

# Volume Minimization of a Closed Coil Helical Spring Using ALO, GWO, DA, FA, FPA, WOA, CSO. BA, PSO and GSA

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**Abstract** Springs are important members often used in machines to exert force, absorb energy and provide flexibility. In mechanical systems, wherever flexibility or relatively a large load under the given circumstances is required, some form of spring is used. In this paper, non-traditional optimization algorithms, namely, Ant Lion Optimizer, Grey Wolf Optimizer, Dragonfly optimization algorithm, Firefly algorithm, Flower Pollination Algorithm, Whale Optimization Algorithm, Cat Swarm Optimization, Bat Algorithm, Particle Swarm Optimization, Gravitational Search Algorithm are proposed to get the global optimal solution for the closed coil helical spring design problem. The problem has three design variables and eight inequality constraints and three bounds. The mathematical formulation of the objective function  $U$  is to minimize the volume of closed coil helical spring subject to constraints. The design variables considered are Wire diameter  $d$ , Mean coil diameter  $D$ , Number of active coils  $N$  of the spring. The proposed methods are tested and the performance is evaluated. Ten non-traditional optimization methods are used to find the minimum volume. The problem is computed in the MATLAB environment. The experimental results show that Particle Swarm Optimization outperforms other methods. The results show that PSO gives better results in terms of consistency and minimum value in terms of time and volume of a closed

coil helical spring compared to other methods. When compared to other Optimization methods, PSO has few advantages like simplicity and efficiency. In the future, PSO could be extended to solve other mechanical element problems.

**Keywords** Non-traditional Optimization, Closed Coil Helical Spring, Volume Minimization

## 1. Introduction

A helical spring or coil spring is a mechanical device, which is normally used to store energy and release it subsequently, to maintain a force between contacting surfaces or to absorb shock.

Helical spring is made by twisting a wire or coil with small diameter in the form of helix having rectangular, square, or circular cross section wrapped around an imaginary cylinder, which can undergo considerable deflection without getting permanently distorted. Two commonly used helical springs are tensile and compression helical spring.

The gap between two adjacent coils of the helix can be varied according to the desired property. Based on this gap,

helical springs are classified as open and close coiled helical spring.

In closely coiled helical spring, wire is tightly wound so that there is no gap between two adjacent coils of the spring. Spring's helix angle is less than 10° or 10°. When the spring wire is closely coiled the plane having each turn is at right angles nearly to the axis of the helix and the wire is subjected to torsion. Shear stresses due to twisting are the major stress produced in helical springs. The load applied is along the axis or parallel to the spring [5].

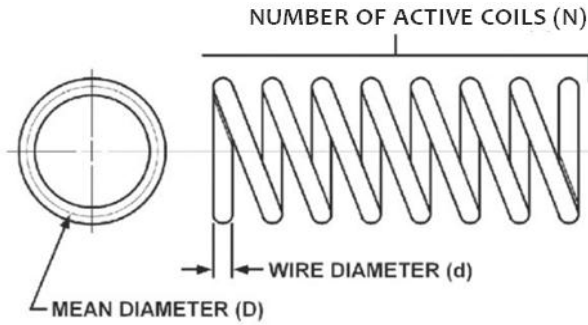


Figure 1. Closed Coil Helical Spring [6]

1.1. Formulation of Problem

The design of the closed coil helical spring is considered with the Number of active coils(N), wire diameter(d), mean coil diameter of spring (D), minimum wire diameter(d<sub>min</sub>), maximum working load (F<sub>max</sub>), preload compressive force(F<sub>p</sub>),allowable shear stress(S), Spring index (C),modulus of rigidity(G), perturbation factor (δ), Stress factor or Wahl factor(C<sub>f</sub>), maximum perturbation factor (δ<sub>max</sub>), maximum outside diameter of spring (D<sub>max</sub>), spring stiffness (K), Free length (l<sub>f</sub>),deflection under the maximum load (δ<sub>l</sub>), deflection under preload (δ<sub>p</sub>), deflection from preload to maximum load (δ<sub>w</sub>) and allowable maximum deflection under preload (δ<sub>pm</sub>) respectively[2,3]. The objective is to volume minimization U of a coil spring under a constant tension/compression load.

$$\text{Minimize } U = \frac{\pi^2}{4}(N + 2)Dd^2$$

1.2. Constants

F <sub>max</sub>	453.6 kg
S	13288.02 kgf/cm <sup>2</sup>
G	808543.6 kgf/cm <sup>2</sup>
l <sub>max</sub>	35.56 cm
d <sub>min</sub>	0.508 cm
D <sub>max</sub>	7.62 cm
F <sub>p</sub>	136.08 kg
δ <sub>w</sub>	3.175 cm
δ <sub>pm</sub>	15.24cm

1.3. Design Variables

The design variables are as follows  
Wire diameter, d = x<sub>1</sub>  
Mean coil diameter, D = x<sub>2</sub>  
Number of active coils, N = x<sub>3</sub>

1.4. Constraints [4]

Stress Constraint

The specified value must be greater than shear stress and can be given as

$$S - \frac{8C_f F_{max} D}{\pi d^3} \geq 0 \tag{1}$$

where

$$C_f = \frac{4(C)-1}{4(C)-4} + \frac{0.615}{C}, C = \frac{D}{d} \tag{2}$$

where allowable shear stress, S = 13288.02 kgf/cm<sup>2</sup> and maximum working load, F<sub>max</sub> = 453.6 kg respectively.

Configuration Constraint

The maximum specified value must be greater than the free length of the spring. The following expression can be used to determine the spring constant (K)

$$K = \frac{Gd^4}{8N D^3} \tag{3}$$

shear modulus G =808543.6 kgf/cm<sup>2</sup>.

δ<sub>l</sub> is expressed as

$$\delta_l = \frac{F_{max}}{K} \tag{4}$$

It is assumed that the solid length is 1.05 times greater than spring length under F<sub>max</sub>. Thus, l<sub>f</sub> is given by

$$l_f = \delta_l + 1.05(N + 2) d \tag{5}$$

The constraint is given by

$$l_{max} - l_f \geq 0 \tag{6}$$

$$l_{max} = 35.56 \text{ cm.}$$

The specified minimum value must be less than wire diameter and the below condition must be satisfied:

$$d - d_{min} \geq 0 \tag{7}$$

$$d_{min} = 0.508 \text{ cm.}$$

The maximum specified must be greater than coil's outside diameter and is given by

$$D_{max} - (D + d) \geq 0 \tag{8}$$

$$D_{max} = 7.62 \text{ cm.}$$

At least 3d times must be mean coil diameter to make sure that the spring is firmly wound and given by

$$C - 3 \geq 0 \tag{9}$$

The maximum specified must be greater than deflection under preload. The  $\delta_p$  is given by

$$\delta_p = \frac{F_p}{K} \tag{10}$$

$$F_p = 136.08 \text{ kg.}$$

The constraint are

$$\delta_{pm} - \delta_p \geq 0 \tag{11}$$

$$\delta_{pm} = 15.24 \text{ cm.}$$

The combined  $\delta_p$  and the length must be consistent and it is expressed by

$$l_f - \delta_p = \frac{(F_{\max} - F_p)}{K} - 1.05(N + 2)d \geq 0 \tag{12}$$

This constraint should be equality. The constraint function will always be zero at convergence.

Specified value must be equal to  $\delta_w$ . Inequality constraint made by the two should converge to zero always. It is expressed by

$$\frac{(F_{\max} - F_p)}{K} - \delta_w \geq 0 \tag{13}$$

$$\delta_w = 3.175 \text{ cm.}$$

**1.5. Variables Bounds**

The lower and upper bounds of design variables mean coil diameter of spring (D, cm), wire diameter (d, cm) and Number of active coils (N) are

$$0.508 \leq d \leq 1.016$$

$$1.270 \leq D \leq 7.620$$

$$15 \leq N \leq 25$$

**1.6. Mathematical Formulation**

The mathematical formulation of the objective function U is to volume minimization of closed coil helical spring subject to the constraints.

$$\text{Minimize } U = 2.46740(x_3 + 2)x_2x_1^2$$

Subject to constraints:

$$13288.02 - \left[ \frac{1155.0829x_2}{x_1^3} \left[ \frac{4x_2 - x_1}{4x_2 - 4x_1} + \frac{0.615x_1}{x_2} \right] \right] \geq 0 \tag{1}$$

$$35.56 - \left[ \frac{0.004488x_3x_2^3}{x_1^4} \right] - (1.05(x_3 + 2)x_1) \geq 0 \tag{2}$$

$$x_1 - 0.508 \geq 0 \tag{3}$$

$$7.62 - (x_2 + x_1) \geq 0 \tag{4}$$

$$\frac{x_2}{x_1} - 3.0 \geq 0 \tag{5}$$

$$15.24 - \frac{0.0013464x_3x_2^3}{x_1^4} \geq 0 \tag{6}$$

$$\frac{0.00314165x_3x_2^3}{x_1^4} - (1.05(x_3 + 2)x_1) \geq 0 \tag{7}$$

$$\frac{0.00314165x_3x_2^3}{x_1^4} - 3.175 \geq 0 \tag{8}$$

and  $x_1, x_2, x_3 \geq 0$

the ranges for different variables are:

$$0.508 \leq x_1 \leq 1.016,$$

$$1.270 \leq x_2 \leq 7.620,$$

$$15 \leq x_3 \leq 25.$$

where

$x_1$  is Wire diameter, d

$x_2$  is Mean coil diameter, D and

$x_3$  is Number of active coils, N

**The ten Non Traditional Optimization Methods used are**

1. Ant Lion Optimizer
2. Grey Wolf Optimizer
3. Dragonfly optimization algorithm
4. Firefly algorithm
5. Flower Pollination algorithm
6. Whale optimization Algorithm
7. Cat Swarm Optimization
8. Bat Algorithm
9. Particle Swarm Optimization
10. Gravitational Search Algorithm

**Table 1.** Comparative Result of 10 Non-traditional Optimization Methods

	ALO	GWO	DA	FA	FPA	WOA	CSO	BA	PSO	GSA
d, cm	0.711175	0.711255	0.715463	0.684105	0.676615	0.71067	0.76394	0.672691	0.7102	0.709845
D, cm	1.318	1.3115	1.3255	1.33755	1.31	1.3205	1.281	1.3555	1.27	1.309
N	17.55	19.9	18.7	18.2	19.95	19.4	18.6	18.45	15	18.7
Time, seconds	0.89865	0.922	0.893	0.9	0.912	0.9	0.91	0.894	0.88	0.9012
Volume, cm <sup>3</sup>	48.12016	47.51955	48.34264	47.96541	49.61859	48.42831	47.60091	47.71368	46.0968	47.85545

## 2. Comparative Results

The 10 methods are run for 20 trails each and the final results were compared.

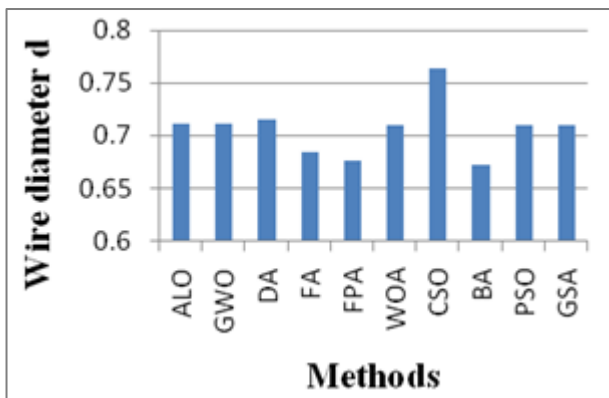
Wire diameter  $d = x_1$

Mean coil diameter  $D = x_2$

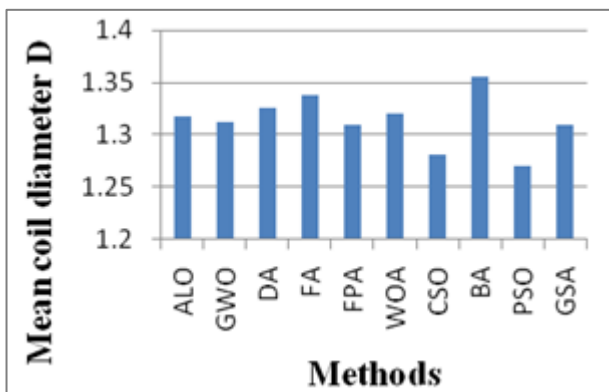
Number of active coils  $N = x_3$

**Table 2.** Boundary values of Closed Coil Helical Spring

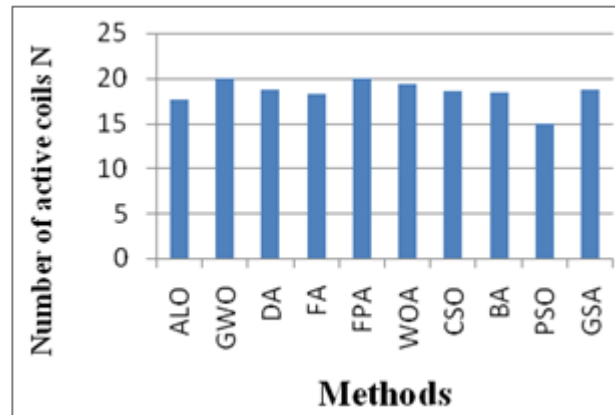
	d (= x <sub>1</sub> )		D (= x <sub>2</sub> )		N (= x <sub>3</sub> )
	cm	mm	cm	mm	No unit
Upper Bound	1.016	10.16	7.62	76.2	25
Lower Bound	0.508	5.08	1.27	12.7	15
Optimum	0.7102	7.102	1.27	12.7	15



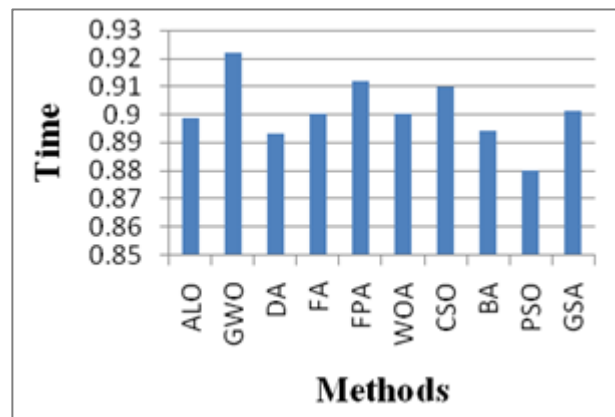
**Figure 2.** Results of 10 Methods for d



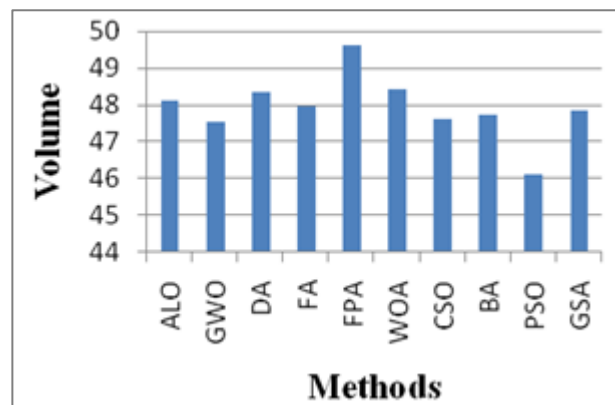
**Figure 3.** Results of 10 Methods D



**Figure 4.** Results of 10 Methods for N



**Figure 5.** Results of 10 Methods for Time



**Figure 6.** Results of 10 Methods for Volume

From the above graphs and comparative table 1, we know that the elapsed time is minimum in Particle Swarm Optimization(0.88 seconds) compared to Dragonfly Optimization Algorithm (0.893 seconds) and Bat Algorithm(0.894 seconds).Comparative result for volume is relatively less in the Particle Swarm Optimization( $46.0968\text{cm}^3$ ) when compared to Grey Wolf Optimizer ( $47.51955\text{cm}^3$ ) and Cat Swarm Optimization ( $47.60091\text{cm}^3$ ).

### 3. Results and Discussion

The optimization is carried out with different methods with the extreme two values of the parameters. The problem is run for 20 trials. Three different criteria are used to compare the methods they are:

#### 3.1. Consistency

The volume is minimum and consistency in the Particle Swarm Optimization ( $46.0968\text{cm}^3$ ) when compared to Grey Wolf Optimizer ( $47.51955\text{cm}^3$ ) and Cat Swarm Optimization ( $47.60091\text{cm}^3$ ).

#### 3.2. Minimum Run Time

Particle Swarm Optimization (0.88 seconds) has the minimum run time compared to Dragonfly Optimization Algorithm (0.893 seconds) and Bat Algorithm (0.894 seconds).

#### 3.3. The Simplicity of Algorithm

Particle Swarm Optimization minimizes the volume, run time and simplicity compared to Grey Wolf Optimizer and Cat Swarm Optimization. The PSO algorithm has the desirable characteristic in solving problems in engineering with higher computational effort [1].

## 4. Conclusions

In the present work, volume minimization of a closed coil helical spring has been investigated. Based on this a computer code was developed in Matlab, and solved using ten non-traditional optimization methods. The results show that PSO gives better results in terms of consistency and minimum value in terms of time and volume of a closed coil helical spring compared to other methods.

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