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Effect of Magnetic Field on Three Dimensional Fluctuating Couette Slip Flow Past Porous Plates

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Abstract

An analysis of the three dimensional flow of an viscous incompressible fluid between two horizontal porous flat plates separated by a finite distance in a slip flow regime is carried out under following conditions: the fluid is electrically conducting, the free stream velocity is uniform, the plate is subjected to a sinusoidal transverse suction velocity distribution and a magnetic field of uniform strength is applied in the direction normal to the plate. The influences of the various parameters on the main flow and cross flow velocity and skin friction are discussed with the help of graphs.

Keywords: Slip flow regime, MHD, porous medium, Couette flow, periodic suction

1. Introduction

The Phenomenon of MHD flow with heat transfer play an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aerodynamics Gersten and Gross (1974) studied the effect of slightly sinusoidal transverse suction velocity distribution on the flow and heat transfer over a plane wall. This suction velocity distribution leads to a cross flow and hence to a three dimensional flow over the surface.

Das et al (2008) investigated three dimensional Couette flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid.

A study on the effect of constant suction and sinusoidal injection on three dimensional Couette flow of a viscous incompressible electrically conducting fluid through a porous medium between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field was investigated by Das (2009).

Ahmed Sahin (2010) analysed the Magnetohydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime.

The effect of heat source on free convective flow of a incompressible, viscous, electrically conducting fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime in presence of a transverse magnetic field was studied by Das, Mishra and Mishra (2011).

Khem chand (2011) investigated the heat transfer and hydromagnetic boundary layer flow of an electrically conducting viscous, incompressible fluid over a continuous flat surface moving in a parallel free stream.

Das, Maity and Das (2012) analysed the unsteady free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate in a porous medium with constant suction in presence of a uniform transverse magnetic field.

The unsteady free convection flow and heat transfer of a viscous incompressible fluid past a vertical porous plate embedded in a porous medium was studied by Mishra (2014).

Guria and Jana (2006) analyzed the three dimensional fluctuating Couette flow through the porous plates with heat transfer. The plates are considered to be at a distance '*d*' apart. The stationary plate is subjected to a periodic suction and plate in uniform motion is subjected to uniform injection.

The aim of the present investigation is to study the effect of magnetic field and velocity slip on flow characteristic of an unsteady Couette flow between two horizontal parallel porous flat plates with periodic suction at the stationary plate and constant injection at the plate in motion. The periodic suction is assumed to be time dependent and perpendicular to the flow direction. This makes the flow to be three dimensional. The main flow velocity and cross flow velocity are calculated and plotted. The results obtained are validated for vanishing slip parameter with the results obtained by Guria and Jana (2006) for no slip condition.

2. Flow Description and Governing Equations

The flow under investigation has been modelled as an unsteady three dimensional flow of a viscous incompressible electrically conducting fluid between two horizontal flat porous plates separated by a distance d in a slip flow regime in presence of a transverse magnetic field B_0. We assume that the upper plate is moving with uniform velocity U in the direction of the flow. We choose a Cartesian coordinate system with its origin on the lower stationary plate, x^*-axis is in the direction of the flow, y^*-axis taken perpendicular to the plate and directed into fluid flowing laminarly with a uniform free stream velocity U and z^*-axis is normal to the $x^* y^*$-plane is introduced.

The upper plate is subjected to a constant injection $-V_0$ and the lower plate to a transverse sinusoidal time dependent suction velocity distribution of the form

$$v^* = -V_0\left[1 + \in Cos\left(\frac{\pi z^*}{d^*} - ct^*\right)\right] \tag{1}$$

where $\in$ is the amplitude of the suction velocity. The negative sign in equation (1) indicates that the suction is towards the plate.

Denoting dimensional velocity components as u^*, v^* and w^* in the directions x^*, y^* and z^* axes respectively, the governing equations are given by

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{2}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\sigma B_0^2 (u^* - U)}{\rho} \tag{3}$$

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) \tag{4}$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\sigma B_0^2 w^*}{\rho} \tag{5}$$

where ν is kinematic viscosity, ρ is density and p^* is fluid pressure.
The corresponding boundary conditions are

$$u^* = L_1^* \frac{\partial u^*}{\partial y^*};\; v^* = -V_0\left[1 + \in Cos\left(\frac{\pi z^*}{d^*} - ct^*\right)\right];\; w^* = L_2^* \frac{\partial w^*}{\partial y^*} \quad \text{at } y^* = 0$$

$$u^* = U;\quad v^* = -V_0;\quad w^* = 0;\quad \text{at } y^* = d \tag{6}$$

where $L_1^*, L_2^* = \left(\frac{2-m}{m}\right)L$ and $L = \mu\left(\frac{\pi}{2P\rho}\right)^{1/2}$ is the mean free path, m the Maxwell's reflection coefficient

By introducing the following non-dimensional parameters

$$y = \frac{y^*}{d};\; z = \frac{z^*}{d};\; t = ct^*;\; p = \frac{P^*}{\rho U^2};\; u = \frac{u^*}{U};\; v = \frac{v^*}{U};\; w = \frac{w^*}{U}; \tag{7}$$

$\mathrm{Re} = \frac{Ud}{\nu}$; the Reynolds number; $\lambda = \frac{cd^2}{\nu}$, the frequency Parameter;

$S = \frac{V_0}{U}$, the Suction Parameter; $M = \frac{\sigma B_0^2 d^2}{\rho\nu}$, Hartmann Number;

$h_1 = \frac{L_1^*}{d}$, velocity slip parameter; $h_2 = \frac{L_2^*}{d}$, velocity slip parameter;

The governing equations (2)-(6) can be rewritten in non-dimensional form as follows

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8}$$

$$\lambda\frac{\partial u}{\partial t} + \mathrm{Re}\left(v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - M(u-1) \tag{9}$$

$$\lambda\frac{\partial v}{\partial t} + \mathrm{Re}\left(v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\mathrm{Re}\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{10}$$

$$\lambda\frac{\partial w}{\partial t} + \mathrm{Re}\left(v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\mathrm{Re}\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - Mw \tag{11}$$

The corresponding boundary conditions are

$$u = h_1\frac{\partial u}{\partial y};\quad v = -S[1 + \in Cos(\pi z - t)];\; w = h_2\frac{\partial w}{\partial y};\quad \text{at } y = 0$$

$$u = 1;\quad v = -S;\quad w = 0;\quad \text{at } y = 1 \tag{12}$$

Here Re, λ denote Reynolds Number and frequency parameter respectively.

3. Solution of the Problem

When the amplitude of oscillation in the suction velocity is small $\in (<< 1)$, we assume u, v, w and p in the following form to solve the differential equations (8)-(11).

$$
\begin{aligned}
u(y,z,t) &= u_0(y) + \in u_1(y,z,t) + \in^2 u_2(y,z,t) + \ldots\ldots\ldots \\
v(y,z,t) &= v_0(y) + \in v_1(y,z,t) + \in^2 v_2(y,z,t) + \ldots\ldots\ldots \\
w(y,z,t) &= w_0(y) + \in w_1(y,z,t) + \in^2 w_2(y,z,t) + \ldots\ldots\ldots \\
p(y,z,t) &= p_0(y) + \in p_1(y,z,t) + \in^2 p_2(y,z,t) + \ldots\ldots\ldots
\end{aligned} \tag{13}
$$

When $\in = 0$, the differential equations (8)-(11) pertaining to two dimensional flow are obtained as

$$v_0' = 0$$

$$u_0'' + \mathrm{Re}\, S u_0' - M u_0 = 0 \tag{14}$$

$$w_0'' + \mathrm{Re}\, S w_0' - M w_0 = 0 \tag{15}$$

Subject to the boundary conditions

$$u_0 = h_1 \frac{\partial u_0}{\partial y}; \quad v_0 = -S; \quad w_0 = h_2 \frac{\partial w_0}{\partial y}; \quad \text{at } y = 0$$

$$u_0 = 1; \quad v_0 = -S; \quad w_0 = 0; \quad \text{at } y = 1 \tag{16}$$

The solutions for the equations (14) are

$$v_0(y) = -S \tag{17}$$

$$u_0(y) = A_1 e^{-m_1 y} + A_2 e^{-m_2 y} \tag{18}$$

$$w_0(y) = 0 \tag{19}$$

The unsteady state equations are

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{20}$$

$$\lambda \frac{\partial u_1}{\partial t} + \mathrm{Re}\left(-S \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} \right) = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} - M u_1 \tag{21}$$

$$\lambda\frac{\partial v_1}{\partial t}-\operatorname{Re}S\frac{\partial v_1}{\partial y} = -\operatorname{Re}\frac{\partial p_1}{\partial y}+\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2} \quad (22)$$

$$\lambda\frac{\partial w_1}{\partial t}-\operatorname{Re}S\frac{\partial w_1}{\partial y} = -\operatorname{Re}\frac{\partial p_1}{\partial z}+\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}-Mw_1 \quad (23)$$

The boundary conditions become

$$u_1 = h_1\frac{\partial u_1}{\partial y};\quad v_1=-S\left[Cos(\pi z-t)\right];\quad w_1=h_2\frac{\partial w_1}{\partial y};\quad \text{at } y=0$$
$$u_1=0;\quad v_1=0;\quad w_1=0;\quad \text{at } y=1 \quad (24)$$

In order to solve this set of differential equations u_1, v_1, w_1 and p_1 are assumed in terms of complex notations as below, real part will have physical significance.

$$u_1(y,z,t)=u_{11}(y)e^{i(\pi z-t)}$$
$$v_1(y,z,t)=v_{11}(y)e^{i(\pi z-t)}$$
$$w_1(y,z,t)=\frac{i}{\pi}v_{11}{}'(y)e^{i(\pi z-t)}$$
$$p_1(y,z,t)=p_{11}(y)e^{i(\pi z-t)} \quad (25)$$

Now using (25) in equations (20) - (23) we get

$$v''_{11}+\operatorname{Re}Sv'_{11}-(\pi^2-i\lambda)v_{11} = \operatorname{Re}p'_{11} \quad (26)$$

$$v'''_{11}+\operatorname{Re}Sv''_{11}-(\pi^2-i\lambda+M)v'_{11} = \pi^2\operatorname{Re}p_{11} \quad (27)$$

$$u''_{11}+\operatorname{Re}Su'_{11}-(\pi^2-i\lambda+M)u_{11} = \operatorname{Re}v_{11}u_0{}' \quad (28)$$

Boundary conditions are

$$u_{11}=h_1\frac{\partial u_{11}}{\partial y};\quad v_{11}=-S;\quad v'_{11}=h_2\frac{\partial^2 v_{11}}{\partial y^2}\quad \text{at}\quad y=0$$
$$u_{11}=0;\quad v_{11}=0;\quad v'_{11}=0;\quad \text{at}\quad y=1 \quad (29)$$

The solutions of the equations (26)-(28) subject to boundary conditions (29) are

$$u_1(y,z,t)=(A_7e^{-m_7y}+A_8e^{-m_8y}+A_9e^{-(m_1+m_3)y}+A_{10}e^{-(m_1+m_4)y}+A_{11}e^{-(m_1+m_5)y}$$

$$+A_{12}e^{-(m_1+m_6)y}+A_{13}e^{-(m_2+m_3)y}+A_{14}e^{-(m_2+m_4)y}+A_{15}e^{-(m_2+m_5)y}$$
$$+A_{16}e^{-(m_2+m_6)y})e^{i(\pi z-t)}$$

$$v_1(y,z,t) = (A_3e^{-m_3y}+A_4e^{-m_4y}+A_5e^{-m_5y}+A_6e^{-m_6y})e^{i(\pi z-t)}$$

$$w_1(y,z,t) = \frac{-i}{\pi}(A_3m_3e^{-m_3y}+A_4m_4e^{-m_4y}+A_5m_5e^{-m_5y}+m_6A_6e^{-m_6y})e^{i(\pi z-t)}$$

where m_3, m_4, m_5 and m_6 are the roots of the equation

$$m^4+S\,\mathrm{Re}\,m^3-(2\pi^2-i\lambda+M)m^2-S\,\mathrm{Re}\,\pi^2m+(\pi^2-i\lambda)\pi^2=0 \quad (30)$$

Skin Friction

The skin friction at the wall is given by

$$\tau=\left(\frac{du}{dy}\right)_{y=0}=\left(\frac{du_0}{dy}\right)_{y=0}+\in\left(\frac{du_1}{dy}\right)_{y=0}+O(\in^2)=DX1+\in DX2+O(\in^2)$$

$$\tau=\left(\frac{du}{dy}\right)_{y=1}=\left(\frac{du_0}{dy}\right)_{y=1}+\in\left(\frac{du_1}{dy}\right)_{y=1}+O(\in^2)=DX3+\in DX4+O(\in^2)$$

For the sake of brevity, the constants are given in Appendix

4. Numerical Results

In order to get the physical insight of the problem we have studied the main flow, cross flow and skin friction as function of various non dimensional parameters such as Reynolds number, Suction parameter, Frequency parameter and Magnetic parameter. The effects of flow parameters on velocity field and skin friction are calculated numerically and discussed with the help of graphs.

To check the validity of the analytical expressions derived in the previous section we have shown main flow velocity u, cross flow velocity w through figures 1 and 2 for vanishing Magnetic field and slip parameter. It can be seen from these figures that the main flow velocity increases as Reynolds number and Suction parameter increases. The cross flow velocity increases with the increase in Suction Parameter. Due to Suction at the stationary plate and injection at the moving plate, the cross flow velocity is found to be increasing near the stationary plate and decreasing near the upper plate. These results are in agreement with Guria and Jana (2006).

In figures 3 and 4, the main flow velocity is plotted as a function of y . It could be seen from these figures that the main flow velocity increases with increase in either Re or S while it decrease with increase in frequency parameter.

From figures 5 and 6, it can be seen that increasing Hartmann number M and velocity slip parameter h_1 increase the velocity profile.

Figures 8-12 illustrate the behaviour of the cross flow velocity as a function of y and various non dimensional parameters. It is observed that the magnitude of the cross flow velocity w increases with the increase in either S or λ, but it increases near the stationary plate and decreases near the moving plate with increases in Re. This is due to fact that suction at the stationary plate and injection at the moving plate are two exactly opposite processes.

From Figures 13-18, Skin friction is found to be increasing at stationary plate and decreasing at the moving plate with increasing Magnetic Parameter M , velocity slip parameters h_1 and h_2 , Reynolds number Re and Suction parameter. Skin friction is found to be decreasing at the stationary plate and increasing at the moving plate with increasing M , h_1 , h_2 and λ .

5. Conclusion

Motivated by the industrial applications of MHD flows past porous plates, we have extended the work of Guria and Jana (2006) to study the effect of magnetic field and slip parameter on the three dimensional unsteady Couette flow of viscous incompressible fluid between two horizontal porous flat plates. The stationary plate is subjected to a periodic suction and the plate in uniform motion is subjected to uniform injection. The conclusions of the study are as follows.

- Velocity profile increases with Reynolds number and Suction parameter while it decreases with increase in frequency parameter.

- The effect of Hartmann number and velocity slip parameter is to increase the main flow velocity.

- The cross flow velocity increases with the increase in either S or λ, but it increases near the stationary plate and decreases near the moving plate with increase in Re.

- The cross flow velocity decreases with increasing Re while it increases with increase in Suction parameter.

- Skin friction is found to be increasing at stationary plate and decreasing at the moving plate due to Magnetic Parameter and velocity slip parameter h_1 and h_2.

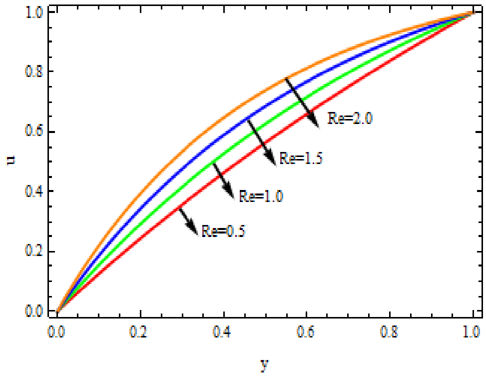

Fig 1: Main Velocity u versus y for $S = 1.0$, $M = 0$, $\lambda = 6$, $z = 0.0$, $\epsilon = 0.2$, $t = 0.2$, $h_1 = 0$, $h_2 = 0$

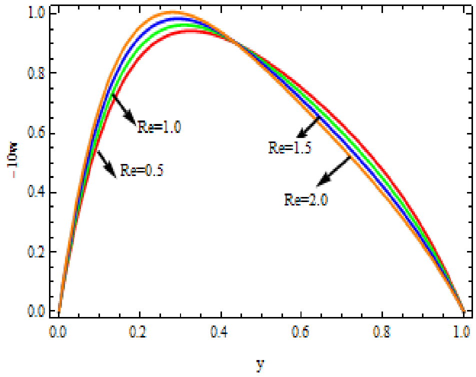

Fig 2: Cross velocity -10w versus y for $S = 1.0$ $M = 0$, $\lambda = 6$, $z = 0.5$ $\epsilon = 0.2$, $t = 0.2$ $h_1 = 0$ $h_2 = 0$

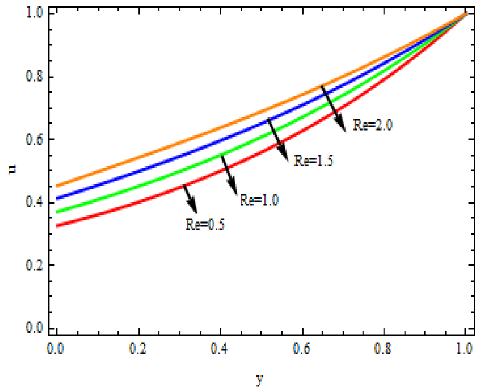

Fig 3: Main Velocity u versus y for $S = 1.0$, $M = 2$, $\lambda = 6$, $z = 0.0$, $\epsilon = 0.2$, $t = 0.2$, $h_1 = 1$, $h_2 = 1$

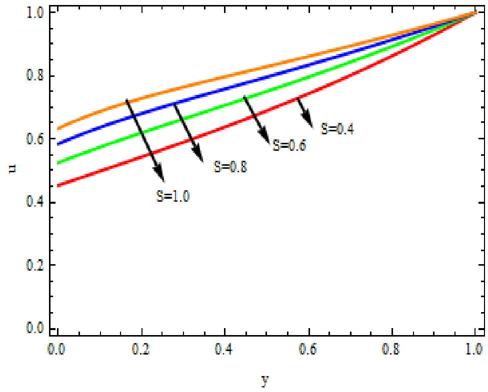

Fig 4: Main Velocity u as a function of y for $M = 2$, $\mathrm{Re} = 5$, $\lambda = 6$, $z = 0.0$, $\epsilon = 0.2$, $t = 0.2$, $h_1 = 1$ $h_2 = 1$

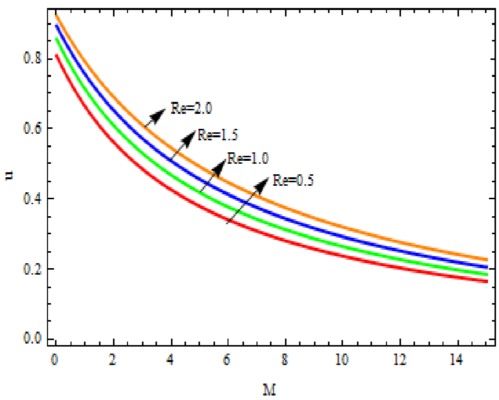

Fig 5:Main velocity u as a function of M for $S = 1.0$ $\lambda = 6$, $z = 0.0$, $\epsilon = 0.2$, $t = 0.2$, $h_1 = 1$ $h_2 = 1$ *at* $y = 0.5$

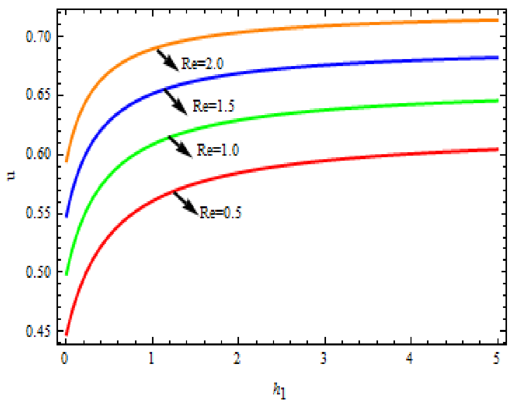

Fig 6: Main velocity u versus h_1 *for* $S = 1.0$, $\lambda = 6$, $z = 0.0$, $\epsilon = 0.2$, $t = 0.2$, $M = 2$, $h_2 = 1$ *at* $y = 0.5$

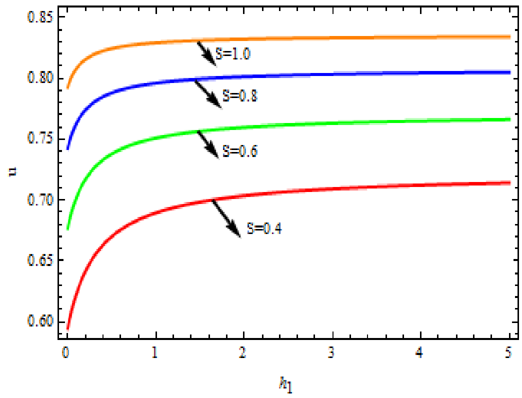

Fig 7:Main velocity u versus h_1 for $\mathrm{Re}=5$, $\lambda=6$, $z=0.0$, $\in=0.2$, $t=0.2$, $M=2$, $h_2=1$ at $y=0.5$

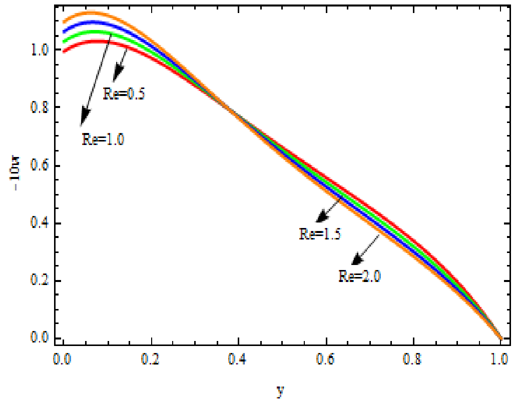

Fig 8: Cross velocity -10w versus y for $S=1.0$ $M=2$, $\lambda=6$, $z=0.5$, $\in=0.2$, $t=0.2$, $h_1=1$, $h_2=1$

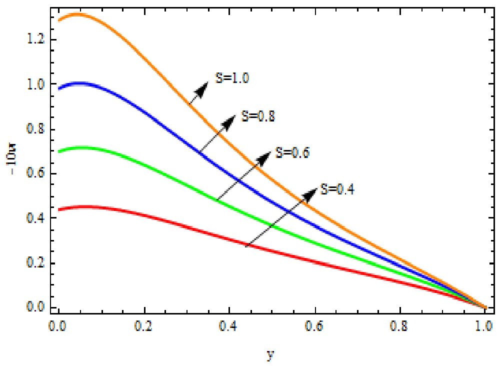

Fig 9 Cross velocity -10w versus y for $\mathrm{Re}=5$ $M=2$ $\lambda=6$, $z=0.5$, $\in=0.2$, $t=0.2$, $h_1=1$, $h_2=1$

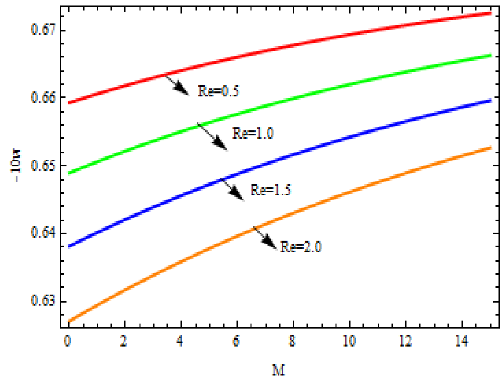

Fig 10: Cross velocity -10w versus M for $S=1.0$, $\lambda=6$, $z=0.5$, $\in=0.2$, $t=0.2$, $h_1=1$, $h_2=1$ at $y=0.5$

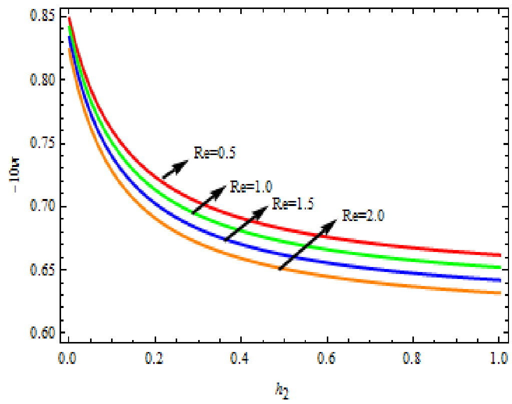

Fig 11: Cross velocity -10w versus h_2 for $S=1.0$ $\lambda=6$, $z=0.5$, $\in=0.2$, $t=0.2$, $M=2$, $h_1=1$ at $y=0.5$

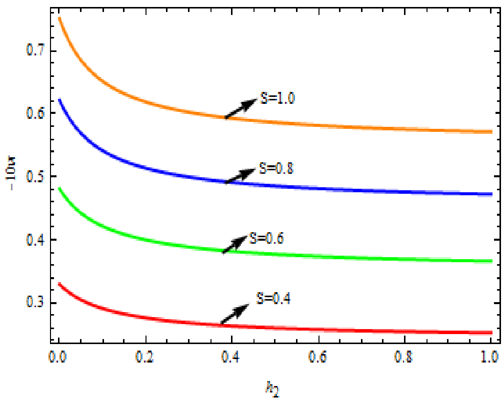

Fig 12: Cross velocity -10w versus h_2 for $\mathrm{Re}=5$ $\lambda=6$, $z=0.5$, $\in=0.2$, $t=0.2$, $M=2$, $h_1=1$ at $y=0.5$

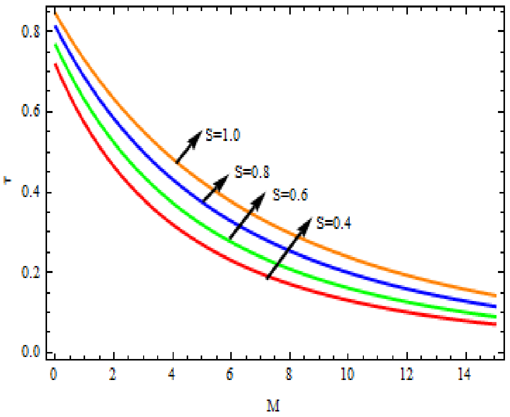

Fig 13: Skin Friction τ *versus M for* $\mathrm{Re} = 5$, $\lambda = 6$, $z = 0.0$, $\in = 0.2$, $t = 0.2$, $h_1 = 1$, $h_2 = 1$ *at* $y = 0$

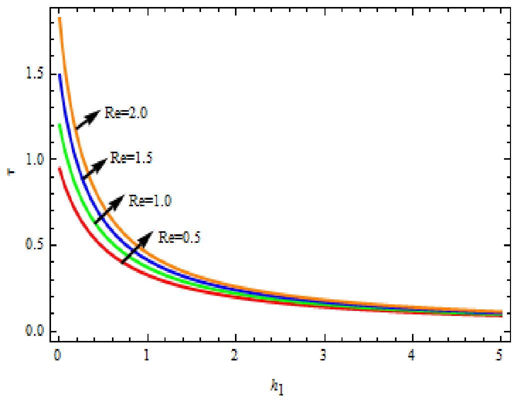

Fig 14: Effect of h_1 *on Skin Friction* τ *for* $S = 1.0$ $\lambda = 6$, $z = 0.0$ $\in = 0.2$, $t = 0.2$ $M = 2$ $h_2 = 1$ *at* $y = 0$

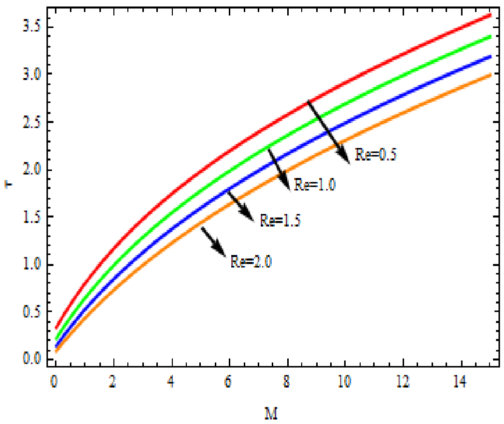

Fig 15: Skin Friction τ *versus M for* $S = 1.0$, $\lambda = 6$ $z = 0.0$, $\in = 0.2$, $t = 0.2$, $h_1 = 1$, $h_2 = 1$ *at* $y = 1$

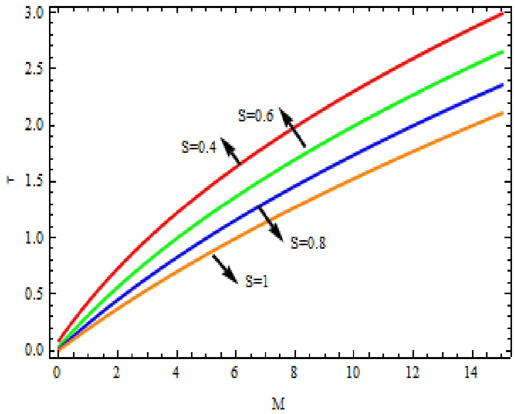

Fig 16: Skin Friction τ *versus M for* $\mathrm{Re} = 5$, $\lambda = 6$, $z = 0.0$, $\in = 0.2$, $t = 0.2$, $h_1 = 1$, $h_2 = 1$ *at* $y = 1$

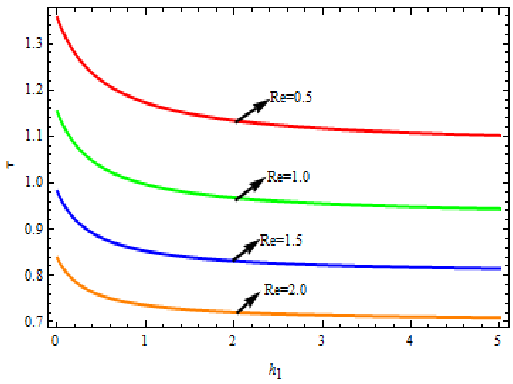

Fig 17: Skin Friction τ *versus* h_1 *for* $S = 1.0$, $\lambda = 6$ $z = 0.0$ $\in = 0.2$, $t = 0.2$, $M = 2$, $h_2 = 1$ *at* $y = 1$

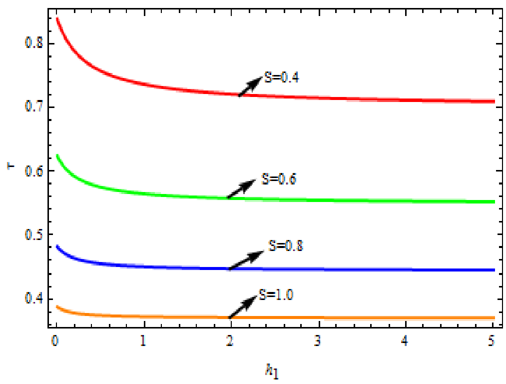

Fig 18: Skin Friction τ *versus* h_1 *for* $\mathrm{Re} = 5$, $\lambda = 6$ $z = 0.0$, $\in = 0.2$, $t = 0.2$, $M = 2$ $h_2 = 1$ *at* $y = 1$

References

[1] Ahmed Sahin, Influence of chemical reaction on transient MHD free convective flow over a vertical plate in slip-flow regime", Emirates Journal for Engineering Research, 15(1), (2010) 25-34.

[2] S. S. Das, Effect of suction and injection on MHD three dimensional coquette flow and heat transfer through a porous medium, Journal of Naval Architecture and Marine Engineering, 6, (2009), 41-51.

[3] S. S. Das, M. Mohanty, J.P. Panda and S. K. Sahoo., Hydromagnetic three dimensional couette flow and heat transfer, Journal of Naval Architecture and Marine Engineering, 5, (2008), 1-10. http://dx.doi.org/10.3329/jname.v5i1.1784

[4] S. S. Das, M. Maity and J. K. Das, Unsteady hydromagnetic convective flow past an infinite vertical porous plate in a porous medium, International Energy and Environment Foundation, 3 (1), (2012), 109 - 118.

[5] S. S. Das, L. K. Mishra, P. K. Mishra, Effect of heat source on MHD free convection flow past an oscillating porous plate in the slip flow regime, Int. J. Energy and Environment, 2(5), (2011), 945-952.

[6] K. Gersten and J. F. Gross, Flow and heat transfer along a plane wall with periodic suction, Z. Angew. Math. Phy., 25,(1974), 399-408. http://dx.doi.org/10.1007/bf01594956

[7] M. Guria and N. Jana, Three -Dimensional fluctuating Couette flow through the porous plates with heat transfer, Int. J. Mathematics and mathematical Sciences,(2006), 1-18. http://dx.doi.org/10.1155/ijmms/2006/61023

[8] Khem Chand, Heat transfer in three dimensional MHD boundary layer flow over a continuous porous surface moving in a parallel free stream, Int. J. Engineering Science and Technology, 3, No.7, (2011), 6058-6063.

[9] A. K. Mishra, Transient free convection flow past a vertical plate through porous medium with variation in slip flow regime, Int. J. Advanced Technology and Engineering Research, 4 (1), (2014), 59 - 63.

Appendix

$$m_1 = \frac{\operatorname{Re} S + \sqrt{(\operatorname{Re} S)^2 + 4M}}{2}; \quad m_2 = \frac{\operatorname{Re} S - \sqrt{(\operatorname{Re} S)^2 + 4M}}{2}; \quad A_1 = \frac{-(1+h_1 m_2)}{e^{-m_2}(1+h_1 m_1) - e^{-m_1}(1+h_1 m_2)}$$

$$A_2 = \frac{(1+h_1 m_1)}{e^{-m_2}(1+h_1 m_1) - e^{-m_1}(1+h_1 m_2)}; \quad A_3 = \frac{p q_4 - q p_4}{p_3 q_4 - p_4 q_3}; \quad A_4 = \frac{p_3 q - q_3 p}{p_3 q_4 - p_4 q_3};$$

$$p = S m_6 (1+h_2 m_6) e^{-m_5}; \; q = S m_5 (1+h_2 m_5) e^{-m_6};$$

$$p_3 = (m_3 - m_6)[(1+h_2(m_3+m_6))e^{-m_5} - (1+h_2(m_5+m_6))e^{-m_3}];$$

$$p_4 = (m_4 - m_6)[(1+h_2(m_4+m_6))e^{-m_5} - (1+h_2(m_5+m_6))e^{-m_4}];$$

$$q_3 = (m_3 - m_5)[(1+h_2(m_3+m_5))e^{-m_6} - (1+h_2(m_5+m_6))e^{-m_3}];$$

$$q_4 = (m_4 - m_5)[(1+h_2(m_4+m_5))e^{-m_6} - (1+h_2(m_5+m_6))e^{-m_4}];$$

$$A_5 = \frac{A_3(m_3-m_6)e^{-m_3} + A_4(m_4-m_6)e^{-m_4}}{(m_6-m_5)e^{-m_5}}; \quad A_6 = \frac{A_3(m_3-m_5)e^{-m_3} + A_4(m_4-m_5)e^{-m_4}}{(m_5-m_6)e^{-m_6}};$$

$$m_7 = \frac{\operatorname{Re} S + \sqrt{(\operatorname{Re} S)^2 + 4(\pi^2 - i\lambda + M)}}{2}; \quad m_8 = \frac{\operatorname{Re} S - \sqrt{(\operatorname{Re} S)^2 + 4(\pi^2 - i\lambda + M)}}{2};$$

$$A_7 = \frac{-P_1 e^{-m_8} + P_2(1+h_1 m_8)}{e^{-m_8}(1+h_1 m_7) - e^{-m_7}(1+h_1 m_8)}; \quad A_8 = \frac{P_1 e^{-m_7} - P_2(1+h_1 m_7)}{e^{-m_8}(1+h_1 m_7) - e^{-m_7}(1+h_1 m_8)};$$

$$A_9 = \frac{-m_1 A_1 A_3 \operatorname{Re}}{(m_1+m_3)^2 - \operatorname{Re} S(m_1+m_3) - (\pi^2 - i\lambda + M)}; \quad A_{10} = \frac{-m_1 A_1 A_4 \operatorname{Re}}{(m_1+m_4)^2 - \operatorname{Re} S(m_1+m_4) - (\pi^2 - i\lambda + M)};$$

$$A_{11} = \frac{-m_1 A_1 A_5 \operatorname{Re}}{(m_1+m_5)^2 - \operatorname{Re} S(m_1+m_5) - (\pi^2 - i\lambda + M)}; \quad A_{12} = \frac{-m_1 A_1 A_6 \operatorname{Re}}{(m_1+m_6)^2 - \operatorname{Re} S(m_1+m_6) - (\pi^2 - i\lambda + M)};$$

$$A_{13} = \frac{-m_2 A_2 A_3 \operatorname{Re}}{(m_2+m_3)^2 - \operatorname{Re} S(m_2+m_3) - (\pi^2 - i\lambda + M)}; \quad A_{14} = \frac{-m_2 A_2 A_4 \operatorname{Re}}{(m_2+m_4)^2 - \operatorname{Re} S(m_2+m_4) - (\pi^2 - i\lambda + M)};$$

$$A_{15} = \frac{-m_2 A_5 A_2 \operatorname{Re}}{(m_2+m_5)^2 - \operatorname{Re} S(m_2+m_5) - (\pi^2 - i\lambda + M)}; \quad A_{16} = \frac{-m_2 A_2 A_6 \operatorname{Re}}{(m_2+m_6)^2 - \operatorname{Re} S(m_2+m_6) - (\pi^2 - i\lambda + M)};$$

$$P_1 = (A_9(1+h_1(m_1+m_3)) + A_{10}(1+h_1(m_1+m_4)) + A_{11}(1+h_1(m_1+m_5)) + A_{12}(1+h_1(m_1+m_6))$$
$$+A_{13}(1+h_1(m_2+m_3)) + A_{14}(1+h_1(m_2+m_4)) + A_{15}(1+h_1(m_2+m_5)) + A_{16}(1+h_1(m_2+m_6)));$$

$$P_2 = (A_9 e^{-(m_1+m_3)} + A_{10} e^{-(m_1+m_4)} + A_{11} e^{-(m_1+m_5)} + A_{12} e^{-(m_1+m_6)} + A_{13} e^{-(m_2+m_3)} + A_{14} e^{-(m_2+m_4)}$$
$$+ A_{15} e^{-(m_2+m_5)} + A_{16} e^{-(m_2+m_6)});$$

$$DX1 = -m_1 A_1 - m_2 A_2;$$

$$DX2 = [-m_7 A_7 - m_8 A_8 - (m_1+m_3)A_9 - (m_1+m_4)A_{10} - (m_1+m_5)A_{11} - (m_1+m_6)A_{12}$$
$$-(m_2+m_3)A_{13} - (m_2+m_4)A_{14} - (m_2+m_5)A_{15} - (m_2+m_6)A_{16}]e^{i(\pi z - t)};$$

$$DX3 = -m_1 A_1 e^{-m_1} - m_2 A_2 e^{-m_2};$$

$$DX4 = [-m_7 A_7 e^{-m_7} - m_8 A_8 e^{-m_8} - (m_1+m_3)A_9 e^{-(m_1+m_3)} - (m_1+m_4)A_{10} e^{-(m_1+m_4)}$$
$$-(m_1+m_5)A_{11} e^{-(m_1+m_5)} - (m_1+m_6)A_{12} e^{-(m_1+m_6)} - (m_2+m_3)A_{13} e^{-(m_2+m_3)}$$
$$-(m_2+m_4)A_{14} e^{-(m_2+m_4)} - (m_2+m_5)A_{15} e^{-(m_2+m_5)} - (m_2+m_6)A_{16} e^{-(m_2+m_6)}]e^{i(\pi z - t)};$$

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