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Maximum Independent Seidel Energy of a Graph

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Maximum Independent Seidel Energy of a Graph

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Abstract-- The concept of Maximum Independent Seidelenergy EImaxS(G)ofasimpleconnectedgraphGhasbeeninnovatedby us and we have calculated the maximum independent seidel energies of some standard graphs. Also we have discovered few basicpropertiesrelatedtomaximumindependentseidelenergy.

Index Terms--Maximum independent seidel eigenvalues, Maximum independent seidel energy, Maximum independent seidel matrix, Maximum independent seidel set

1. Introduction

Let G be a simple connected graph with order p and size q which doesn't have any loops, no multiple nor directed edges. I. Gutman [1] laid foundation to the concept of Energy of a graph G in the year 1978. Let A =(a_{ij}) shall be considered as the adjacency matrix of the graph G. The eigenvalues $\rho_1,\rho_2,\rho_3,....,\rho_n$ of G are obtained from its characteristic polynomial A - IG. The eigenvalues of A are real with its total sum being equal to zero. The energy E(G) of Graph G is defined to be the sum of the absolute values of all the eigenvalues obtained from G i.e., $E(G)=\sum_{i=1}^{n}|\rho_i|$. The seidel matrix [2] of G was formulated as the square matrix $A_S(G)$ of order p whose each entry is defined as (s_{ij})= 1 if $v_i v_j \epsilon E$
-1 if $v_i v_j \notin E$
0 otherwise

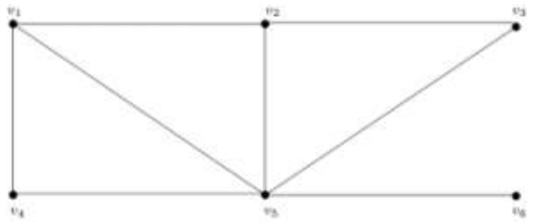
Let ω1, ω2, ω3, ..., ωn be the eigenvalues obtained from the seidel matrix $A_S(G)$ of G. Then SE(G) = $\sum_{i=1}^{n}|\omega_i|$ gives the the Seidel energy SE(G) of a graph G. K.B. Murthy et al [3] ideated Maximum independent vertex Energy in the year 2016. A set I ⊆ G can be called as the independent set if for any x,y I, they are not adjacent to each other. The largest cardinality of such an independent set in G is called the independence number of G and it is denoted by β (G). Motivated by the above concepts, we came up with the idea of maximum independent seidel energy EImaxS(G)of a graph G and thus computed maximum independent seidel energies of some standard graphs. Other similar studies related to energy are [4].

2. Illustration

Consider a Graph G with the vertex set= {v_1, v_2, v_3, v_4, v_5, v_6} and its independent set I = {v_1, v_3, v_6} as in the below figure. Then its maximum seidel adjacency matrix is

$$A_{\mathrm{Im}axS}(G)=\begin{bmatrix} 1 & -1 & 1 & -1 & -1 & 1 \\ -1 & 0 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 0 & -1 & 1 \\ -1 & -1 & -1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

The characteristic polynomial of $A_{\mathrm{Im}axS}(G)$ is $f_{\rho}\left(G, \lambda_{\mathrm{Im}axS}\right)$= and the maximum independent seidel eigen values are 4.09295, 2.70725, -2.19524, -1.8249, 0.720327 and -0.500387. Thus the maximum independent seidel energy of G is EImaxS(G) =12.041054.



3. MAXIMUM INDEPENDENT SEIDEL ENERGY OF A GRAPH

Let G be a graph composed with p vertices and q edges. An independent set I of a graph G with maximum number of members containing in it is called a maximum independent set. Then the maximum independent seidel matrix of G is the p × p matrix $A_{\mathrm{Im}axS}(G)$

where (a_{ij})= $\begin{cases} 1 & \text{if } v_i v_j \epsilon E \\ -1 & \text{if } v_i v_j \notin E \\ 1 & \text{if } i=j \text{ and } v_i \in I \\ 0 & \text{Otherwise} \end{cases}$

The characteristic polynomial of $A_{\mathrm{Im}axS}(G)$ denoted by

fn(G, λ) isdefined as det (λI - $A_{\mathrm{Im}axS}(G)$). The maximum independent seidel eigenvalues of the graphG are the eigenvalues of $A_{\mathrm{Im}axS}(G)$. Since $A_{\mathrm{Im}axS}(G)$ contains real and symmetric entries, its eigenvalues are real and we represent them in non-increasing order $\lambda_{IS1}, \lambda_{IS2}, \lambda_{IS3}$, , λ_{ISn}.. The minimum hub seidel energy is defined as: EImaxS(G) $= \sum_{i=1}^{n} \left|\lambda_{ISi}\right|$

4. SOME BASIC PROPERTIES OF MAXIMUM INDEPENDENT SEIDEL ENERGY

Theorem4.1:ForagraphGwithindependencenumber$\beta(G)$, its maximum independence seidel matrix yeilds a characteris- tic polynomial $f_n\left(G, \lambda_{IS}\right) = a_0 \lambda_{IS}^{n} + a_1 \lambda_{IS}^{n-1} + ... + a_n$

which is obtained from the maximum independent seidel matrix ofgraph G. Then

(1) $a_0 = 1$

(2) $a_1 = -\beta(G)$

(3) $a_2 = \binom{\beta(G)}{2} - \frac{n^2 - n}{2}$

Proof:

(1) From the definition of $f_p\left(G, \lambda_{IS}\right)$, it is clear that $a_0 = 1$

(2) The sum of determinants of principal 1×1 submatrices of the adjacency matrix $A_{\mathrm{Im}axS}(G)$ obtained from matrix $A_{\mathrm{Im}axS}(G)$ is equal to $\beta(G)$. Thus $(-1)a_1 = \beta(G)$.

(3) The sum of determinants of all principal 2×2 submatrices of $A_{\mathrm{Im}axS}(G)$ is equal to $(-1)^2 C_2$, that is

$$a_2 = \sum_{1 \le i < j \le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}$$

$$= \sum_{1\le i<j\le n}\left(a_{ii}a_{jj}-a_{ij}a_{ji}\right)$$

$$= \sum_{1\le i<j\le n}\left(a_{ii}a_{jj}\right)-\sum_{1\le i<j\le n}\left(a_{ij}\right)^2$$

$$= \binom{\beta(G)}{2}-\frac{n^2-n}{2}$$

Theorem 4.2. For a simple connected graph G with p vertices, its maximum independent seidel adjacency matrix $A_{\mathrm{Im}axS}(G)$ yields p eigen values which satisfies

(1) $\sum_{i=1}^{n}\lambda_{ISi}=\beta(G)$

(2) $\sum_{i=1}^{n}\lambda_{ISi}^2=\beta(G)+n^2-n$

Proof. (1) Trace of $A_{\mathrm{Im}axS}(G)$ equals sum of its eigenvalues , then $\sum_{i=1}^{n}\lambda_{ISi}=\sum_{i=1}^{n}a_{ii}=\beta(G)$ where β (G) represents the cardinality of maximum independent set I of G .

(2) Similarly the trace of $\left(A_{\mathrm{Im}axS}(G)\right)^2$ equals the sum of squares of eigenvalues of $A_{\mathrm{Im}axS}(G)$. Thus

$$\sum_{i=1}^{n}\lambda_{ISi}^2=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}a_{ji}$$

$$=\sum_{i=1}^{n}a_{ii}^2+\sum_{i\neq j}^{n}a_{ij}a_{ji}$$

$$=\sum_{i=1}^{n}a_{ii}^2+\sum_{i<j}^{n}a_{ij}^2$$

Therefore, $\sum_{i=1}^{n}\lambda_{ISi}^2=\beta(G)+2\left(m[1]^2+\left(\frac{n^2-n}{2}-m\right)(1)^2\right)$

V. MAXIMUM INDEPENDENT SEIDEL ENERGY OF FEW STANDARD GRAPH

Theorem 5.1

For $n\ge 2$, the maximum independent Seidel energyof the Star graph of order n is $\sqrt{n^2+2n-3}$

Proof.

Let $K_{1,n-1}$ be the Star graph with n vertices $\{v_0,v_1,v_2,....,v_{n-1}\}$ containing maximum independent set $I=\{v_0,v_1,v_2,....,v_{n-1}\}$

Since independence number $\beta(K_{1,n-1})$ = *n-1*, we get $A_{\mathrm{Im}axS}(K_{1,n-1})=\begin{pmatrix}0 & -1 & \dots & -1 & -1\\ -1 & 1 & \dots & 1 & 1\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ -1 & 1 & \dots & 1 & 1\\ -1 & 1 & \dots & 1 & 1\end{pmatrix}_{n\times n}$

Then the Characteristic polynomial is $(\lambda_{IS})^{n-2}\left(\lambda_{IS}^2-(n-1)\lambda_{IS}-(n-1)\right)$

Spectrum, $Spec_{\operatorname{Im}axS}(K_{1,n-1}) = \begin{pmatrix} 0 & \frac{n-1\pm\sqrt{n^2+2n-3}}{2} \\ n-2 & 1 \end{pmatrix}$

Therefore, maximum independent Seidel energy is $E_{\operatorname{Im}axS}(K_{1,n-1}) = \sum_{i=1}^{n}|\lambda IS_i|$

$$=|0|(n-2)+\left|\frac{(n-1)\pm\sqrt{n^2+2n-3}}{2}\right|1$$

$$=\sqrt{n^2+2n-3}$$

$\therefore$ The maximum independent Seidel energy of the Star graph is $\sqrt{n^2+2n-3}$

Theorem 5.2

For $n \geq 2$, the maximum independent Seidel energyof the Complete Bipartite graph of order *2n*is $(n-1)+\sqrt{4n^2+1}$

Proof.

Let $K_{n,n}$ be the Complete Bipartite graph with Vertex set V=. $\{u_1,u_2,....,u_n,v_1,v_2,....,v_n\}$ containing themaximum independent set $I=\{v_1,v_2,....,v_n\}$

Since independence number $\beta(K_{n,n})$ = *n*, we get

$$A_{\operatorname{Im}axS}(K_{n,n}) = \begin{pmatrix} 0 & 1 & \ldots & -1 & -1 \\ 1 & 0 & \ldots & -1 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & \ldots & 1 & 1 \\ -1 & -1 & \ldots & 1 & 1 \end{pmatrix}_{2n\times 2n}$$

Then the Characteristic polynomial is $\lambda_{IS}^{n-1}(\lambda_{IS}+1)^{n-1}\left[\lambda_{IS}{}^2+(2n-1)\lambda_{IS}-n\right]$

Spectrum, $Spec_{\operatorname{Im}axS}(K_{n,n}) = \begin{pmatrix} 0 & -1 & \frac{2n-1\pm\sqrt{4n^2+1}}{2} \\ n-1 & n-1 & 1 \end{pmatrix}$

The maximum independent Seidel energy is, $E_{\operatorname{Im}axS}(K_{n,n}) = \sum_{i=1}^{n}|\lambda IS_i|$

$$=|0|(n-1)+|-1|(n-1)+\left|\frac{2n-1\pm\sqrt{4n^2+1}}{2}\right|1$$

$$=(n-1)+\sqrt{4n^2+1}$$

$\therefore$ The maximum independent Seidel energyof the Complete Bipartite graph is $(n-1)+\sqrt{4n^2+1}$

Theorem 5.3

For $n \geq 2$, the maximum independent Seidel energyof the Crown graph of order $2n$ is $\sqrt{17}(n-1)+\sqrt{4n^2-16n+17}$

Proof.

Let S_n^0 be the Crown graph with Vertex set V= $\{u_1,u_2,....,u_n,v_1,v_2,....,v_n\}$ containing the maximum independent set $I=\{u_1,u_2,....,u_n\}$.

Since independence number $\beta(S_n^0)$ =n , we get

$$A_{\mathrm{Im}axS}(S_n^0)=\begin{pmatrix} 1 & 1 & 1 & \cdots & \cdots & -1 & -1 \\ 1 & 1 & 1 & \cdots & \cdots & 1 & -1 \\ 1 & 1 & 1 & \cdots & \cdots & -1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ -1 & 1 & -1 & \cdots & \cdots & 0 & 1 \\ -1 & -1 & 1 & \cdots & \cdots & 1 & 0 \end{pmatrix}_{(2n)\times(2n)}$$

The Characteristic polynomial is $\left[\lambda_{IS}^2-(2n-1)\lambda_{IS}-(3n-4)\right]\left[\lambda_{IS}^2+\lambda_{IS}-4\right]^{n-1}$

Spectrum, $Spec_{\mathrm{Im}axS}(S_n^0)=\begin{pmatrix} \frac{-1\pm\sqrt{17}}{2} & \frac{(2n-1)\pm\sqrt{4n^2-16n+17}}{2} \\ n-1 & 1 \end{pmatrix}$

The maximum independent Seidel energy is ,

$$E_{\mathrm{Im}axS}\left(S_n^0\right)=\sum_{i=1}^{n}\left|\lambda IS_i\right|$$

$$=\left|\frac{-1\pm\sqrt{17}}{2}\right|(n-1)+\left|\frac{(2n-1)\pm\sqrt{4n^2-16n+17}}{2}\right|1$$

$$=\sqrt{17}(n-1)+\sqrt{4n^2-16n+17}$$

$\therefore$ The maximum independent Seidel energyof the Crown graph is $\sqrt{17}(n-1)+\sqrt{4n^2-16n+17}$

CONCLUSION

The Maximum independent Seidel energy of Complete Bipartite, Star and Crown Graphs are computed in this paper. From the results, it is found that the maximum independent Seidel energy of a graph changesupon the choice of its maximum independent set.

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