an exact similarity solution of the Navier-Stokes equations.

An exact analysis of a generalized MHD Couette flow, on taking into account the Hall current was studied by Soundalgekar and Uplekar (1986). Expressions for the axial and transverse components of velocity and the skin friction, were derived. He assumed the linear variation of the temperature along the plate and derived the exact solution to the energy equation.

The effect of Hall current on an unsteady free convection flow of a viscous incompressible and electrically conducting fluid, in presence of foreign gases (such as H_2, CO_2, H_2O, NH_3) along an infinite vertical porous flat plate subjected to a transpiration velocity inversely proportional to the square-root of time in the presence of a uniform transverse magnetic field was studied by Hossain and Rashid (1987).

Later Hossain and Arbad (1988) analyzed the effect of Hall current on the hydromagnetic free convection flow of an electrically conducting viscous incompressible fluid past an impulsively accelerated porous plate in the presence of uniform magnetic field. Numerical solutions were obtained for the axial and transverse components of velocity and skin-friction using Crank-Nicolson implicit finite-difference method.

Three-dimensional unsteady flow with heat and mass transfer over a continuous stretching surface was studied by Lakshmisha *et al*. (1988). He considered both constant temperature and constant heat flux conditions at the wall. Self-similar solutions are shown to exist when either the rate of stretching decreases with time or is constant. Situations studied include deceleration in stretching of the boundary, mass transfer at the surface, saddle and nodal point flows and the effect of a magnetic field.

Effect of an external uniform magnetic field on the flow due to a rotating disk was studied by Attia (1995). The effect of radiation on the forced and free convection flow of an optically dense viscous incompressible fluid along a heated vertical flat plate with uniform free stream and uniform surface temperature with Rosseland diffusion approximation was studied by Hossain and Takhar (1996). With appropriate transformations the boundary layer equations governing the flow were reduced to local non-similarity equations. Solutions of the governing equations were obtained by employing the implicit finite difference methods together with Keller box scheme.

Anjalidevi and Kandasamy (2000) explored the effects of magnetic field on heat and mass transfer flow along a semi-infinite horizontal plate. They assumed the fluid to be viscous, incompressible and electrically conducting. An approximate solution for the steady laminar flow along a semi-infinite horizontal plate was obtained using numerical technique.

Chamkha (2000) studied the problem of transient, unsteady three-dimensional natural convection laminar fluid flow over an inclined permeable surface in the presence of MHD and resulting IVP was solved using implicit finite-difference method. Combined effects of Hall and ion-slip currents on free convective flow past a semi-infinite vertical plate was investigated by Abo-Eldahab and Aziz (2000). He found the numerical approach for deriving fundamental equations on the assumption of small magnetic Reynolds number. He considered the effect of internal heat generation or absorption.

Miklavcic and Wang (2004) extended Von Karman's (1921) problem by studying the effects of slip where the surface of the rotating disk admits partial slip. The nonlinear similarity equations are integrated accurately for the full range of slip coefficients using numerical approach. Pop *et al.* (2004) analyzed the radiation effects on the flow of an incompressible viscous fluid over a flat sheet near the stagnation point. The system of ordinary differential equations was solved numerically using the Runge-Kutta method coupled with the shooting technique.

Later Attia (2009) studied the steady flow of an incompressible viscous fluid along with heat transfer due to an infinite rotating disk in a porous medium. He considered the effect of porosity of the medium on the velocity and temperature distributions. Nonlinear governing equations were solved numerically using Crank-Nicolson method.

Hydromagnetic mixed convective heat and mass transfer over a vertical plate with a convective surface boundary condition was studied by Aziz (2009). He considered the classical problem of hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid. He used numerical approach for attaining the possible solutions.

The effect of temperature-dependent viscosity on free convective flow past a vertical porous plate was studied by Makinde (2010) in the presence of a magnetic field without heat source and with heat source. He used Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method to solve the reduced system of equations.

Steady laminar three-dimensional magneto hydrodynamic boundary layer flow and heat transfer over a stretching surface in a viscoelastic fluid was studied by Ahmad and Nazar (2011). They used finite-difference scheme known as the Keller-box method for solving the governing equations.

Effect of thermal radiation and magneto hydrodynamics on heat and mass transfer flow of viscous and incompressible fluid near the stagnation point towards a stretching sheet in porous medium was carried out by Salem and Fathy (2012). The stretching velocity and the ambient fluid velocity are assumed to vary linearly with the distance from the stagnation point. The Rosseland approximation is used to describe the radiative heat flux in the energy equation.

Krishnendu Bhattacharyya *et al.* (2012) studied the effect of chemical reaction on boundary layer stagnation point of an electrically conducting viscous, incompressible fluid over a shrinking sheet with suction or blowing in the presence of magnetic field using quasilinearization technique. The flow is permeated by an externally applied magnetic field normal to the plane of flow. Finite difference method is used to solve the self-similar equations.

Asim Aziz *et al.* (2014) studied a simplified model of an incompressible fluid flow along with heat and mass transfer past a porous flat plate embedded in a Darcy type porous medium. The resulting system of ordinary differential equations reduced

to a system of first order differential equations was solved numerically using Matlab bvp4c code. He observed that the increase in permeability of the porous medium increase the velocity and decrease the temperature profile. This happens due to a decrease in drag of the fluid flow.

Shampa Ghosh (2015) studied magneto hydrodynamic boundary layer flow over a stretching sheet with chemical reaction. The surface velocity of the stretching sheet and the transverse magnetic field were assumed to vary as a linear function of the distance from the origin. The systems of equations were solved numerically by using a finite difference scheme. He revealed that the effect of magnetic field and chemical reaction have significant role in controlling the rate of heat transfer in the boundary layer region.

Since the better and more realistic approach to obtain the flow behavior is to study the two and three dimensional flows, this thesis aims at presenting the analysis of various two and three dimensional flow scenarios through porous medium.

Motivated by the above interesting characteristics of flow behavior, in the following chapters we have discussed the heat and mass transfer of convective flow through porous medium to include the effect of magnetic field. The flow behavior is analyzed numerically. The similarity transformations are used to transform the governing partial differential equations into nonlinear ordinary differential equations. The resulting system of Ordinary differential equations is then reduced to a system of first order differential equations which are solved by shooting procedure using fourth order Runge-Kutta Method.