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# STABILITY OF NON-PARALLEL STRATIFIED SHEAR FLOWS WITH HALL EFFECT

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**ABSTRACT**

*In this paper, we study the effect of Hall current for the case of three-dimensional non-parallel stratified shear flow of an inviscid, incompressible perfectly conducting fluid. The non-linear equations of the flow and the magnetic induction equation are obtained with the uniform applied magnetic field. These equations are linearized by assuming the* perturbation *from the undisturbed flow to be small. Numerical computations are carried out for the non-dimensional parameters. The effect of different physical parameters such as Magnetic Reynolds number, Magnetic pressure number, Hall parameter, Richardson number, Brunt-Vaisala frequency, longitudinal and transverse wave number are discussed with the help of graphs.*

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## 1. INTRODUCTION

Shear instability caused by velocity shear is one of most important factors in flow instabilities. Even though the mechanism of shear instability are yet to be fully revealed, it has been applied to analyze instability in mixing layers, jets in pipes, wakes behind cylinders, etc. Some simple models have been employed to study shear instability, including the Kelvin-Helmholtz (K-H) model, piecewise linear velocity profile, continuous arbitrary velocity profile U(y) by Rayleigh (1880, 1894). Deardorff (1965), Gallagher and Mercer (1965) and Ingersoll (1966) investigated the stability of plane Couette flow. Ling and Reynolds (1973) focused their attention in analyzing the non-parallel flow corrections for the stability of shear flows. Long-wave instability and growth rate of the inviscid shear flows was examined by Liang Sun (2011).

Magnetohydrodynamic (MHD) shear flows are common in space plasmas. Well-known examples of such flows are the flows near the magnetopauses of the Earth and other planets, the flows close to the heliopause and the flows at the boundaries of the fast and slow streams of the solar wind. Also some flows observed in the solar atmosphere can be treated as shear flows. Studying stability of MHD shear flows is of considerable importance for the understanding of the physical processes in space and for correct interpretation of the observations.

Lerner and Knobloch (1985) analysed the stability of dissipative magnetohydrodynamic shear flows using linearized perturbations to an unbounded, plane Couette flow in a parallel magnetic field. The stability against small disturbances of the plane laminar motion of an electrically conducting fluid between parallel plates in relative motion under a transverse magnetic field was investigated by Takashima (1998).

The effect of Hall currents on thermal instability has received the attention of several authors namely Raptis and Ram (1984), Sharma and Rani (1988), Sunil *et al.* (2005), Sharma and Kumar (2000), Gupta and Agarwal (2011). Hall effects on unsteady hydromagnetic flow of an electrically conducting fluid bounded by a non-conducting plate were investigated by Prasada Rao and Krishna (1981).

In this paper, the work of Padmini and Subbiah (1995) is extended to analyze the effect of Hall current on the linear stability of stratified shear fluid in the presence of uniform horizontal magnetic field. Here, the stability of stratified shear flow of an unsteady, incompressible, inviscid electrically conducting fluid confined between two rigid planes at $z = \pm L$ is taken into consideration. The magnetic field is assumed to be large enough to produce significant Hall current. The fluid layer is permeated by a uniform external magnetic induction field $\vec{H} = (H_x,\ H_y, 0)$. The plates at $z = \pm L$ are assumed to be electrically non-conducting.

## 2. MATHEMATICAL FORMULATION

Consider the unsteady three dimensional stratified flow of an incompressible, inviscid, perfectly conducting Boussinesq fluid in the presence of a uniform magnetic field. The basic state nonparallel shear layer is characterized by arbitrary velocities in the horizontal and longitudinal direction. The equilibrium state velocity is taken as *(U(z), V(z), 0).* The governing equations are linearized using long wave approximation. The fluid is confined between two plates at $z = \pm L$. A uniform magnetic field $\vec{H} = \left(H_x, H_y, 0\right)$ is applied.

Based on the assumptions taken physical model of the problem is presented in Figure 1.

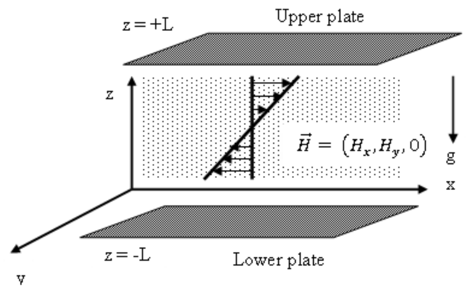


**Fig 1.** Flow configuration

The fundamental equations relevant to the present problem are
The equation of motion governing the fluid

$$\rho\left(\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right) = -\nabla p-\rho g\hat{z}+\mu_m(\nabla\times\vec{H})\times\vec{H} \qquad (1)$$

The equation of continuity for an incompressible fluid

$$\nabla.\vec{q} = 0 \qquad (2)$$

$$\frac{\partial\rho}{\partial t}+\vec{q}.\nabla\rho = 0 \qquad (3)$$

Maxwell's equations

$$\nabla\times\vec{H} = 4\,\pi\,\vec{J} \qquad (4)$$

$$\nabla\times\vec{E} = -\mu_m\frac{\partial\vec{H}}{\partial t} \qquad (5)$$

$$\nabla.\overrightarrow{H} = 0 \qquad (6)$$

Taking Hall current into account the generalized Ohm's law is

$$\vec{J}=\sigma\left(\vec{E}+\mu_m\left(\vec{q}\times\vec{H}\right)\right)-\frac{\omega\tau}{H_0}\mu_m\left(\vec{J}\times\vec{H}\right) \qquad (7)$$

Displacement current is neglected and all quantities are measured in terms of electromagnetic units.

Simplifying equations (4), (5) and (7) we get

$$\frac{\partial\vec{H}}{\partial t}=\nabla\times\left(\vec{q}\times\vec{H}\right)+\eta\,\nabla^2\vec{H}-\frac{\omega\tau}{H_0}.\frac{1}{4\,\pi\,\sigma}\left(\nabla\times\left(\nabla\times\vec{H}\right)\times\vec{H}\right) \qquad (8)$$

Where $\eta=\frac{1}{4\,\pi\,\sigma}$ is the magnetic resistivity of the fluid

Based on the boundary condition that the velocity must vanish at the boundaries (i.e)

$$\vec{q}=0 \text{ at } z=\pm L \qquad (9)$$

The basic flow variables are given by

$$\vec{q}=(U(z),V(z),0),\ \rho_0=\rho_0(z),\ p_0=p_0(z) \text{ and } \vec{H}=\left(H_x,H_y,0\right)$$

which satisfies the governing equations and boundary conditions provided

$$\frac{\partial p_0}{\partial z}=-\rho_0 g \qquad (10)$$

where *U(z), V(z)*, $\rho_0(z)$, $p_0(z)$ are continuously differential functions of z in the flow domain.

Introducing the nondimensional quantities

$$t=\frac{L\,t^*}{U_0},\qquad p=\rho_0\,U_0{}^2p^*,\qquad \rho=\frac{\rho_0\,U_0{}^2N_0{}^2}{L\,g}\rho^*,$$
$$N^2=-\frac{g}{\rho_0}\left(\frac{d\rho}{dz}\right),\ \vec{H}=H_0\vec{H}^*,\qquad (x,y,z) = L(x^*,y^*,z^*)$$

where $N_0$ is a typical value of Brunt-Vaisala frequency in the flow domain, *L* is the characteristic length and $U_0$ is the characteristic velocity.

By substituting the above nondimensional quantities into equations (1), (2), (3), (6) and (8), it reduces to the form (after removing asterisks)

$$\nabla.\vec{q} = 0 \qquad (11)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\nabla p - Ri\, g\hat{k} + S(\nabla \times \vec{H}) \times \vec{H} \tag{12}$$

$$\frac{\partial \rho}{\partial t} + \vec{q}.\nabla \rho = 0 \tag{13}$$

$$\frac{\partial \vec{H}}{\partial t} = \frac{1}{Rm}\nabla^2 \vec{H} + \nabla \times \left(\vec{q} \times \vec{H}\right) + M\left(\nabla \times \left(\left(\nabla \times \vec{H}\right) \times \vec{H}\right)\right) \tag{14}$$

$$\nabla.\vec{H} = 0 \tag{15}$$

Where $S = \frac{\mu H_0^2}{\rho U_0^2}$, Magnetic pressure number

$Rm = \frac{LU_0}{\eta}$, Magnetic Reynolds number

$Ri = \frac{g\beta L^2}{\rho_0 U_0^2}$, Richardson number

$M = \frac{\omega\tau}{\mu_m U_0 \eta}$ Hall parameter

The relevant boundary conditions in dimensionless form is written as

$$\vec{q} = 0 \text{ on } z = \pm 1 \tag{16}$$

On this unsteady flow we superpose infinitesimal disturbances of the form

$u = U(z) + u'$, $v = V(z) + v'$, $w = w'$

$p = p_0(z) + p'$, $\rho = \rho_0(z) + \rho'$

$$H_x = H_x + h_x{}', \quad H_y = H_y + h_y{}', \quad H_z = h_z{}' \tag{17}$$

The nondimensional perturbations in the form of normal modes is of the form

$$f(z)e^{i(kx+kly-k\sigma t)} \tag{18}$$

where $f(z)$, function of $z$ stands for perturbed velocity, pressure and magnetic field, $k$ and $l$ are wave numbers in the $x$ and $y$ direction respectively and $\sigma$ is the growth rate of the disturbance which in general is a complex constant.

Substituting equations (17) and (18) into equations (10) - (15) leads to

$$iku + iklv + \frac{\partial w}{\partial z} = 0$$

$$ik\left(-\sigma + U(z) + l\,V(z)\right)u + w.\frac{\partial U(z)}{\partial z} = -ikp - SH_y\, ik\left(h_y - lh_x\right)$$

$$ik\left(-\sigma + U(z) + l\,V(z)\right)v + w.\frac{\partial V(z)}{\partial z} = -iklp + SH_x\, ik\left(h_y - lh_x\right)$$

$$ik\left(-\sigma + U(z) + l\,V(z)\right)w = -\frac{\partial p}{\partial z} - R_i\rho - S\left(H_y\left(\frac{\partial h_y}{\partial z} - iklh_z\right) + H_x\left(\frac{\partial h_x}{\partial z} - ikh_z\right)\right)$$

$$ik\left(-\sigma + U(z) + l\,V(z)\right)\rho - \frac{N^2}{N_0^2}w = 0$$

$$ikh_x + iklh_y + \frac{\partial h_z}{\partial z} = 0$$

$$\left(-ik\sigma - \frac{1}{Rm}\left((ik)^2 + (ikl)^2 + \frac{\partial^2}{\partial z^2}\right)\right)h_x = ikl\left(U(z)h_y + u\,H_y - V(z)h_x - v\,H_x\right)$$

$$-H_x\left(\frac{\partial w}{\partial z}\right)+h_z\frac{\partial U(z)}{\partial z}+U(z)\frac{\partial h_z}{\partial z}$$

$$+M\left(H_y ikl\left(iklh_z-\frac{\partial h_y}{\partial z}\right)-H_x ik\left(\frac{\partial h_y}{\partial z}-iklh_z\right)\right)$$

$$\left(-ik\sigma-\frac{1}{Rm}\left((ik)^2+(ikl)^2+\frac{\partial^2}{\partial z^2}\right)\right)h_y=\frac{\partial V(z)}{\partial z}\ h_z+V(z)\frac{\partial h_z}{\partial z}-H_y\frac{\partial w}{\partial z}-U(z)\frac{\partial h_y}{\partial x}$$

$$+ik\big(V(z)h_x+vH_x-uH_y\big)$$

$$+M\ \left(H_x(ik)\left(\frac{\partial h_x}{\partial z}-ikh_z\right)+H_y(ikl)\left(\frac{\partial h_x}{\partial z}-ikh_z\right)\right)$$

$$\left(-ik\sigma-\frac{1}{Rm}\left((ik)^2+(ikl)^2+\frac{\partial^2}{\partial z^2}\right)\right)h_z=ik(wH_x-U(z)h_z)+ikl\big(wH_y-V(z)h_z\big)$$

$$+M\left(\left(H_x(ik)^2\big(h_y-lh_x\big)\right)+H_y(ik)^2l\big(h_y-lh_x\big)\right) \qquad (19)$$

Imposing no slip condition on the flow fields, the boundary conditions becomes

u = v = w = 0 on z = ±1 (20)

## 3. STABILITY ANALYSIS

To make the equation mathematically tractable, we assume the velocity profile to be linear and the perturbations are restricted to long waves.

Hence, the above set of equations can be modified to the form

$$iku+iklv+\frac{\partial w}{\partial z} \qquad =0$$

$$ik(-\sigma+(1+l)z)u+w \qquad = -ikp-SH_y\,ik\big(h_y-lh_x\big)$$

$$ik(-\sigma+(1+l)z)v+w \qquad = -iklp+SH_x\,ik[h_y-lh_x]$$

$$ik(-\sigma+(1+l)z)w \qquad = -\frac{\partial p}{\partial z}-\ R_i\rho-S\left(H_y\left(\frac{\partial h_y}{\partial z}-iklh_z\right)+H_x\left(\frac{\partial h_x}{\partial z}-ikh_z\right)\right)$$

$$ik(-\sigma+(1+l\ )z)\rho-\frac{N^2}{N_0^2}w=0$$

$$ikh_x+\ iklh_y+\frac{\partial h_z}{\partial z} \qquad =0$$

$$\left(-ik\sigma-\frac{1}{Rm}\left((ik)^2+(ikl)^2+\frac{\partial^2}{\partial z^2}\right)\right)h_x=\ ikl\big(z(h_y-h_x)+u\,H_y-v\,H_x\big)$$

$$-H_x\left(\frac{\partial w}{\partial z}\right)+h_z+z\frac{\partial h_z}{\partial z}$$

$$+M\left(H_y(ikl)\left(h_z-ikl\ \frac{\partial h_y}{\partial z}\right)-H_x\,(ik)\left(\frac{\partial h_y}{\partial z}-ikl\,h_z\right)\right)$$

$$\left(-ik\sigma-\frac{1}{Rm}\left((ik)^2+(ikl)^2+\frac{\partial^2}{\partial z^2}\right)\right)h_y=\ h_z+z\frac{\partial h_z}{\partial z}-H_y\frac{\partial w}{\partial z}-z\,(ik)h_y$$

$$+ik\left(zh_x + vH_x - uH_y\right)$$
$$+M\left(H_x(ik)\left(\frac{\partial h_x}{\partial z} - ikh_z\right) + H_y(ikl)\left(\frac{\partial h_x}{\partial z} - (ik)h_z\right)\right)$$

$$\left(-ik\sigma - \frac{1}{Rm}\left((ik)^2 + (ikl)^2 + \frac{\partial^2}{\partial z^2}\right)\right)h_z = ik(wH_x - zh_z) + ikl\left(wH_y - zh_z\right)$$
$$+M\left((ik)^2\left(h_y - lh_x\right)\left(H_x - lH_y\right)\right) \tag{21}$$

By assuming the series expansions with respect to k in the form

$$f = f_0 + kf_1 + k^2 f_2 + \cdots \tag{22}$$

where $f = (u, v, w, \sigma, \rho, p, h_x, h_y, h_z)$

Substituting equation (22) into above set of equations and retaining the quantities of the zeroth order, we get

$$iu_0 + ilv_0 + \frac{\partial w_0}{\partial z} = 0$$
$$iT(z)u_0 + w_0 = -ip_0$$
$$iT(z)v_0 + w_0 = -ilp_0$$
$$-\frac{\partial p_0}{\partial z} - Ri\,\rho_0 = 0$$
$$iT(z)\rho_0 - \frac{N^2}{{N_0}^2}w_0 = 0 \tag{23}$$
$$ih_{x0} + ilh_{y0} + \frac{\partial h_{z0}}{\partial z} = 0$$

$$il\left(u_0 H_y - v_0 H_x\right) - H_x\left(\frac{\partial w_0}{\partial z}\right) = -\frac{1}{Rm}\frac{\partial^2 h_{x0}}{\partial z^2}$$
$$i\left(v_0 H_x - u_0 H_y\right) - H_y\left(\frac{\partial w_0}{\partial z}\right) = -\frac{1}{Rm}\frac{\partial^2 h_{y0}}{\partial z^2}$$
$$iw_0\left(H_x + l\,H_y\right) = -\frac{1}{Rm}\frac{\partial^2 h_{z0}}{\partial z^2} \tag{24}$$

where $T(z) = (1 + l)z - \sigma_0$

with the relevant boundary condition that

$$u_0 = v_0 = w_0 = 0 \qquad \text{at } z = \pm 1 \tag{25}$$

By considering the coefficients of first order in $k$, we get

$$iu_1 + ilv_1 + \frac{\partial w_1}{\partial z} = 0$$
$$iT(z)u_1 - i\sigma_1 u_0 + w_1 = -ip_1 - iS\,H_y\left(h_{y0} - lh_{x0}\right)$$
$$iT(z)v_1 - i\sigma_1 v_0 + w_1 = -ilp_1 + i\,S\,H_x\left(h_{y0} - lh_{x0}\right)$$
$$-\frac{\partial p_1}{\partial z} - Ri\,\rho_1 - SH_y\frac{\partial h_{y0}}{\partial z} - SH_x\frac{\partial h_{x0}}{\partial z} = 0$$
$$iT(z)\rho_1 - i\sigma_1\rho_0 - \frac{N^2}{{N_0}^2}w_1 = 0 \tag{26}$$
$$ih_{x1} + ilh_{y1} + \frac{\partial h_{z1}}{\partial z} = 0$$

$$-\frac{1}{Rm}\frac{\partial^2 h_{x1}}{\partial z^2} - i\sigma_0 h_{x0} = ilz\left(h_{y0} - h_{x0}\right) + ilu_1 H_y - ilv_1 H_x - H_x\left(\frac{\partial w_1}{\partial z}\right) + h_{z0}$$

$$+z\frac{\partial h_{z0}}{\partial z} - Mi\frac{\partial h_{y0}}{\partial z}\left(H_x + lH_y\right)$$

$$-\frac{1}{Rm}\frac{\partial^2 h_{y1}}{\partial z^2} - i\sigma_0 h_{y0} = iz\left(h_{x0} - h_{y0}\right) - iu_1 H_y + iv_1 H_x - H_y\frac{\partial w_1}{\partial z} + h_{z0}$$

$$+z\frac{\partial h_{z0}}{\partial z} + Mi\frac{\partial h_{x0}}{\partial z}\left(H_x + lH_y\right)$$

$$-\frac{1}{Rm}\frac{\partial^2 h_{z1}}{\partial z^2} - i\sigma_0 h_{z0} = iw_1\left(H_x + lH_y\right) - iz(1+l)h_{z0}$$

$$+M\left(h_{y0} - lh_{x0}\right)\left(H_x + l\,H_y\right) \qquad (27)$$

The appropriate boundary conditions are given by

$$u_1 = v_1 = w_1 = 0 \qquad \text{at } z = \pm 1 \qquad (28)$$

By considering the quantities of second order in $k$, we get

$$iu_2 + ilv_2 + \frac{\partial w_2}{\partial z} = 0$$

$$iT(z)u_2 - i\sigma_1 u_1 - i\sigma_2 u_0 + w_2 = -ip_2 - iS\,H_y\left(h_{y1} - lh_{x1}\right)$$

$$iT(z)v_2 - i\sigma_1 v_1 - i\sigma_2 v_0 + w_2 = -ilp_2 + i\,S\,H_x\left(h_{y1} - lh_{x1}\right)$$

$$iT(z)w_0 - \frac{\partial p_2}{\partial z} - Ri\,\rho_2 - SH_y\frac{\partial h_{y1}}{\partial z} - SH_x\frac{\partial h_{x1}}{\partial z} = 0$$

$$iT(z)\rho_2 - i\sigma_1\rho_1 - i\sigma_2\rho_0 - \frac{N^2}{{N_0}^2}w_2 = 0 \qquad (29)$$

$$ih_{x2} + ilh_{y2} + \frac{\partial h_{z2}}{\partial z} = 0$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{x2}}{\partial z^2} - (1+l^2)h_{x0}\right) - i\sigma_1 h_{x0} - i\sigma_0 h_{x1} = ilz\left(h_{y1} - h_{x1}\right) + ilu_2 H_y - ilv_2 H_x$$

$$-H_x\left(\frac{\partial w_2}{\partial z}\right) + h_{z1} + z\frac{\partial h_{z1}}{\partial z} - Mi\frac{\partial h_{y1}}{\partial z}\left(H_x + lH_y\right)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{y2}}{\partial z^2} - (1+l^2)h_{y0}\right) - i\sigma_1 h_{y0} - i\sigma_0 h_{y1} = iz\left(h_{x1} - h_{y1}\right) - iu_2 H_y + iv_2 H_x$$

$$-H_y\frac{\partial w_2}{\partial z} + h_{z1} + z\frac{\partial h_{z1}}{\partial z} + Mi\frac{\partial h_{x1}}{\partial z}\left(H_x + lH_y\right)$$

$$-\frac{1}{Rm}\left(\frac{\partial^2 h_{z2}}{\partial z^2} - (1+l^2)h_{z0}\right) - i\sigma_1 h_{z0} - i\sigma_0 h_{z1} = iw_2\left(H_x + lH_y\right) - iz(1+l)h_{z1}$$

$$-M\left(h_{y0}-lh_{x0}\right)\left(H_x+l\,H_y\right) \tag{30}$$

Corresponding boundary conditions are

$$u_2=v_2=w_2=0 \qquad \text{at } z=\pm 1 \tag{31}$$

Eliminating $\rho_0,\ p_0, u_0, v_0$in favour of $w_0$ from (23) we obtain

$$T(z)^2\frac{\partial^2 w_0}{\partial z^2}+\frac{Ri\,N^2}{{N_0}^2}(1+l^2)w_0=0 \tag{32}$$

The solution of equation (28) is given as

$$w_0=A\,T(z)^{m_1}+B\,T(z)^{m_2} \tag{33}$$

where $m_{1,2}=\frac{1\pm\sqrt{\lambda}}{2}$, $\lambda=1-4\,Ri\,\frac{N^2}{N_0^2}\frac{(1+l^2)}{(1+l)^2}$, A and B are constants of integration.

To determine the arbitrary constants, we impose the boundary condition that the velocity should vanish at the boundaries (i.e) $w_0=0\ \ at\ z=\pm 1$, yields

$$\begin{vmatrix}(1+l-\sigma_0)^{m_1} & (1+l-\sigma_0)^{m_2}\\ (-(1+l)-\sigma_0)^{m_1} & (-(1+l)-\sigma_0)^{m_2}\end{vmatrix}=0$$

By solving the above determinant the value of $\sigma_0$ can be obtained as

$$\sigma_0=\ (1+l)\frac{1+e^{\frac{2n\pi i}{m_1-m_2}}}{1-e^{\frac{2n\pi i}{m_1-m_2}}} \tag{34}$$

The solution of equations (23) and (24) are given by

$$\begin{aligned}
u_0&=B_5T(z)^{m_1-1}+B_6T(z)^{m_2-1}\\
v_0&=B_7T(z)^{m_1-1}+B_8T(z)^{m_2-1}\\
w_0&=\ T(z)^{m_1}+B\,T(z)^{m_2}\\
\rho_0&=B_1T(z)^{m_1-1}+B_2T(z)^{m_2-1}\\
p_0&=\ B_3T(z)^{m_1}+B_4\,T(z)^{m_2}
\end{aligned} \tag{35}$$

$$\begin{aligned}
h_{x0}&=-Rm\ (B_9T(z)^{m_1+1}+B_{10}T(z)^{m_2+1})\\
h_{y0}&=-Rm\ (B_{11}T(z)^{m_1+1}+B_{12}T(z)^{m_2+1})\\
h_{z0}&=-Rm\ (B_{13}T(z)^{m_1+2}+B_{14}T(z)^{m_2+2})
\end{aligned}$$

By simplifying equ (26) interms of $w_1$, we get

$$T(z)^2\frac{\partial^2 w_1}{\partial y^2}+\frac{Ri\,N^2}{{N_0}^2}(1+l^2)w_1=\sigma_1\left((C_{15}-Ri\,C_{16})T(z)^{m_1-1}+(C_{17}-Ri\,C_{18})T(z)^{m_2-1}\right)$$

$$+SRm\ (C_{19}T(z)^{m_1+1}+C_{20}T(z)^{m_2+1}) \tag{36}$$

The solution of equ (36) is obtained in the form

$$w_1=\ CT(z)^{m_1}+DT(z)^{m_2}+\sigma_1\left((B_{21}-Ri\,B_{22})T(z)^{m_1-1}+(B_{23}-Ri\,B_{24})T(z)^{m_2-1}\right)$$

$$+SRm\ (B_{25}T(z)^{m_1+1}+B_{26}T(z)^{m_2+1})$$

By applying the boundary condition $w_1(\pm 1)=0$ and simplifying for $\sigma_1$, we obtain

$$\sigma_1=\frac{S\,Rm\,B_{41}}{B_{42}-Ri\,B_{43}} \tag{37}$$

By solving the set of equs (26) and (27), we get

$$u_1 = \left(S\,Rm\,B_{58} - \sigma_1(B_{59} - Ri\,B_{60})\right)T(z)^{m_1-1} + \left(S\,Rm\,B_{61} - \sigma_1(B_{62} - Ri\,B_{63})\right)T(z)^{m_2-1}$$
$$+ \sigma_1\left((B_{64} - Ri\,B_{65})T(z)^{m_1-2} + (B_{66} - Ri\,B_{67})T(z)^{m_2-2}\right)$$
$$+ SRm\,(B_{68}T(z)^{m_1} + B_{69}T(z)^{m_2})$$

$$v_1 = \left(S\,Rm\,B_{70} - \sigma_1(B_{71} - Ri\,B_{72})\right)T(z)^{m_1-1} + \left(S\,Rm\,B_{73} - \sigma_1(B_{74} - Ri\,B_{75})\right)T(z)^{m_2-1}$$
$$+ \sigma_1\left((B_{76} - Ri\,B_{77})T(z)^{m_1-2} + (B_{78} - Ri\,B_{79})T(z)^{m_2-2}\right)$$
$$+ SRm\,(B_{80}T(z)^{m_1} + B_{81}T(z)^{m_2})$$

$$w_1 = \left(S\,Rm\,B_{35} - \sigma_1(B_{36} - Ri\,B_{37})\right)T(z)^{m_1} + \left(S\,Rm\,B_{38} - \sigma_1(B_{39} - Ri\,B_{40})\right)T(z)^{m_2}$$
$$+ \sigma_1\left((B_{21} - Ri\,B_{22})T(z)^{m_1-1} + (B_{23} - Ri\,B_{24})T(z)^{m_2-1}\right)$$
$$+ SRm\,(B_{25}T(z)^{m_1+1} + B_{26}T(z)^{m_2+1})$$

$$\rho_1 = \left(S\,Rm\,B_{35} - \sigma_1(B_{36} - Ri\,B_{37})\right)T(z)^{m_1-1} + \left(S\,Rm\,B_{38} - \sigma_1(B_{39} - Ri\,B_{40})\right)T(z)^{m_2-1}$$
$$+ \sigma_1\left((B_{44} - Ri\,B_{22})T(z)^{m_1-2} + (B_{45} - Ri\,B_{24})T(z)^{m_2-2}\right)$$
$$+ SRm\,(B_{25}T(z)^{m_1} + B_{26}T(z)^{m_2})$$

$$p_1 = \left(S\,Rm\,B_{46} - \sigma_1(B_{47} - Ri\,B_{48})\right)T(z)^{m_1} + \left(S\,Rm\,B_{49} - \sigma_1(B_{50} - Ri\,B_{51})\right)T(z)^{m_2}$$
$$+ \sigma_1\left((B_{52} - Ri\,B_{53})T(z)^{m_1-1} + (B_{54} - Ri\,B_{55})T(z)^{m_2-1}\right)$$
$$+ SRm\,(B_{56}T(z)^{m_1+1} + B_{57}T(z)^{m_2+1})$$

$$h_{x1} = Rm(\sigma_1(B_{82} - Ri\,B_{83})T(z)^{m_1} + \sigma_1(B_{84} - Ri\,B_{85})T(z)^{m_2}$$
$$+(S\,Rm\,B_{86} - \sigma_1(B_{87} - Ri\,B_{88}) + B_{101} + Rm\,B_{102})T(z)^{m_1+1}$$
$$+(S\,Rm\,B_{89} - \sigma_1(B_{90} - Ri\,B_{91}) + B_{108} + Rm\,B_{109})T(z)^{m_2+1}$$
$$+(S\,Rm\,B_{92} + M\,B_{93} + B_{99} + Rm\,B_{100})T(z)^{m_1+2}$$
$$+(S\,Rm\,B_{94} + M\,B_{95} + B_{106} + Rm\,B_{107})T(z)^{m_2+2}$$
$$+\left(B_{96} + z(B_{97} + Rm\,B_{98})\right)T(z)^{m_1+3} + \left(B_{103} + z(B_{104} + Rm\,B_{105})\right)T(z)^{m_2+3}$$
$$+Rm(B_{110}T(z)^{m_1+4} + B_{111}T(z)^{m_2+4})\Big)$$

$$h_{y1} = Rm(\sigma_1(B_{112} - Ri\,B_{113})T(z)^{m_1} + \sigma_1(B_{114} - Ri\,B_{115})T(z)^{m_2}$$
$$+(S\,Rm\,B_{116} - \sigma_1(B_{117} - Ri\,B_{118}) + RmB_{131}z^3)T(z)^{m_1+1}$$
$$+(S\,Rm\,B_{119} - \sigma_1(B_{120} - Ri\,B_{121}) + RmB_{133}z^3)T(z)^{m_2+1}$$
$$+(S\,Rm\,B_{122} + Rm\,B_{130}z^2)T(z)^{m_1+2} + (S\,Rm\,B_{123} + Rm\,B_{132}z^2)T(z)^{m_2+2}$$
$$+Rm\big((\,B_{124} + z\,B_{125} + M\,B_{126})T(z)^{m_1+3} + (\,B_{127} + z\,B_{128} + M\,B_{129})T(z)^{m_2+3}$$
$$+Rm(B_{134}T(z)^{m_1+4} + B_{135}T(z)^{m_2+4})\big)$$

$$h_{z1} = Rm(\sigma_1(B_{136} - Ri\,B_{137})T(z)^{m_1+1} + \sigma_1(B_{138} - Ri\,B_{139})T(z)^{m_2+1}$$

$$+\left(S\,Rm\,B_{140}-\sigma_1(B_{141}-Ri\,B_{142})\right)T(z)^{m_1+2}$$
$$+\left(S\,Rm\,B_{143}-\sigma_1(B_{144}-Ri\,B_{145})\right)T(z)^{m_2+2}$$
$$+Rm\big((M\,B_{148}-SB_{146})T(z)^{m_1+3}+(M\,B_{149}-S\,B_{147})T(z)^{m_2+3}$$
$$+Rm(B_{150}T(z)^{m_1+5}+B_{151}T(z)^{m_2+5})\big) \tag{38}$$

The simplified form of equation (29) in terms of $w_2$ is obtained as

$$T(z)^2\frac{\partial^2 w_2}{\partial y^2}+\frac{Ri\,N^2}{{N_0}^2}(1+l^2)w_2=B_{152}T(z)^{m_1-2}+B_{153}T(z)^{m_2-2}+\left(B_{154}-\sigma_2(d_3-Ri\,d_4)\right)T(z)^{m_1-1}+$$
$$\left(B_{155}-\sigma_2(d_5-Ri\,d_6)\right)T(z)^{m_2-1}+B_{156}T(z)^{m_1}+B_{157}T(z)^{m_2}+B_{158}T(z)^{m_1+1}+$$
$$B_{159}T(z)^{m_2+1}+B_{160}T(z)^{m_1+2}+B_{161}T(z)^{m_2+2}$$
$$+B_{162}T(z)^{m_1+3}+B_{163}T(z)^{m_2+3}+B_{164}T(z)^{m_1+4}+B_{165}T(z)^{m_2+4} \tag{39}$$

The solution of equation (39) is obtained as

$$w_2=(E+B_{174})T(z)^{m_1}+(F+B_{175})DT(z)^{m_2}$$
$$+B_{166}T(z)^{m_1-2}+B_{167}T(z)^{m_2-2}+\left(B_{168}-\sigma_2(B_{169}-Ri\,B_{170})\right)T(z)^{m_1-1}$$
$$+\left(B_{171}-\sigma_2(B_{172}-Ri\,B_{173})\right)T(z)^{m_2-1}+B_{176}T(z)^{m_1+1}+B_{177}T(z)^{m_2+1}$$
$$+B_{178}T(z)^{m_1+2}+B_{179}T(z)^{m_2+2}+B_{180}T(z)^{m_1+3}+B_{181}T(z)^{m_2+3}$$
$$+B_{182}T(z)^{m_1+4}+B_{183}T(z)^{m_2+4} \tag{40}$$

By applying the boundary condition that $w_2(\pm 1)=0$ and simplifying for $\sigma_2$, we obtain

$$\sigma_2=\frac{B_{189}}{B_{190}-Ri\,B_{191}} \tag{41}$$

For the sake of brevity the constants are given in A*ppendix*.

## 4. RESULT AND DISCUSSION

In this paper, we have made an attempt to analyze the stability of stratified non-parallel shear flow with Hall effect. A numerical computation is done to analyze the nature of various physical quantities which describes the stability characteristics. To understand the effect of various nondimensional parameters, the growth rate is plotted as a function of these parameters. Figures (2) – (8) present the growth rate as a function of wave number for different dimensionless quantities present in the problem when $\lambda > 0$. The results are discussed as follows.

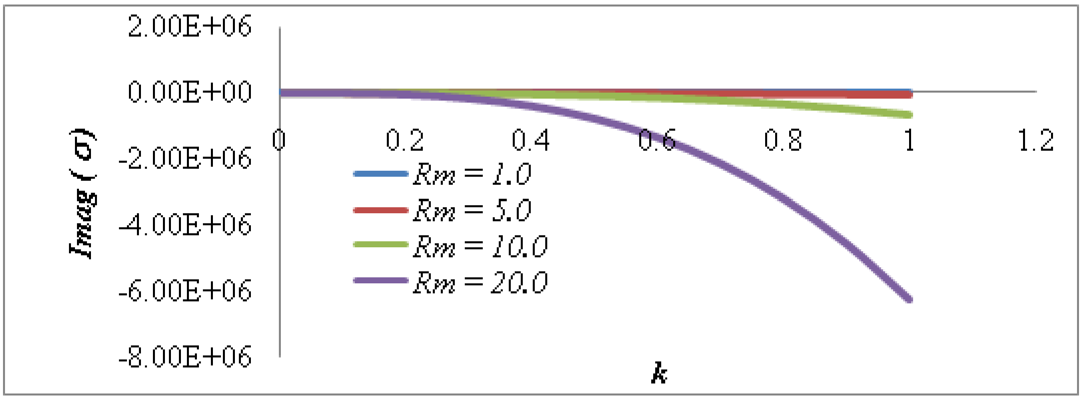


**Figure 2.** Growth rate vs wave number for various *Rm*

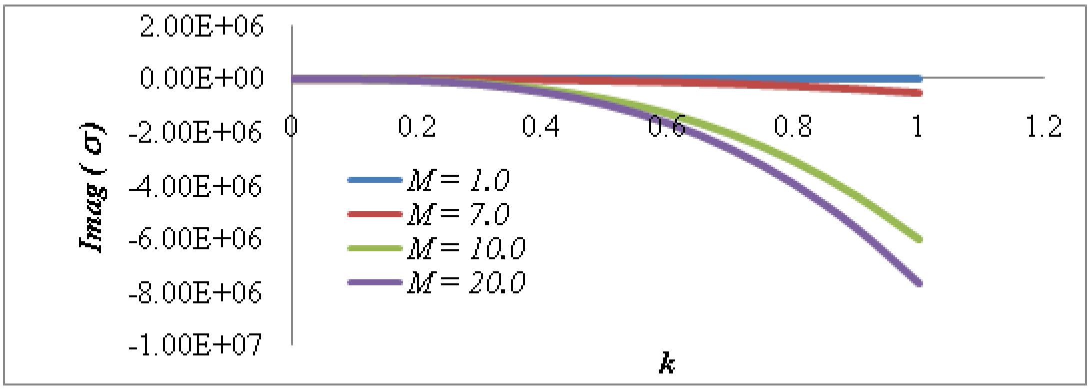


**Figure 3.** Growth rate vs wave number for various *M*

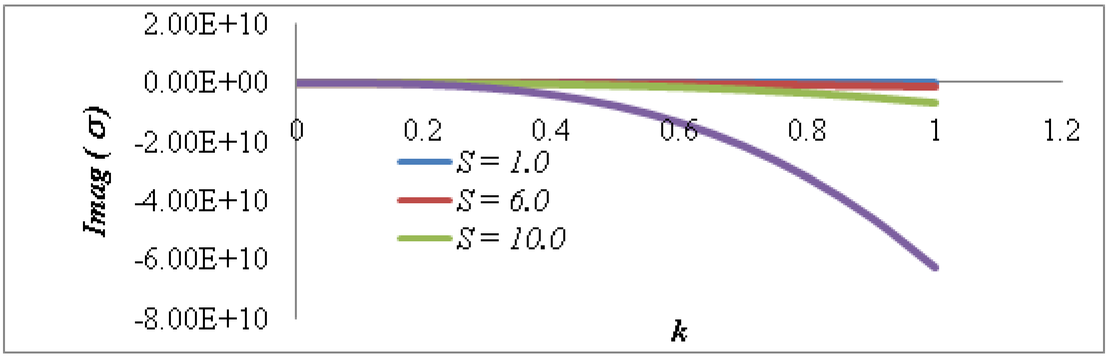


**Figure 4.** Growth rate vs wave number for various *S*

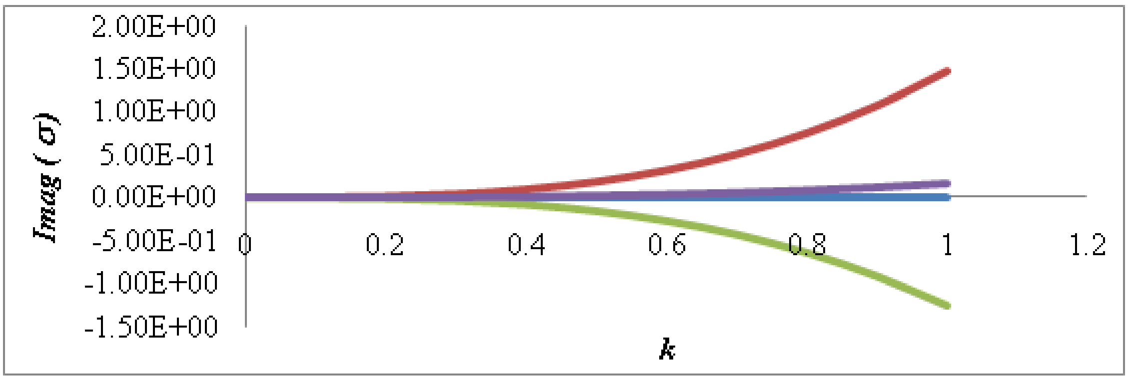


**Figure 5.** Growth rate vs wave number for various $N^2$

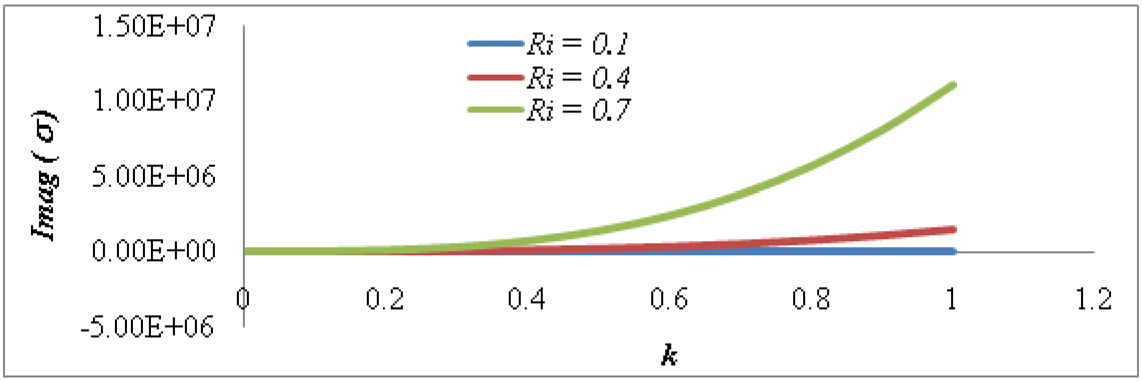


**Figure 6.** Growth rate vs wave number for various *Ri*

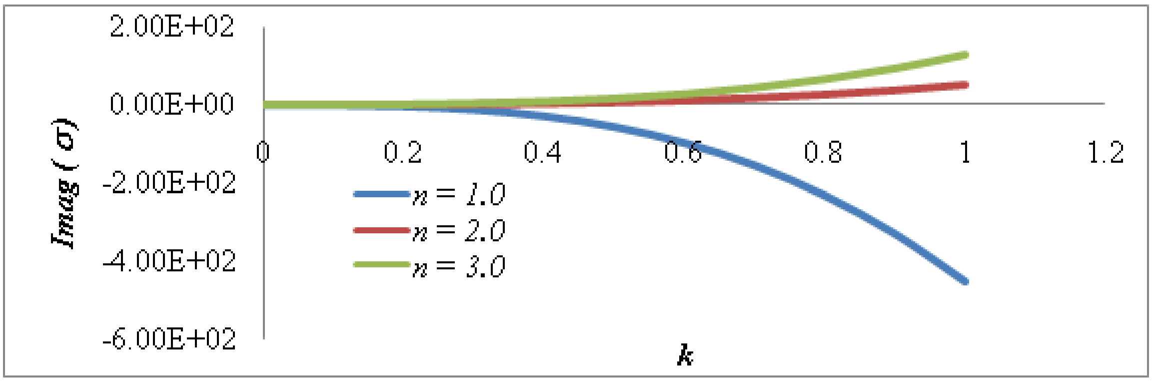


**Figure 7.** Growth rate wave number for various *n*

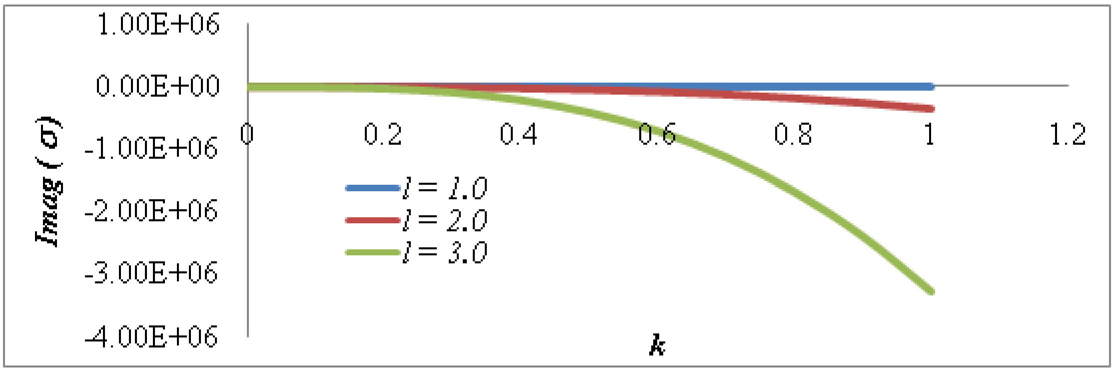


**Figure 8.** Growth rate vs wave number for various *l*

Figure 2 presents growth rate as a function of wave number for various Magnetic Reynolds number. It is understood that increase in Magnetic Reynolds number decreases the growth rate with increasing wave number. Thus, we may conclude that increase in Magnetic Reynolds number contributes more to the flow stability. Growth rate as a function of wave number with increasing Hall parameter is shown in Fig. 3. From Figure 3, it is clear that increase in Hall parameter decreases the growth rate with the increase in wave number thereby making the system stable.

Figure 4 explains the variation of growth rate as a function of wave number for various Magnetic pressure number. It is observed that increase in magnetic pressure number decreases the growth rate with the increase in wave number. From this, we conclude that with the increase in wave number, the growth rate increases and leads to decay of disturbances. The growth rate as a function of wave number is shown through Figure 5 with various Brunt

Vaisala frequency. We observe from the figure that, initially the system is stable and Brunt – Vaisala frequency plays a key role on the stability of the system. We can observe that for smaller Brunt - Vaisala frequency, the system is unstable and as the Brunt – Vaisala frequency increases, the disturbances tend to decay thereby stabilizing the system.

Figure 6 presents the behavior of growth rate as a function of wave number for different Richardson number. On careful observation, it can be inferred that with the increase in Richardson number the growth rate also increases thereby contributes more to the instability of the flow. Figure 7 depicts the behavior of growth rate with respect to various *n*. It is concluded that, infinite number of modes exists for the given stability problem. In the case of increasing transverse wave number the behavior of growth rate is discussed in Fig 8. It is noted that increase in transverse wave number decreases the growth rate with the increase in k and results in the stabilization of the system.

Growth rate vs Brunt Vaisala frequency for various Hall parameter, Magnetic Reynolds number, Magnetic pressure number and longitudinal wave number is demonstrated in Figures (9) – (12). From these figures, it is clear that growth rate decreases with the increase in Hall parameter, Magnetic Reynolds number and longitudinal wave number, increases with increase in Magnetic pressure number. From all the above cases it can be inferred that the system is unstable, becomes stable with the increase in the Brunt Vaisala frequency. Figure (4.13) presents the behavior of growth rate vs Richardson number for various wave number *k*. It is concluded that, with the increase in Richardson number growth rate decreases with the increase in *k*. this makes the system more stable.

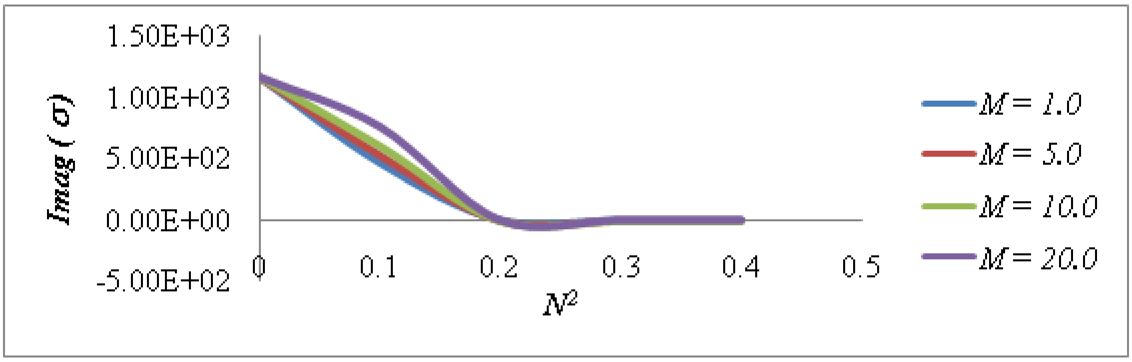


**Figure 9.** Growth rate vs Brunt vaisala frequency for various *M*

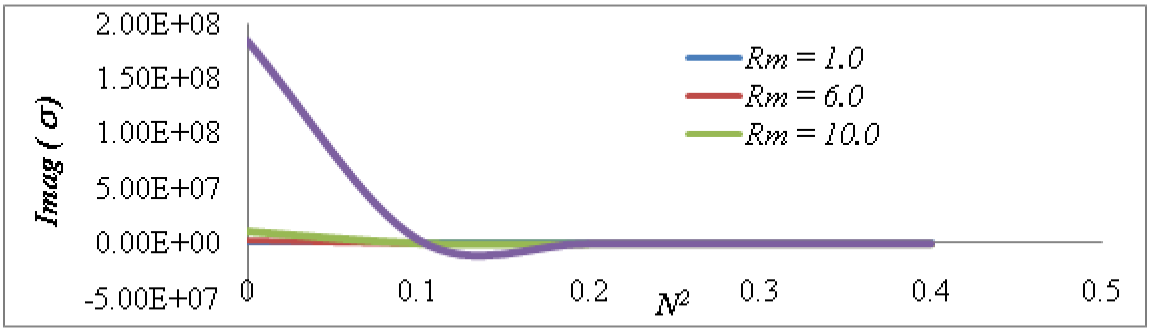


**Figure 10.** Growth rate vs Brunt vaisala frequency for various *Rm*

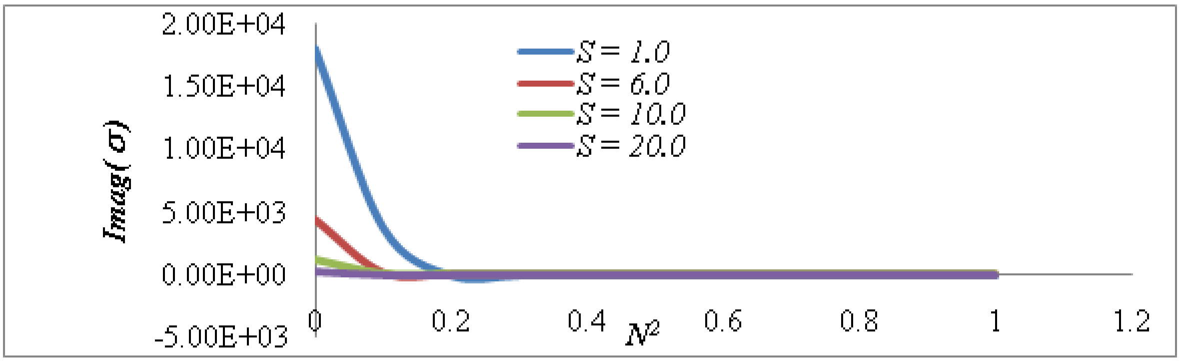


**Figure 11.** Growth rate vs Brunt vaisala frequency for various *S*

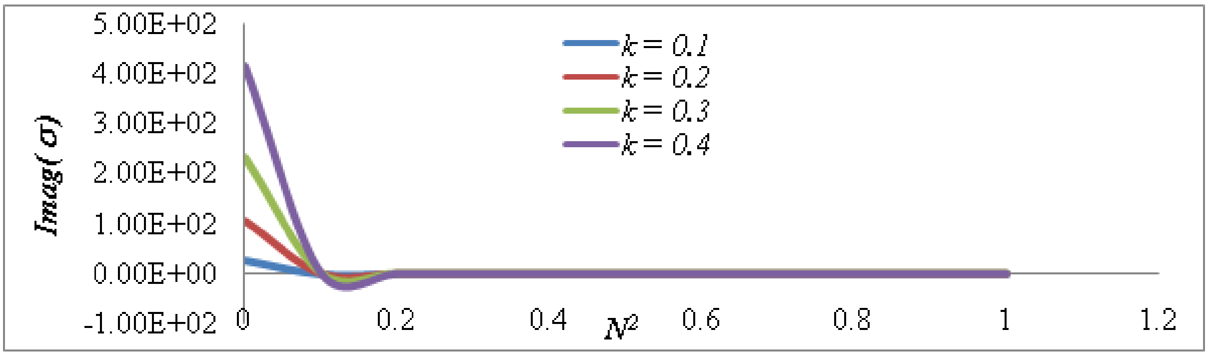


**Figure 12.** Growth rate vs Brunt vaisala frequency for various *k*

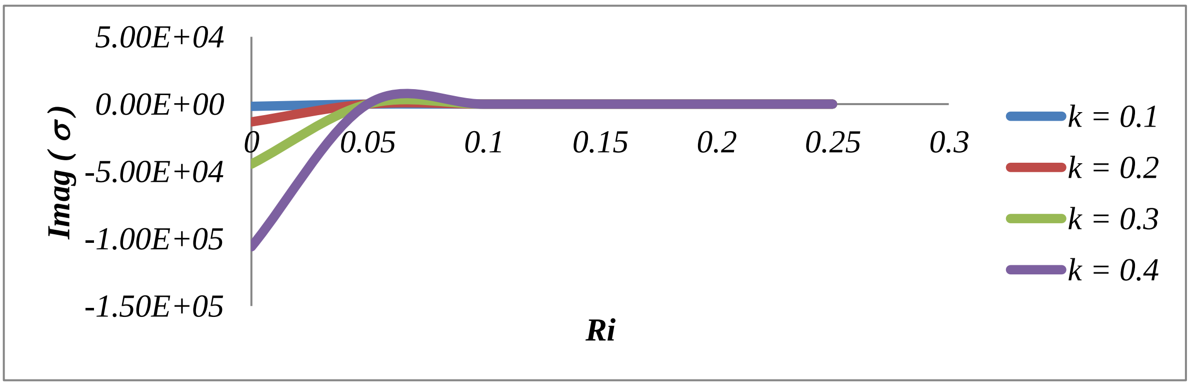


**Figure 13**. Growth rate vs Richarson number for various *k*

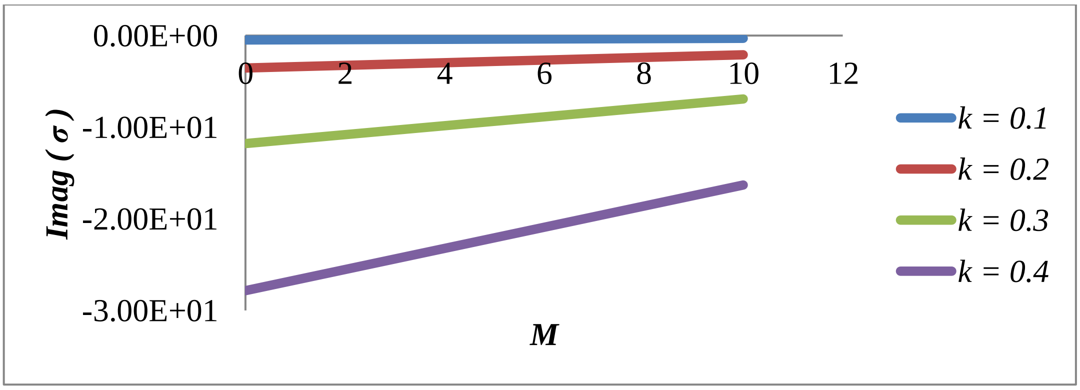


**Figure 14.** Growth rate vs Hall parameter for various *k*

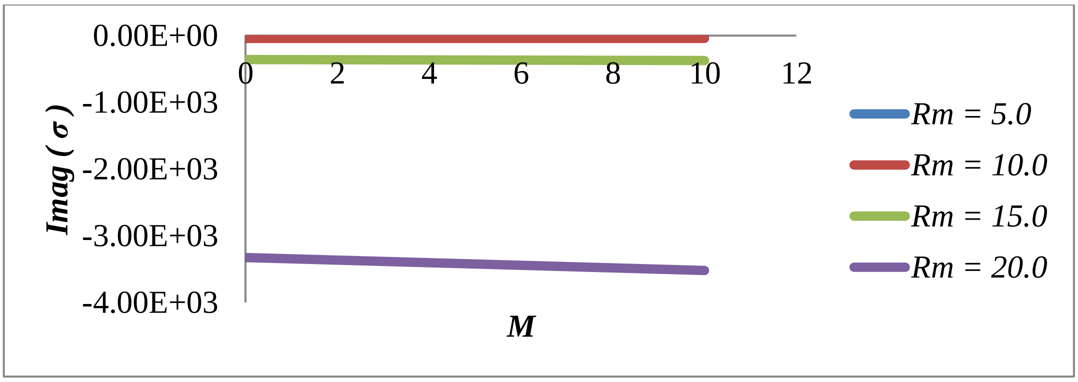


**Figure 15.** Growth rate vs Hall parameter for various *Rm*

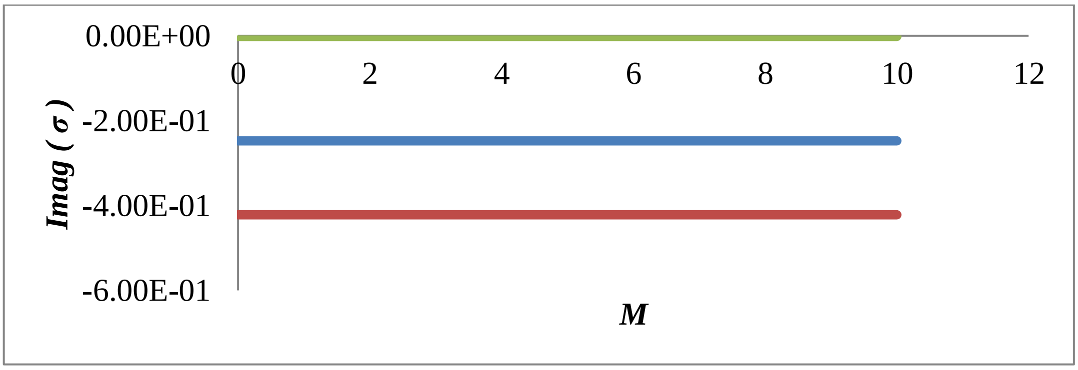


**Figure 16.** Growth rate vs Hall parameter for various $N^2$

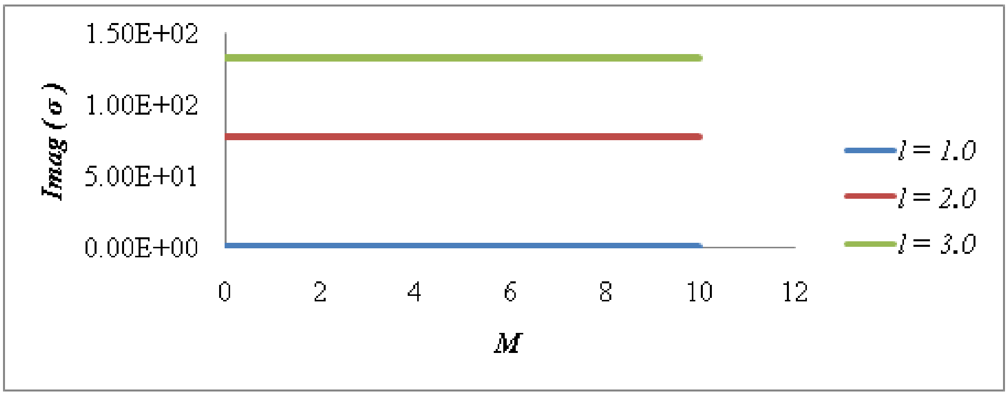


**Figure 17.** Growth rate vs Hall parameter for various *l*

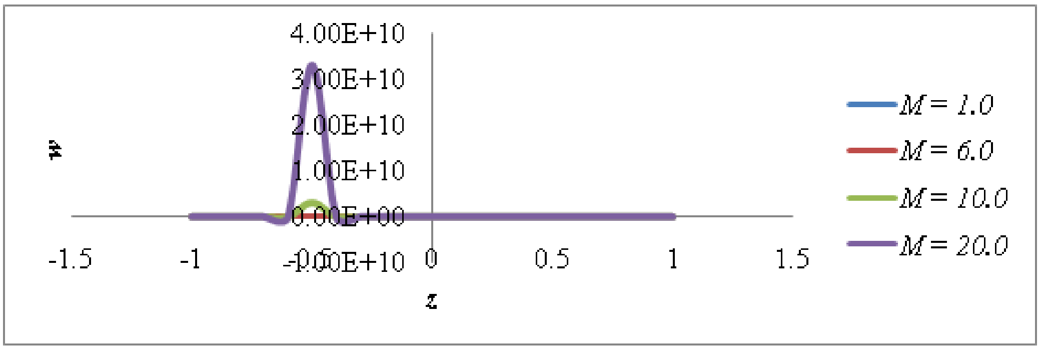


**Figure 18** Velocity profile for various *M*

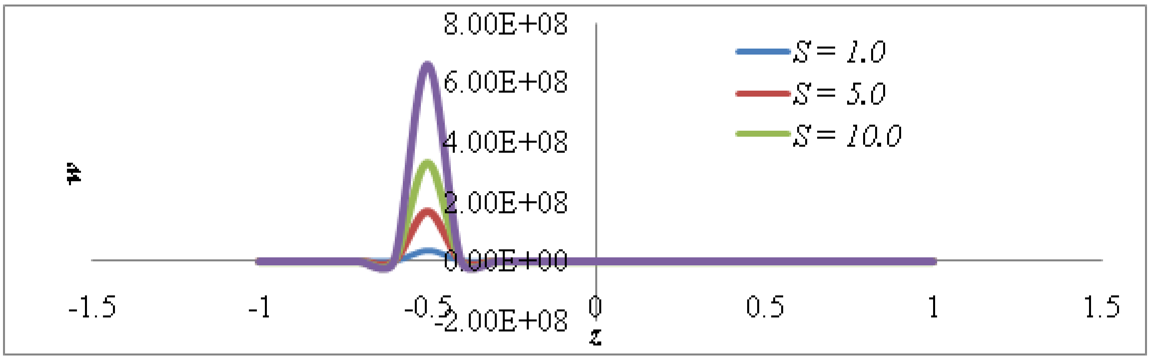


**Figure 19.** Velocity profile for various *S*

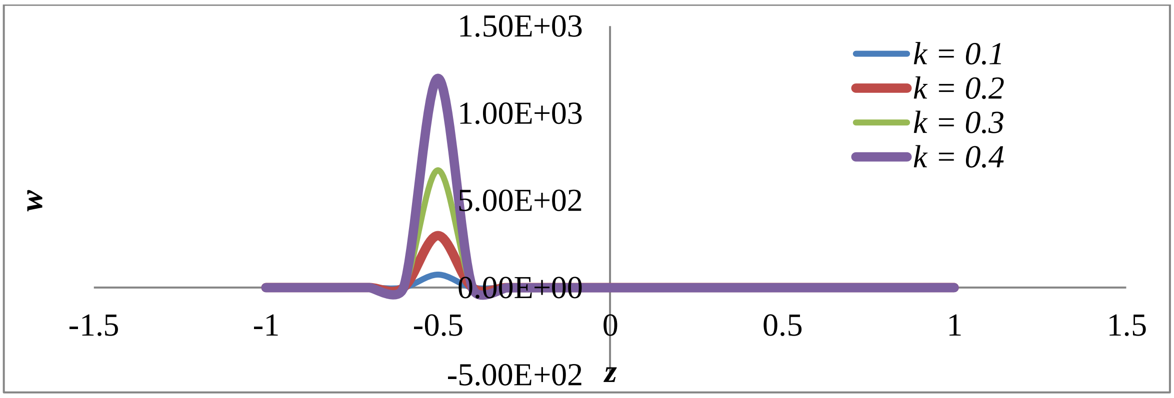


**Figure 20.** Velocity profile for various *k*

Figures (14) – (17) portray the nature of growth rate with respect to Hall parameter. From these Figures, it is clear that growth rate decreases with the increase in wave number, magnetic Reynolds number and Brunt – Vaisala frequency making the system more stable, growth of disturbances increases with the increase in and transverse wave number. The system becomes unstable with increasing transverse wave number.

Figures (18) - (20) give the velocity profile for various Hall parameter, Magnetic pressure number and wave number. From these Figures, we can conclude that velocity increases with the increase in the above said parameters.

## 5. CONCLUSION

The effect due to the inclusion of Hall current on the linear stability of inviscid, incompressible nonparallel stratified shear flow of a perfectly conducting fluid is analyzed. Series expansion method is used to solve the equations governing the flow. A theory for non-parallel stratified shear flow is developed formally and applied in detail for three dimensional Cartesian coordinate system for $\lambda > 0$. From the results obtained from the previous section, following conclusions can be drawn.

- The flow field is stable with the increase in Magnetic Reynolds number, Hall parameter, transverse wave number and Magnetic pressure number.
- The system becomes unstable with the increase in Brunt-Vaisala frequency.
- Increase in Richardson number destabilizes the field of flow.
- The system becomes unstable for various Hall parameter, Magnetic Reynolds number, Magnetic pressure number and longitudinal wave number with the increase in Brunt Vaisala frequency.

- The system is stable with the increase in Richardson number for various wave number.
- Increase in wave number, magnetic Reynolds number and Brunt – Vaisala frequency results in the stability of the system, the system becomes unstable with the increase in transverse wave number.
- Velocity profile increases with the increase in Hall parameter, Magnetic Pressure number and wave number.

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**Appendix**

$B = -(1-\sigma_0)^{m_1-m_2}$ $\quad C_1 = \frac{N^2}{i\,{N_0}^2}$

$C_2 = \frac{N^2 B}{i\,{N_0}^2}$ $\quad C_3 = \frac{-i(1+l)(m_1-1)}{(1+l^2)}$

$C_4 = \frac{-i(1+l)B(m_2-1)}{(1+l^2)}$

$C_5 = -C_3 + i$ $\quad C_6 = -C_4 + Bi$

$C_7 = -l\,C_3 + i$ $\quad C_8 = -l\,C_4 + Bi$

$C_9 = \frac{-i\,lH_yC_5 - ilH_xC_7 - H_xm_1(1+l)}{m_1\,(m_1+1)(1+l)^2}$

$C_{10} = \frac{i\,lH_yC_6 - ilH_xC_8 - H_xBm_2(1+l)}{m_2\,(m_2+1)(1+l)^2}$

$C_{11} = \frac{i\,H_xC_7 - iH_yC_5 - H_ym_1}{m_1\,(m_1+1)(1+l)^2}$

$C_{12} = \frac{i\,H_xC_8 - iH_yC_6 - H_yBm_2}{m_2\,(m_2+1)(1+l)^2}$

$C_{13} = \frac{i(H_x+lH_y)}{(m_1+1)(m_1+2)(1+l)^2}$

$C_{14} = \frac{i(H_x+lH_y)B}{(m_2+1)(m_2+2)(1+l)^2}$

$C_{15} = \frac{-(1+l)^2 m_1\,(m_1-1)}{(1+l^2)}$

$C_{16} = C_1$ , $\quad C_{17} = \frac{-(1+l)^2 Bm_2\,(m_2-1)}{(1+l^2)}$, $\ C_{18} = C_2$

$$C_{19} = \left(-\left(\frac{lH_y}{(1+l^2)} - \frac{l^2H_x}{(1+l^2)} - lH_x\right)C_9 + \left(\frac{H_y}{(1+l^2)} - \frac{lH_x}{(1+l^2)} - H_y\right)C_{11}\right)i(1+l)(m_1+1)$$

$$C_{20} = \left(-\left(\frac{lH_y}{(1+l^2)} - \frac{l^2H_x}{(1+l^2)} - H_x\right)C_{10} + \left(\frac{H_y}{(1+l^2)} - \frac{lH_x}{(1+l^2)} - H_y\right)C_{12}\right)i(1+l)(m_2+1)$$

$$C_{21} = \frac{C_{15}}{(m_1-1)^2 - (m_1-1) + \frac{Ri(1+l^2)}{(1+l)^2}\frac{N^2}{N_0^2}}$$

$$C_{22} = \frac{C_{16}}{(m_1-1)^2 - (m_1-1) + \frac{Ri(1+l^2)}{(1+l)^2}}$$

$$C_{23} = \frac{C_{17}}{(m_2-1)^2 - (m_2-1) + \frac{Ri(1+l^2)}{(1+l)^2}}$$

$$C_{24} = \frac{C_{18}}{(m_2-1)^2 - (m_2-1) + \frac{Ri(1+l^2)}{(1+l)^2}}$$

$$C_{25} = \frac{C_{19}}{(m_1+1)^2 - (m_1+1) + \frac{Ri(1+l^2)}{(1+l)^2}}$$

$$C_{26} = \frac{C_{20}}{(m_2+1)^2 - (m_2+1) + \frac{Ri(1+l^2)}{(1+l)^2}}$$

$a_1 = (1+l) - \sigma_0$

$a_2 = -(1+l) - \sigma_0$

$C_{27} = a_1^{m_2}a_2^{m_1} - a_1^{m_1}a_2^{m_2}$

$C_{28} = a_1^{m_1-1}a_2^{m_1} - a_2^{m_1-1}a_1^{m_1}$

$C_{29} = a_1^{m_2-1}a_2^{m_1} - a_1^{m_1}a_2^{m_2-1}$

$$C_{30} = C_{25}\left(a_1^{m_1+1}a_2^{m_1} - a_1^{m_1}a_2^{m_1+1}\right) + C_{26}\left(a_1^{m_2+1}a_2^{m_1} - a_1^{m_1}a_2^{m_2+1}\right)$$

$C_{31} = a_1^{m_1}a_2^{m_2} - a_1^{m_2}a_2^{m_1}$

$C_{32} = a_1^{m_1-1}a_2^{m_2} - a_2^{m_1-1}a_1^{m_2}$

$C_{33} = a_1^{m_2-1}a_2^{m_2} - a_1^{m_2}a_2^{m_2-1}$

$$C_{34} = C_{25}\left(a_1^{m_1+1}a_2^{m_2} - a_1^{m_2}a_2^{m_1+1}\right) + C_{26}\left(a_1^{m_2+1}a_2^{m_2} - a_1^{m_2}a_2^{m_2+1}\right)$$

$C_{35} = \frac{C_{34}}{C_{31}}$, $\quad C_{36} = \frac{C_{21}C_{32}+C_{23}C_{33}}{C_{31}}$

$C_{37} = \frac{C_{22}C_{32}+C_{24}C_{33}}{C_{31}}$, $\quad C_{38} = \frac{C_{30}}{C_{27}}$

$C_{39} = \frac{C_{21}C_{28}+C_{23}C_{29}}{C_{27}}$, $\quad C_{40} = \frac{C_{22}C_{28}+C_{24}C_{29}}{C_{27}}$

$C_{41} = C_{35}a_1^{m_1} + C_{38}a_1^{m_2} - C_{25}a_1^{m_1+1} - C_{26}a_1^{m_2+1}$

$C_{42} = C_{36}a_1^{m_1} + C_{39}a_1^{m_2} - C_{21}a_1^{m_1-1} - C_{23}a_1^{m_2-1}$

$C_{43} = C_{37}a_1^{m_1} + C_{40}a_1^{m_2} - C_{22}a_1^{m_1-1} - C_{24}a_1^{m_2-1}$

$C_{44} = C_1 - \frac{iN^2}{{N_0}^2}C_{21}$ , $\quad C_{45} = C_2 - \frac{iN^2}{{N_0}^2}C_{23}$

$$C_{46} = \frac{1}{i(1+l^2)}C_{35}(m_1-1)(1+l)$$

$$C_{47} = \frac{1}{i(1+l^2)}C_{36}(m_1-1)(1+l)$$

$$C_{48} = \frac{1}{i(1+l^2)}C_{37}(m_1-1)(1+l)$$

$$C_{49} = \frac{-1}{i(1+l^2)}C_{38}(m_2-1)(1+l)$$

$$C_{50} = \frac{-1}{i(1+l^2)}C_{39}(m_2-1)(1+l)$$

$C_{51} = \frac{-1}{i(1+l^2)}C_{40}(m_2-1)(1+l)$

$C_{52} = \frac{C_{21}(m_1-2)(1+l)}{i(1+l^2)} - \frac{m_1(1+l)}{i(1+l^2)}$

$C_{53} = \frac{C_{22}(m_1-2)(1+l)}{i(1+l^2)}$

$C_{54} = \frac{C_{23}(m_2-2)(1+l)}{i(1+l^2)} - \frac{Bm_2(1+l)}{i(1+l^2)}$

$C_{55} = \frac{C_{24}(m_2-2)(1+l)}{i(1+l^2)}$

$C_{56} = \frac{-C_{25}m_1(1+l)}{i(1+l^2)} + \frac{(H_y+lH_x)(C_{11}-lC_9)}{(1+l^2)}$

$C_{57} = \frac{-C_{26}m_2(1+l)}{i(1+l^2)} + \frac{(H_y+lH_x)(C_{12}-lC_{10})}{(1+l^2)}$

$C_{58} = -C_{46} + iC_{35}$ $\quad C_{59} = -C_{47} + iC_{36}$

$C_{60} = -C_{48} + iC_{37}$ $\quad C_{61} = -C_{49} + iC_{38}$

$C_{62} = -C_{50} + iC_{39}$ $\quad C_{63} = -C_{51} + iC_{40}$

$C_{64} = -C_{52} + iC_{21} + C_5$, $\ C_{65} = -C_{53} + iC_{22}$

$C_{66} = -C_{54} + iC_{23} + C_6$, $\ C_{67} = -C_{55} + iC_{24}$

$C_{68} = -C_{56} + iC_{25} + H_y(C_{11} - lC_9)$

$C_{69} = -C_{57} + iC_{26} + H_y(C_{12} - lC_{10})$

$C_{70} = -lC_{46} + iC_{35}$, $\quad C_{71} = -lC_{47} + iC_{36}$

$C_{72} = -lC_{48} + iC_{37}$, $\quad C_{73} = -lC_{49} + iC_{38}$

$C_{74} = -lC_{50} + iC_{39}$, $\quad C_{75} = -lC_{51} + iC_{40}$

$C_{76} = -lC_{52} + iC_{21} + C_7,\ C_{77} = -lC_{53} + iC_{22}$

$C_{78} = -lC_{54} + iC_{23} + C_8,\ C_{79} = -lC_{55} + iC_{24}$

$C_{80} = -lC_{56} + iC_{25} - H_x(C_{11} - lC_9)$

$C_{81} = -lC_{57} + iC_{26} - H_x(C_{12} - lC_{10})$

$$C_{82} = \frac{\left(-ilH_yC_{64} + ilH_xC_{76} + H_xC_{21}(m_1-1)(1+l)\right)(1+l)^2}{m_1(m_1-1)}$$

$$C_{83} = \frac{\left(-ilH_yC_{65} + ilH_xC_{77} + H_xC_{22}(m_1-1)(1+l)\right)(1+l)^2}{m_1(m_1-1)}$$

$$C_{84} = \frac{\left(-ilH_yC_{66} + ilH_xC_{78} + H_xC_{23}(m_2-1)(1+l)\right)(1+l)^2}{m_2(m_2-1)}$$

$$C_{85} = \frac{\left(-ilH_yC_{67} + ilH_xC_{79} + H_xC_{24}(m_2-1)(1+l)\right)(1+l)^2}{m_2(m_2-1)}$$

$$C_{86} = \frac{\left(-ilH_yC_{58} + ilH_xC_{70} + H_xC_{35}m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$$

$$C_{87} = \frac{\left(-ilH_yC_{59} + ilH_xC_{71} + H_xC_{36}m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$$

$$C_{88} = \frac{\left(-ilH_yC_{60} + ilH_xC_{72} + H_xC_{37}m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$$

$$C_{89} = \frac{\left(-ilH_yC_{61} + ilH_xC_{73} + H_xC_{38}m_1(1+l)\right)(1+l)^2}{m_2(m_2+1)}$$

$$C_{90} = \frac{\left(-ilH_yC_{62} + ilH_xC_{74} + H_xC_{39}m_2(1+l)\right)(1+l)^2}{m_2(m_2+1)}$$

$$C_{91} = \frac{\left(-ilH_yC_{63} + ilH_xC_{75} + H_xC_{40}m_2(1+l)\right)(1+l)^2}{m_2(m_2+1)}$$

$$C_{92} = \frac{\left(C_{25}(m_1+1)(1+l)H_x - ilH_yC_{68}\right)(1+l)^2}{(m_1+1)(m_1+2)}$$

$C_{93} = \frac{i(H_x+lH_y)C_{11}(1+l)^3}{(m_1+2)}$

$C_{94} = \frac{(C_{26}(m_2+1)(1+l)H_x - ilH_yC_{69})(1+l)^2}{(m_2+1)(m_2+2)}$

$C_{95} = \frac{i(H_x+lH_y)C_{12}(1+l)^3}{(m_2+2)}, \quad C_{96} = \frac{i\sigma_0C_9(1+l)^2}{(m_1+2)(m_1+3)}$

$C_{97} = \frac{(ilC_{11}-ilC_9)(1+l)^2}{(m_1+2)(m_1+3)}, \quad C_{98} = \frac{C_{13}(1+l)^3}{(m_1+3)}, \quad C_{99} = \frac{(ilC_{11}-ilC_9)(1+l)}{2(m_1+2)}, \quad C_{100} = \frac{C_{13}(1+l)^2}{2}$

$C_{101} = \frac{(ilC_{11}-ilC_9)}{6}, \quad C_{102} = \frac{C_{13}(m_1+2)(1+l)}{6}$

$C_{103} = \frac{i\sigma_0C_{10}(1+l)^2}{(m_2+2)(m_2+3)}, \quad C_{104} = \frac{(ilC_{12}-ilC_{10})(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{105} = \frac{C_{14}(1+l)^3}{(m_2+3)}, \quad C_{106} = \frac{(ilC_{12}-ilC_{10})(1+l)}{2(m_2+2)}$

$C_{107} = \frac{C_{14}(1+l)^2}{2}, \quad C_{108} = \frac{(ilC_{12}-ilC_{10})}{6}$

$C_{109} = \frac{C_{14}(m_2+2)(1+l)}{6}, \quad C_{110} = \frac{C_{13}(1+l)^2}{(m_1+3)(m_1+4)}$

$C_{111} = \frac{C_{14}(1+l)^2}{(m_2+3)(m_2+4)}, \quad C_{112} = \frac{\left(-iH_xC_{76}+iH_yC_{64}+iH_yC_{21}(m_1-1)(1+l)\right)(1+l)^2}{m_1(m_1-1)}$

$C_{113} = \frac{\left(-iH_xC_{77}+iH_yC_{65}+iH_yC_{22}(m_1-1)(1+l)\right)(1+l)^2}{m_1(m_1-1)}$

$C_{114} = \frac{\left(-iH_xC_{78}+iH_yC_{66}+iH_yC_{23}(m_2-1)(1+l)\right)(1+l)^2}{m_2(m_2-1)}$

$C_{115} = \frac{\left(-iH_xC_{79}+iH_yC_{67}+iH_yC_{24}(m_2-1)(1+l)\right)(1+l)^2}{m_2(m_2-1)}$

$C_{116} = \frac{\left(-iH_xC_{70}+iH_yC_{58}+H_y(C_{35}+C_{80})m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$

$C_{117} = \frac{\left(-iH_xC_{71}+iH_yC_{59}+H_yC_{36}m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$

$C_{118} = \frac{\left(-iH_xC_{72}+iH_yC_{60}+H_yC_{37}m_1(1+l)\right)(1+l)^2}{m_1(m_1+1)}$

$C_{119} = \frac{\left(-iH_xC_{73}+iH_yC_{61}+H_y(C_{38}+C_{81})m_2(1+l)\right)(1+l)^2}{m_2(m_2+1)}$

$C_{120} = \frac{\left(-iH_xC_{74}+iH_yC_{62}+H_yC_{39}m_2(1+l)\right)(1+l)^2}{m_2(m_2+1)}$

$C_{121} = \frac{\left(-iH_xC_{75}+iH_yC_{63}+H_yC_{40}m_2(1+l)\right)(1+l)^2}{m_2(m_2+1)}$

$C_{122} = \frac{(-iH_xC_{80}+iH_yC_{68})(1+l)^2}{(m_1+1)(m_1+2)}$

$C_{123} = \frac{(-iH_xC_{81}+iH_yC_{69})(1+l)^2}{(m_2+1)(m_2+2)}$

$C_{124} = \frac{i\sigma_0C_{11}(1+l)^2}{(m_1+2)(m_1+3)}, \quad C_{125} = \frac{(i(C_9-C_{11})+C_{13}(m_1+2)(1+l))(1+l)^2}{(m_1+2)(m_1+3)}$

$C_{126} = \frac{(iH_x+ilH_y)C_9(1+l)^2}{(m_1+2)(m_1+3)}, \quad C_{127} = \frac{i\sigma_0C_{12}(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{128} = \frac{(i(C_{10}-C_{12})+C_{14}(m_2+2)(1+l))(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{129} = \frac{(iH_x+ilH_y)C_{10}(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{130} = \left(\frac{i(C_9-C_{11})}{2} + C_{13}(m_1+2)(1+l)\right)\frac{1+l}{(m_1+2)}$

$C_{131} = \frac{i(C_9-C_{11})+C_{13}(m_1+2)(1+l)}{6}$

$C_{132} = \left(\frac{i(C_{10}-C_{12})}{2} + C_{14}(m_2+2)(1+l)\right)\frac{1+l}{(m_2+2)}$

$C_{133} = \frac{i(C_{10}-C_{12})+C_{14}(m_2+2)(1+l)}{6}$

$C_{134} = \frac{C_{13}(1+l)^2}{(m_1+3)(m_1+4)}, \quad C_{135} = \frac{C_{14}(1+l)^2}{(m_2+3)(m_2+4)}$

$C_{136} = \frac{-C_{21}(1+l)^2}{m_1(m_1+1)}, \quad C_{137} = \frac{-C_{22}(1+l)^2}{m_1(m_1+1)}$

$C_{138} = \frac{-C_{23}(1+l)^2}{m_2(m_2+1)}, \quad C_{139} = \frac{-C_{24}(1+l)^2}{m_2(m_2+1)}$

$C_{140} = \frac{-i(H_x+lH_y)C_{35}(1+l)^2}{(m_1+1)(m_1+2)}$

$C_{141} = \frac{-i(H_x+lH_y)C_{36}(1+l)^2}{(m_1+1)(m_1+2)}$

$C_{142} = \frac{-i(H_x+lH_y)C_{37}(1+l)^2}{(m_1+1)(m_1+2)}$

$C_{143} = \frac{-i(H_x+lH_y)C_{38}(1+l)^2}{(m_2+1)(m_2+2)}$

$C_{144} = \frac{-i(H_x+lH_y)C_{39}(1+l)^2}{(m_2+1)(m_2+2)}$

$C_{145} = \frac{-i(H_x+lH_y)C_{40}(1+l)^2}{(m_2+1)(m_2+2)}$

$C_{146} = \frac{C_{25}(1+l)^2}{(m_1+2)(m_1+3)}, \quad C_{147} = \frac{C_{26}(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{148} = \frac{(H_x+lH_y)(lC_9-C_{11})(1+l)^2}{(m_1+2)(m_1+3)}$

$C_{149} = \frac{(H_x+lH_y)(lC_{10}-C_{12})(1+l)^2}{(m_2+2)(m_2+3)}$

$C_{150} = \frac{-iC_{13}(1+l)^2}{(m_1+4)(m_1+5)}, \quad C_{151} = \frac{-iC_{14}(1+l)^2}{(m_2+4)(m_2+5)}$

$$C_{152} = \sigma_1(C_{21} - RiC_{22})(m_1-1)(m_1-2)(1+l)^2 - i\,Ri(1+l^2)\sigma_1(C_{44} - RiC_{22})$$

$$C_{153} = \sigma_1(C_{23} - RiC_{24})(m_2 - 1)(m_2 - 2)(1 + l)^2 - i\,Ri(1 + l^2)\sigma_1(C_{45} - RiC_{24})$$

$$C_{154} = -\sigma_1\big(S\,Rm\,C_{35} - \sigma_1(C_{36} - RiC_{37})\big)m_1(m_1 - 1)(1 + l)^2 - i\,Ri(1 + l^2)\sigma_1\big(S\,Rm\,C_{35} - \sigma_1(C_{36} - RiC_{37})\big)$$

$$C_{155} = -\sigma_1\big(S\,Rm\,C_{38} - \sigma_1(C_{39} - RiC_{40})\big)m_2(m_2 - 1)(1 + l)^2 - i\,Ri(1 + l^2)\sigma_1\big(S\,Rm\,C_{38} - \sigma_1(C_{39} - RiC_{40})\big)$$

$$C_{156} = SRm\,C_{25}\,m_1(m_1 + 1)(1 + l)^2 - i\,Ri(1 + l^2)S\,Rm\,C_{25}$$
$$-iSRm\sigma_1\,m_1(1 + l)(C_{112} - RiC_{113})\big(-lH_x + (2 + l^2)H_y\big)$$
$$-iSRm\sigma_1\,m_1(1 + l)(C_{82} - RiC_{83})\big(-lH_y + (1 + 2l^2)H_x\big)$$

$$C_{157} = SRm\,C_{26}\,m_2(m_2 + 1)(1 + l)^2 - i\,Ri(1 + l^2)S\,Rm\,C_{26}$$
$$-iSRm\sigma_1\,m_2(1 + l)(C_{114} - RiC_{115})\big(-lH_x + (2 + l^2)H_y\big)$$
$$-iSRm\sigma_1\,m_2(1 + l)(C_{84} - RiC_{85})\big(-lH_y + (1 + 2l^2)H_x\big)$$

$$C_{158} = (-iS)\big(-lH_x + (2 + l^2)H_y\big)Rm\big(SRm\,C_{116} - \sigma_1(C_{117} - RiC_{118})\big)(m_1 + 1)(1 + l)$$
$$-iSRm\big(-lH_y + (1 + 2l^2)H_x\big)(SRm\,C_{86} - \sigma_1(C_{87} - RiC_{88}) + C_{101} + RmC_{102})(m_1 + 1)(1 + l)$$

$$C_{158a} = -iSRm^2\big(-lH_x + (2 + l^2)H_y\big)C_{131}(m_1 + 1)(1 + l)$$

$$C_{159} = (-iS)\big(-lH_x + (2 + l^2)H_y\big)Rm\left(SRm\,C_{119} - \sigma_1\left(C_{120} - RiC_{121}\right)\right)(m_2 + 1)(1 + l)$$
$$-iSRm\big(-lH_y + (1 + 2l^2)H_x\big)(SRm\,C_{89} - \sigma_1(C_{90} - RiC_{91}) + C_{108} + RmC_{109})(m_2 + 1)(1 + l)$$

$$C_{159a} = -iSRm^2\big(-lH_x + (2 + l^2)H_y\big)C_{133}(m_2 + 1)(1 + l)$$

$$C_{160} = -(1 + l^2) + SRm\,C_{122}$$
$$-iS\big(-lH_y + (1 + 2l^2)H_x\big)Rm(SRm\,C_{92} + S_HC_{93} + C_{99} + RmC_{100})(m_1 + 2)(1 + l)$$

$$C_{160a} = Rm(3C_{131} + C_{130})$$

$$C_{161} = -(1 + l^2)D + SRm\,C_{123}$$
$$-iS\big(-lH_y + (1 + 2l^2)H_x\big)Rm(SRm\,C_{94} + S_HC_{95} + C_{106} + RmC_{107})(m_2 + 2)(1 + l)$$

$$C_{161a} = Rm(3C_{133} + C_{132})$$

$$C_{162} = -\left(iS\left((2 + l^2)H_y - lH_x\right)Rm(Rm(C_{124} + S_HC_{126}) + iSRm\left((1 + 2l^2)H_x - lH_y\right)C_{96}\right)(m_1 + 3)(1 + l)$$

$$C_{162a} = -\left(iSRm\left((2 + l^2)H_y - lH_x\right)(2Rm(C_{130} + +iC_{125}) + iSRm\left((1 + 2l^2)H_x - lH_y\right)(C_{97} + RmC_{98})\right)(m_1 + 3)(1 + l)$$

$$C_{163} = -\left(iS\left((2 + l^2)H_y - lH_x\right)Rm(Rm(C_{127} + S_HC_{129}) + iSRm\left((1 + 2l^2)H_x - lH_y\right)C_{103}\right)(m_2 + 3)(1 + l)$$

$$C_{163a} = -\left(iSRm\left((2 + l^2)H_y - lH_x\right)(2Rm(C_{132} + +iC_{128}) + iSRm\left((1 + 2l^2)H_x - lH_y\right)(C_{104} + RmC_{105})\right)(m_2 + 3)(1 + l)$$

$$C_{164} = -iSRm\big(-lH_x + (2 + l^2)H_y\big)iRmC_{125}$$
$$-iSRm\big(-lH_y + (1 + 2l^2)H_x\big)(C_{97} + RmC_{98})$$
$$+RmC_{110}(m_1 + 4)(1 + l)$$

$$C_{165} = -iSRm\big(-lH_x + (2 + l^2)H_y\big)iRmC_{126}$$
$$-iSRm\big(-lH_y + (1 + 2l^2)H_x\big)(C_{104} + RmC_{105})$$
$$+RmC_{111}(m_2 + 4)(1 + l)$$

$$p = Ri\frac{N^2}{N_0^2}\frac{(1+l^2)}{(1+l)^2}$$

$$C_{166} = \frac{C_{152}}{(m_1-2)^2-(m_1-2)+p}$$

$$C_{167} = \frac{C_{153}}{(m_2-2)^2-(m_2-2)+p}$$

$$C_{168} = \frac{C_{154}}{(m_1-1)^2-(m_1-1)+p}$$

$$C_{169} = \frac{C_{155}}{(m_2-1)^2-(m_2-1)+p}$$

$$C_{170} = \frac{d_3}{(m_1-1)^2-(m_1-1)+p}$$

$$C_{171} = \frac{d_4}{(m_2-1)^2-(m_2-1)+p}$$

$$C_{172} = \frac{d_5}{(m_1-1)^2-(m_1-1)+p}$$

$$C_{173} = \frac{d_6}{(m_2-1)^2-(m_2-1)+p}$$

$$C_{174} = \frac{C_{156}}{(m_1)^2-(m_1)+p}$$

$$C_{175} = \frac{C_{157}}{(m_2)^2-(m_2)+p}$$

$$C_{176} = \frac{C_{158}}{(m_1+1)^2-(m_1+1)+p}$$

$$C_{176a} = \frac{C_{158a}}{p}\left(1 + \frac{m_1+1}{p} + (m_1 + 1)^2\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{2(m_1+1)^3}{p^2}\right)$$

$$C_{176b} = \frac{C_{158a}}{p}\left(\frac{3}{p} + 6(m_1 + 1)\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{18(m_1+1)^2}{p^2}\right)$$

$$C_{176c} = \frac{C_{158a}}{p}\left(6\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{12(m_1+2)}{p^2}\right)$$

$$C_{177} = \frac{C_{159}}{(m_2+1)^2-(m_2+1)+p}$$

$$C_{177a} = \frac{C_{159a}}{p}\left(1 + \frac{m_2+1}{p} + (m_2 + 1)^2\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{2(m_2+1)^3}{p^2}\right)$$

$$C_{177b} = \frac{C_{159a}}{p}\left(\frac{3}{p} + 6(m_2 + 1)\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{18(m_2+1)^2}{p^2}\right)$$

$$C_{177c} = \frac{C_{159a}}{p}\left(6\left(\frac{1}{p^2} - \frac{1}{p}\right) - \frac{12(m_2+2)}{p^2}\right)$$

$$C_{178} = \frac{C_{160}}{(m_1+2)^2-(m_1+2)+p}$$

$$C_{178a} = \frac{C_{160a}}{p}\left(1 + \frac{m_1+1}{p}\right)$$

$$C_{178b} = \frac{C_{160a}}{p}\left(\frac{2}{p} + 4(m_1 + 1)\left(\frac{1}{p^2} - \frac{1}{p}\right)\right)$$

$$C_{179} = \frac{C_{161}}{(m_2+2)^2-(m_2+2)+p}$$

$$C_{179a} = \frac{C_{161a}}{p}\left(1 + \frac{m_2+1}{p}\right)$$

$$C_{179b} = \frac{C_{161a}}{p}\left(\frac{2}{p} + 4(m_2 + 1)\left(\frac{1}{p^2} - \frac{1}{p}\right)\right)$$

$$C_{180} = \frac{C_{162}}{(m_1+3)^2-(m_1+3)+p}$$

$$C_{180a} = \frac{C_{162a}}{p}\left(1 + \frac{m_1+3}{p}\right)$$

$$C_{181} = \frac{C_{163}}{(m_2+3)^2-(m_2+3)+p}$$

$$C_{181a} = \frac{C_{163a}}{p}\left(1 + \frac{m_2+3}{p}\right)$$

$$C_{182} = \frac{C_{164}}{(m_1+4)^2-(m_1+4)+p}, \quad C_{183} = \frac{C_{165}}{(m_2+4)^2-(m_2+4)+p}$$

$d_1 = -(1+l) - \sigma_0, \qquad d_2 = (1+l) - \sigma_0$

$d_3 = m_1(m_1-1)(1+l)^2, \quad d_4 = i(1+l^2)C_1$

$d_5 = m_2(m_2-1)(1+l)^2, \quad d_6 = i(1+l^2)C_2$

$$C_{184} = C_{166}{d_1}^{m_1-2} + C_{167}{d_1}^{m_2-2} + C_{168}{d_1}^{m_1-1} + C_{169}{d_1}^{m_2-1} + (C_{176} - C_{176a} + C_{176b} - C_{176c}){d_1}^{m_1+1} + (C_{177} - C_{177a} + C_{177b} - C_{177\,c}){d_1}^{m_2+1} + (C_{178} + C_{178a} - C_{178b}){d_1}^{m_1+2} + (C_{179} + C_{179a} - C_{179b}){d_1}^{m_2+2} + (C_{180} - C_{180a}){d_1}^{m_1+3} + (C_{181} - C_{181a}){d_1}^{m_2+3} + C_{182}{d_1}^{m_1+4} + C_{183}{d_1}^{m_2+4}$$

$$C_{185} = C_{166}{d_2}^{m_1-2} + C_{167}{d_2}^{m_2-2} + C_{168}{d_2}^{m_1-1} + C_{169}{d_2}^{m_2-1} + (C_{176} + C_{176a} + C_{176b} + C_{176c}){d_2}^{m_1+1} + (C_{177} + C_{177a} + C_{177b} + C_{177\,c}){d_2}^{m_2+1} + (C_{178} + C_{178a} + C_{178b}){d_2}^{m_1+2} + (C_{179} + C_{179a} + C_{179b}){d_2}^{m_2+2} + (C_{180} + C_{180a}){d_2}^{m_1+3} + (C_{181} + C_{181a})\,{d_2}^{m_2+3} + C_{182}{d_2}^{m_1+4} + C_{183}{d_2}^{m_2+4}$$

$$C_{186} = d_1^{m_1-1}d_2^{m_1} - d_2^{m_1-1}d_1^{m_1} + d_2^{m_1-1} + \frac{d_2^{m_1-1}d_1^{m_1} - d_1^{m_1-1}d_2^{m_1}}{d_1^{m_2}d_2^{m_1} - d_1^{m_1}d_2^{m_2}}$$

$$C_{187} = d_1^{m_2-1}d_2^{m_1} - d_2^{m_2-1}d_1^{m_1} + d_2^{m_2-1} + \frac{d_2^{m_2-1}d_1^{m_1} - d_1^{m_2-1}d_2^{m_1}}{d_1^{m_2}d_2^{m_1} - d_1^{m_1}d_2^{m_2}}$$

$$C_{188} = C_{184}{d_1}^{m_1} - C_{185}{d_1}^{m_2} + \frac{C_{185}d_1^{m_1} - C_{184}d_2^{m_1}}{d_1^{m_2}d_2^{m_1} - d_1^{m_1}d_2^{m_2}} + C_{185}$$

$$C_{189} = C_{154}C_{186} + C_{155}C_{187} + d_{11}$$

$$C_{190} = d_3C_{186} + d_5C_{187}, \quad C_{191} = d_4C_{186} + d_6C_{187}$$