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MHD FLOWS DUE TO NON-COAXIAL ROTATIONS OF POROUS DISK AND A VISCOUS FLUID AT INFINITY: GRAPHICAL SOLUTIONS USING MATLAB

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ABSTRACT. Fluids play a vital role in many aspects of our daily life. We drink water, breath air, fluids runs through our bodies and it controls the weather. The study of motion of fluids is a complex phenomena. The equations which govern the flows of Newtonian fluids are Navier-Stokes equations. In this paper, the flows which are due to non – coaxial rotations of porous disk and a fluid at infinity are considered. Analytical solution for the velocity field using Laplace transform is derived. MATLAB coding is written to get the graphical solutions. The results are compared with the existing results. MATLAB software provides accurate results depending on the solution we obtained.

1. INTRODUCTION

Disk shaped bodies are often encountered in many engineering applications. It has always been interesting to carry out the flows which are rotating. Examples of such flows are weather patterns, atmospheric fronts and ocean currents. In particular, the MHD fluid flow problem of a rotating disk finds special places in several science and engineering applications, for instance, in turbo machinery, in cosmical fluid dynamics, in gaseous and nuclear reactors, in MHD power generators, flow meters, pumps and so on.

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The flow due to an infinite rotating disk is one of the classical problems which were first introduced by von Karman (1921). Cochran (1934) used the von Karman transformations and obtained asymptotic solutions for the steady hydrodynamic problem. The flow of a conducting fluid above a rotating disk in the presence of an external uniform magnetic field was studied by Mistikawy and Attia (1990, 1991). Erdogan (1995) has studied the unsteady viscous flow between eccentric rotating disks. Hayat et al. (2001) examined Erdogan's work (1997) for a porous disk in the presence of a magnetic field. Ersoy, H.V (2010) studied MHD flow of a second order /grade fluid due to non-coaxial rotation of a disk and the fluid at infinity. Islam S, Harron T, Elahi M, Ullah M, Siddiqui, A.M (2011) worked in steady and unsteady exact inverse solutions for the flow of a viscous fluid ([1-13]).

In this paper, the exact analytical solution (due to arbitrary periodic oscillation) describing the flow at large and small times after the start is obtained. Thus the MHD effect and arbitrary nature of oscillation, the graphical representation of the flow characteristics is the special feature of this chapter.

2. Mathematical formulation

The flow of an incompressible electrically conducting fluid is considered. The fluid is electrically conducting in the presence of a magnetic field. The disk $(z = 0)$ is assumed to be a porous disk. The fluid fills the space $z = 0$ and is in contact with the disk. The axes of rotation of both the disk and the fluid are assumed to be in the plane $x = 0$. The distances between axes are being considered as l_1. Initially the disk and the fluid are rotating about z^1- axis with constant angular velocity Ω. At time $t = 0$, the disk and the fluid start rotate at z and z^1 axes respectively with constant angular velocity Ω. The disk also oscillates its own plane with frequency n, at time $t > 0$(Figure. 1). Under the above assumptions, the equations governing the unsteady motion of the conducting viscous incompressible fluid are those pertaining to the conservation of momentum and of mass which are

$$\frac{dV}{dt} = -\frac{1}{\rho}\nabla\rho_1 + \nu\nabla^2 V + \frac{1}{\rho}(JXB) \tag{2.1}$$

$$divV = 0 \tag{2.2}$$

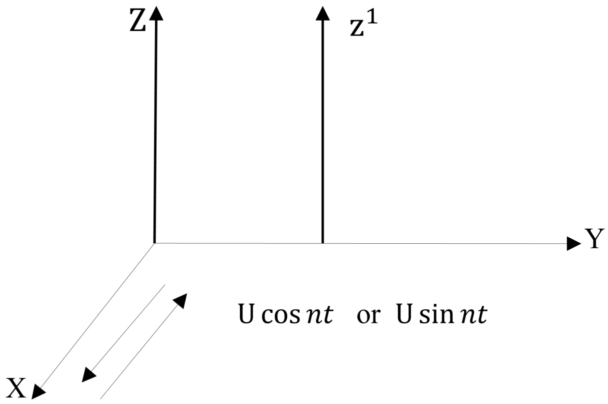

FIGURE 1. Flow Geometry

The equations governing the flow consists of the Maxwell equations and a generalized Ohm's law which after neglecting the displacement currents are

$$divB = 0$$

$$curlB = \mu_m J$$

$$curlE = \frac{(-\partial B)}{\partial t}$$

$$J = \sigma(E + VXB)$$

where $V = (u, v, w)$ is the fluid velocity with $u, v,$ and w as the velocity components in the x, y and z – directions respectively, ρ is the fluid density, ρ_1 is the scalar pressure, $\frac{d}{dt}$ is the material derivative, v is the kinematic viscosity, J is the current density, $B = B_0t + b$ is the total magnetic field which is the sum of applied magnetic field B_0 and induced magnetic field b, μ_m is the magnetic permeability and E is the electric field and σ is the electrical conductivity of the fluid.

For the derivation of Lorentz force in equation (2.1) it is assumed that the magnetic field is normal to the velocity field, the electric field is negligible and the induced magnetic field is small compared with the applied magnetic field. The last assumption is valid when the magnetic Reynolds number is very small and there is no displacement current.

In view of the above assumptions the electromagnetic body force involved in equation (2.1) takes the form

$$\begin{aligned} \frac{1}{\rho}(JXB) &= \frac{\sigma}{\rho}[(VXB)XB] \\ &= \frac{\sigma}{\rho}[B_0(V.B_0) - V(B_0.B_0)] \\ &= -\frac{\sigma B_0^2}{\rho} \end{aligned} \tag{2.3}$$

The relevant boundary and initial conditions are taken in the form

$$\begin{aligned} u &= -\Omega y + Uh(t), \\ v &= \Omega x \quad \text{at} \quad z = 0 \quad \text{for} \quad t > 0, \\ u &= -\Omega(y-1), \\ v &= \Omega x \quad \text{as} \quad z \to \infty \quad \text{for all} \quad t, \\ u &= -\Omega(y-1), \\ v &= x \quad \text{at} \quad t = 0 \quad \text{for} \quad z > 0. \end{aligned} \tag{2.4}$$

where U is the velocity and $h(t)$ is the general periodic oscillation of a disk.

The Fourier series representation of $h(t)$ is given by

$$h(t) = \sum_{k=-\infty}^{\infty} a_k e^{iknt}$$

$$\text{where} \quad a_k = \frac{1}{T_0}\int h(t)e^{-iknt}dt$$

where $n = \frac{2\pi}{T_0}$ is the non zero oscillating frequency.

The coefficients $\{a_k\}$ are Fourier series coefficients or the spectral coefficients of $h(t)$. The boundary and initial conditions show that the motion is a summation of a helical and translatory motion with the velocity profile being

$$u = -\Omega y + f(z,t), v = \Omega x + g(z,t) \tag{2.5}$$

Using equation (2.2), the uniform porous disk is of the form

$$w = -w_0 \tag{2.6}$$

where ($w_0 > 0$ is the suction velocity and $w_0 < 0$ is the corresponding blowing velocity) From the above equation (2.5),

$$f(z,t) = u + \Omega y \quad \& \quad g(z,t) = v - \Omega x.$$

Using equations (2.1), (2.3), (2.5) and (2.6), an equation can be written as

$$v\frac{\partial^3 F}{\partial z^3}+w_0\frac{\partial^2 F}{\partial z^2}-\frac{\partial^2 F}{\partial t\partial z}-\left(i\Omega+\frac{\sigma}{\rho}B_0^2\right)\frac{\partial F}{\partial z}=0 \tag{2.7}$$

$$\text{in which}\quad F=f+ig. \tag{2.8}$$

Using the above equation, the boundary and initial conditions are

$$\begin{aligned} F(0,t) &= f(0,t)+ig(0,t)\\ &= (u+\Omega y)+i(v-\Omega x)\\ &= -\Omega y+\Omega y+Uh(t)+i(\Omega x-\Omega x)\\ &= Uh(t)\quad \text{for all}\quad t>0, \end{aligned} \tag{2.9}$$

$$\begin{aligned} F(\infty,t) &= f(\infty,t)+ig(\infty,t)=(u+\Omega y)+i(v-\Omega x)\\ &= -\Omega(y-1)+\Omega y+i(\Omega x-\Omega x)\\ &= \Omega 1 \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} F(z,0) &= f(z,0)+ig(z,0)\\ &= (u+\Omega y)+i(v-\Omega x)\\ &= -\Omega(y-1)+\Omega y+i(\Omega x-\Omega x)\\ &= \Omega 1. \end{aligned}$$

In order to find the solution of equation (2.7) subject to equations (2.9) and (2.10) the Laplace transform pair can be defined as

$$\overline{H}(z,s)=\int_0^\infty F(z,t)e^{-st}dt$$

$$F(z,t)=\frac{1}{2\pi i}\int_{\overline{\lambda}-i\infty}^{\lambda+i\infty}\overline{H}(z,s)e^{st}dt \tag{2.11}$$

$$\text{Taking}\quad M=i\Omega+\frac{\sigma}{\rho}B_0^2 \tag{2.12}$$

and using the Laplace parameters, the equations (2.7) becomes

$$\begin{aligned} v\frac{\partial^3 F}{\partial z^3}+w_0\frac{\partial^2 F}{\partial z^2}-\frac{\partial^2 F}{\partial t\partial z}-M\frac{\partial F}{\partial z} &= 0\\ \frac{\partial^3 F}{\partial z^3}+\frac{w_0}{v}\frac{\partial^2 F}{\partial z^2}-\frac{1}{v}\frac{\partial^2 F}{\partial t\partial z}-\frac{M}{v}\frac{\partial F}{\partial z} &= 0\\ \frac{\partial^3}{\partial z^3}+\frac{w_0}{v}\frac{\partial^2}{\partial z^2}-\frac{\partial}{\partial z}\left(\frac{M}{v}+\frac{1}{v}\frac{\partial}{\partial t}\right)F &= 0. \end{aligned}$$

Now, let $\frac{\partial}{\partial t}=s$. Then

$$\begin{aligned} \left[\frac{d^3}{dz^3}+\frac{w_0}{v}\frac{d^2}{dz^2}-\frac{d}{dz}\left(\frac{M+s}{v}\right)\right]F &= 0\\ \Rightarrow\left[\frac{d^3}{dz^3}+\frac{w_0}{v}\frac{d^2}{dz^2}-\frac{d}{dz}\left(\frac{M+s}{v}\right)\right]\overline{H}(z,s) &= 0 \end{aligned} \tag{2.13}$$

Also,

$$\begin{aligned} \overline{H}(0,s) &= \int_0^\infty F(0,t)e^{-st}dt=\int_0^\infty Uh(t)e^{-st}dt\\ &= \int_0^\infty \sum_{k=\infty}^{\infty} a_k e^{iknt}e^{-st}dt = U\sum_{k=-\infty}^{\infty} a_k\int_0^\infty e^{[ikn-s]t}dt = U\sum_{k=-\infty}^{\infty} a_k\int_0^\infty e^{[ikn-s]t}dt\\ &= U\sum_{k=-\infty}^{\infty} a_k\int_0^\infty e^{-[s-ikn]t}dt = U\sum_{k=-\infty}^{\infty} a_k\left[\frac{-e^{-[s-ikn]t}}{s-ikn}\right]_0^\infty\\ &= U\sum_{k=-\infty}^{\infty}\frac{a_k}{s-ikn}\left[-e^{-[s-ikn]t}\right]_0^\infty = U\sum_{k=-\infty}^{\infty}.\frac{a_k}{s-ikn}\left[-e^{-\infty}-[-e^0]\right]. \end{aligned}$$

Therefore

$$\overline{H}(0,s)=U\sum_{k=-\infty}^{\infty}\frac{a_k}{s-ikn}=\int_0^\infty F(z,t)e^{-st}dt$$

$$\overline{H}(z,s)=\int_0^\infty \Omega 1 e^{-st}dt=\Omega 1\int_0^\infty e^{-st}dt$$

and

$$\overline{H}(z,s)=\frac{\Omega 1}{s}\quad\text{as}\quad z\to\infty. \tag{2.14}$$

The auxiliary equation of (2.11) is

$$m^3 + \frac{w_0}{v} m^2 - \left(\frac{M+s}{v} \right) m = 0$$
$$m \left(m^2 + \frac{w_0}{v} m - \left(\frac{M+s}{v} \right) \right) = 0$$

$$\Rightarrow m = \frac{-w_0}{2v} \pm \sqrt{\left(\frac{w_0}{2v} \right)^2 + \left(\frac{M+s}{v} \right)}.$$

Therefore the roots are

$$m_1 = 0, m_2 = \frac{w_0}{2v} + \sqrt{\left(\frac{w_0}{2v} \right)^2 + \left(\frac{M+s}{v} \right)}$$

$$m_3 = \frac{w_0}{2v} - \sqrt{\left(\frac{w_0}{2v} \right)^2 + \left(\frac{M+s}{v} \right)},$$

and the general solution of the ordinary differential equation (2.11) is

$$\overline{H}(z,s) = C_1 e^{0.z} + C_2 e^{-\left[\frac{w_0}{2v} + \sqrt{\left(\frac{w_0}{2v} \right)^2 + \left(\frac{M+s}{v} \right)} \right]} + C_3 e^{-\left[\frac{w_0}{2v} - \sqrt{\left(\frac{w_0}{2v} \right)^2 + \left(\frac{M+s}{v} \right)} \right] z},$$

where C_1, C_2 and C_3 arbitrary constants.

Using equation (2.12) and (2.13) and taking $z = 0$ in (2.14),

$$\overline{H}(z,s) = C_1 + C_2 + C_3.$$

Substituting $z = \infty$ in (2.14) follows $C_1 = \frac{\Omega 1}{s}$, $\overline{H}(z,s) = C_1$. Since $w_0 < 0, C_3 = 0$, $\overline{H}(z,s) = C_1 + C_2$ we have

$$U \sum_{k=-\infty}^{\infty} \frac{a_k}{s - ik_n} = \frac{\Omega 1}{s} + C_2,$$

$$C_2 = U \sum_{k=-\infty}^{\infty} \frac{a_k}{s - ik_n} - \frac{\Omega 1}{s}$$

and hence

$$C_1 = \frac{\Omega 1}{s}, \quad C_2 = U \sum_{k=-\infty}^{\infty} \frac{a_k}{s - ik_n} - \frac{\Omega 1}{s}, \quad C_3 = 0.$$

Now substituting C_1, C_2 and C_3 in (2.14), and taking the Laplace transform for the above result, the velocity field is of the form

$$\overline{H}(z,s) = \frac{\Omega l}{s}\left\{\begin{array}{l} 1 - e^{-\left\{\frac{w_0}{2v}+\sqrt{\left(\frac{w_0}{2v}\right)^2+\left(\frac{M+s}{v}\right)}\right\}z} + \\ U\sum_{k=-\infty}^{\infty}\frac{a_k}{s-ik_n}\left(1-e^{-\left\{\frac{w_0}{2v}\sqrt{\left(\frac{w_0}{2v}\right)^2+\left(\frac{M+s}{v}\right)}\right\}z}\right)\end{array}\right\}$$

(2.15)

$$F(z,t) = \Omega 1\left[1-e^{\frac{\frac{-w_0}{2v}z}{2}}\left\{e^{-z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\left(\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}\right)t}\right]+\right.\right.$$
$$\left. e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\left(\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}\right)t}\right]\right\}+$$
$$\left. U\sum_{k=-\infty}^{\infty}a_k e^{\frac{-w_0 z}{2v}}L^{-1}\left[e^{-z\sqrt{\frac{\left(\frac{w_0}{2v}\right)^2+\left(\frac{M+s}{v}\right)}{s-ikn}}}\right]\right],$$

where L^{-1} indicates the inverse Laplace transform and it is known that

$$L^{-1}\left(\frac{e^{-z\sqrt{\left(\frac{w_0}{2v}\right)^2+\frac{s}{v}}}}{s-ikn}\right) = \frac{z}{2\sqrt{\pi vt^3}}e^{-\left\{\left(\frac{w_0}{2v}\right)^2+\frac{s}{v}\right\}vt-\frac{s^2}{4vt}}$$

(2.16)
$$L^{-1}\left[\frac{1}{s-ikn}\right] = e^{iknt}.$$

Using convolution theorem of Laplace transform,

$$L^{-1}\left(\frac{e^{-z\sqrt{\left(\frac{w_0}{2v}\right)^2+\frac{s}{v}}}}{s-ikn}\right) = e^{iknt} * \frac{z}{2\sqrt{\pi vt^3}}e^{-\left\{\left(\frac{w_0}{2v}\right)^2+\frac{s}{v}\right\}vt-\frac{s^2}{4vt}}.$$

Here, "*" in the above equation is indicated for the convolution and

$$L^{-1}\left(\frac{e^{-z\sqrt{\left(\frac{w_0}{2v}\right)^2+\frac{s}{v}}}}{s-ikn}\right)$$

(2.17)
$$= \frac{e^{iknt}}{2}\left(\begin{array}{l} e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}\right]vt+ \\ e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}\right]vt\end{array}\right).$$

Applying (2.17) in (2.16), and using (2.8) then

$$\begin{aligned} &\frac{f}{\Omega 1}+\frac{g}{\Omega 1} \\ &=1-\frac{e^{\frac{-w_0}{2v}z}}{2}\left(e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}\right]t\right. \\ &\left.+e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}+\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i\Omega}{v}}\right]t\right) \\ &+\frac{Ue^{\frac{-w_0 t}{2v}}}{2\Omega 1}\sum_{k=-\infty}^{-\infty}a_k e^{ikw_0 t}X \\ &\cdot\left(e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i(\Omega+kn)t}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+i\left(\frac{\Omega+kn}{v}\right)}\right]t\right. \\ &\left.+e^{z\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+\frac{i(\Omega+kn)t}{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho}+i\left(\frac{\Omega+kn}{v}\right)}\right]t\right) \end{aligned} \tag{2.18}$$

where $erfc(\overline{x})$ is the complementary error function and is defined by

$$erfc(\overline{x})=1-erf(\overline{x})=\int_z^\infty e^{-\tau_1^2}d\tau_1.$$

Clearly the real and imaginary parts of equation (2.17) is $\frac{f}{\Omega 1}$ and $\frac{g}{\Omega 1}$ respectively. Substituting

$$\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho v}+\frac{i\Omega}{v}}=x_1+iy_1, \tag{2.19}$$

$$\sqrt{\frac{w_0^2}{4v}+\frac{\sigma B_0^2}{\rho v}+i\left(\frac{\Omega+kn}{v}\right)}=r_k+i\delta_k, \tag{2.20}$$

the equation (2.18) becomes

$$\begin{aligned} &\frac{f}{\Omega 1}+\frac{g}{\Omega 1}=H^*+\frac{Ue^{-\frac{w_0}{2v}}}{2\Omega i}\sum_{k=-\infty}^{\infty}a_k e^{iknt} \\ &\left(e^{\frac{-z(r_k+i\delta_k)}{\sqrt{v}}}erfc\left[\frac{z}{2\sqrt{vt}}-(r_k+i\delta_k)\sqrt{t}\right]+e^{\frac{z(r_k+i\delta_k)}{\sqrt{v}}}erfc\left[\frac{z}{2\sqrt{vt}}+(r_k+i\delta_k)\sqrt{t}\right]\right), \end{aligned} \tag{2.21}$$

in which

$$H^* = 1 - \frac{e^{-\frac{w_0 z}{2v}}}{2}$$

$$\left(e^{\frac{-z}{\sqrt{v}}(x_1+iy_1)} erfc\left[\frac{z}{2\sqrt{vt}} - (x_1+iy_1)\sqrt{t}\right] + e^{\frac{z}{\sqrt{v}}(x_1+iy_1)} erfc\left[\frac{z}{2\sqrt{vt}} + (x_1+iy_1)\sqrt{t}\right]\right).$$

Using equations (2.19) and (2.20),

$$x_1 = \left(\frac{1}{2}\left\{\sqrt{\left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)^2 + \Omega^2} + \left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)\right\}\right)^{\frac{1}{2}}$$

$$y_1 = \left(\frac{1}{2}\left\{\sqrt{\left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)^2 + \Omega^2} - \left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)\right\}\right)^{\frac{1}{2}}$$

$$r_k = \left(\frac{1}{2}\left\{\sqrt{\left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)^2 + (\Omega+n_k)^2} + \left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)\right\}\right)^{\frac{1}{2}}$$

$$\delta_k = \left(\frac{1}{2}\left\{\sqrt{\left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)^2 + (\Omega+n_k)^2} - \left(\frac{w_0^2}{4v} + \frac{\sigma B_0^2}{\rho}\right)\right\}\right)^{\frac{1}{2}}.$$

The solution obtained in the equation (2.21) is the complete analytic solution for the velocity field due to an arbitrary periodic oscillation in its own plane.

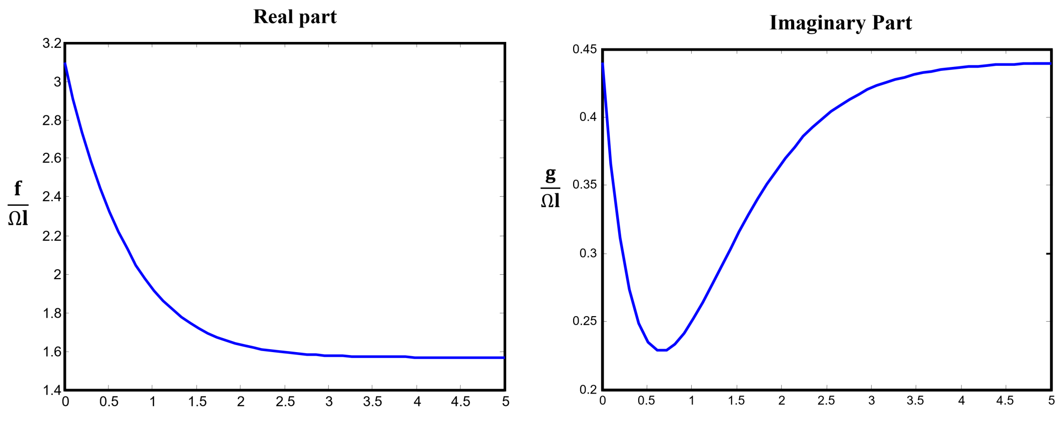

FIGURE 2. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

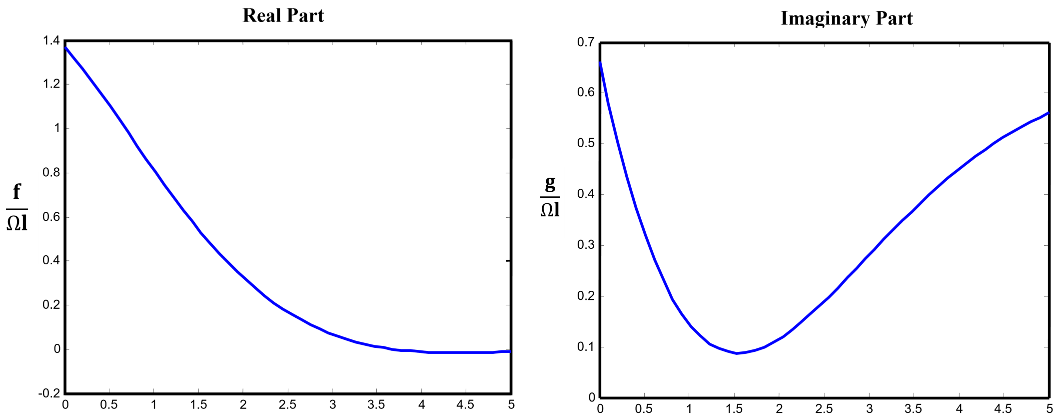

FIGURE 3. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

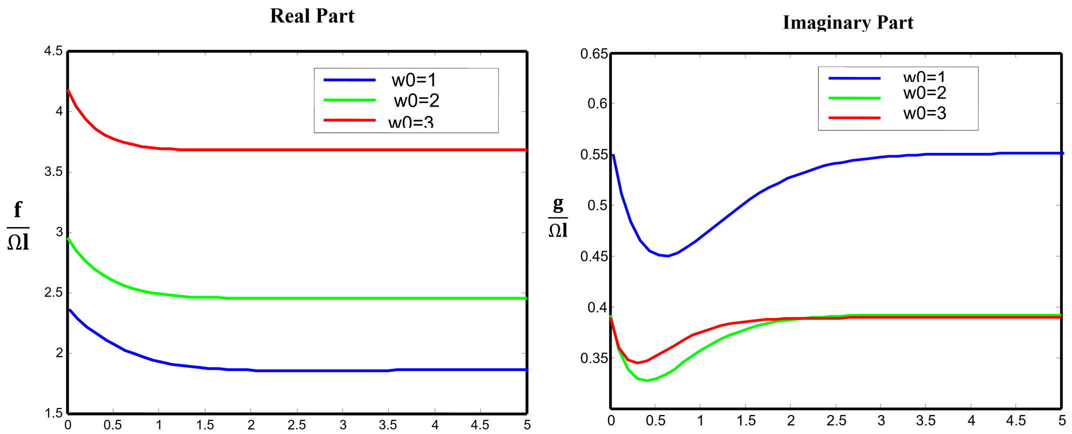

FIGURE 4. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

3. Results and Discussions

In this section figures 3.2 to 3.17 are drawn for the various parameters on the velocity profiles. Figures 3.2 to 3.5 show that the transverse waves occur in both the suction and blowing cases for $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$. Figures 3.6 to 3.9 show that the boundary layer thickness decreases with an increase of the suction / blowing parameter for both $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$. In hydromagnetic situation, figures 3.10 to 3.15

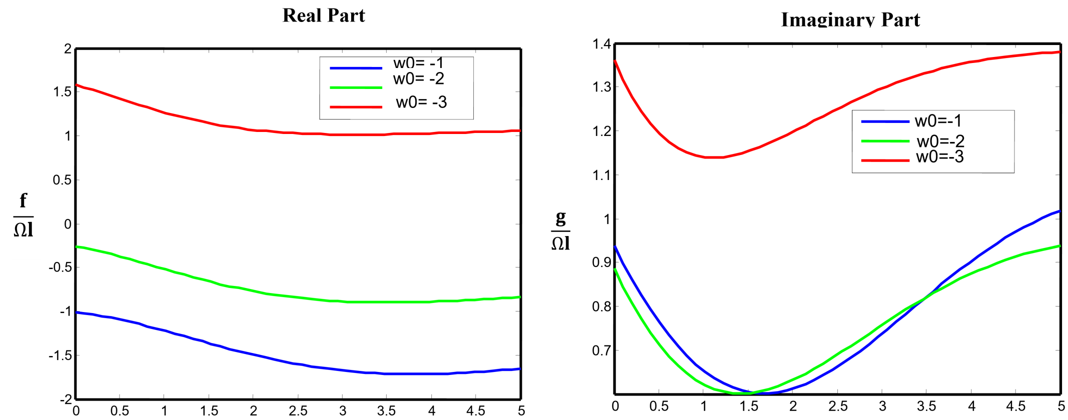

FIGURE 5. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

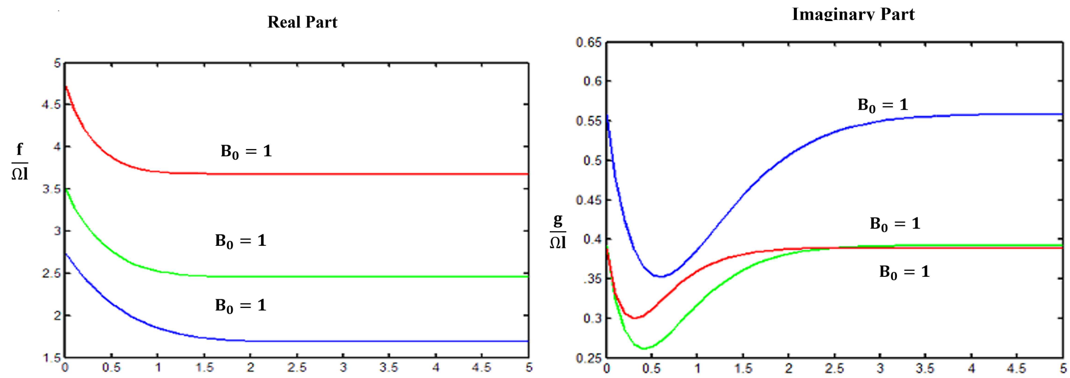

FIGURE 6. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

are drawn for various disk oscillations and it is noted that $\frac{f}{\Omega l}$ and $\frac{g}{\Omega l}$ decreases with the increase of magnetic field. Figures 3.16 to 3.17 are drawn for various disk oscillations to describe the steady state after the large time.

CONCLUSION

(1) The solutions for suction and blowing cases are derived for all values of frequencies including at resonant frequency.

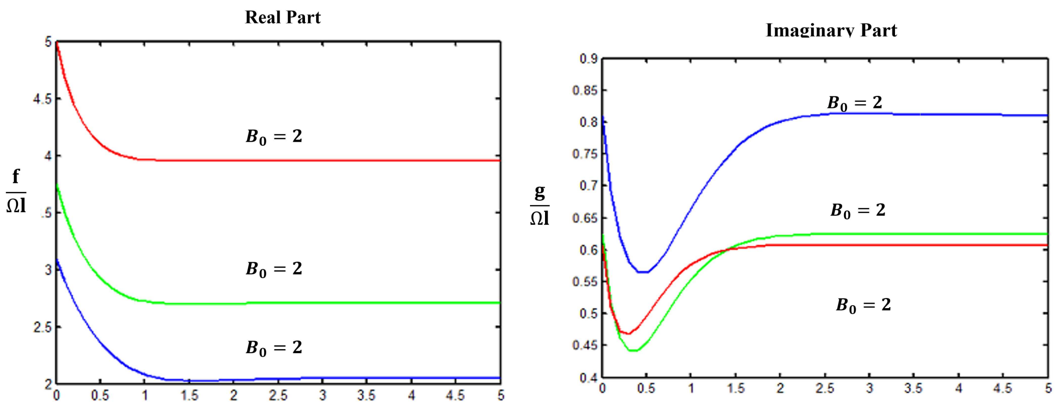

FIGURE 7. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

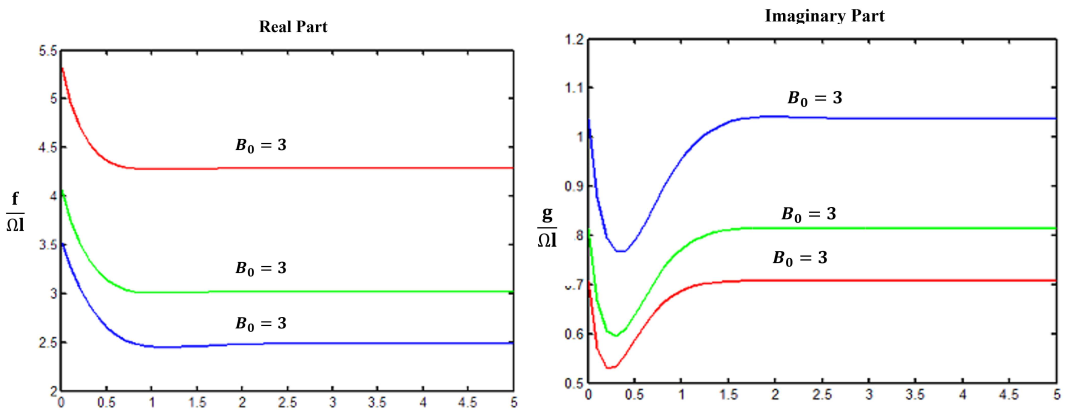

FIGURE 8. The effect of magnetic field on $\frac{f}{\Omega l}$ and $\frac{f}{\Omega l}$ in the presence of suction at $\alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

(2) The effects of the magnetic parameter and suction/blowing parameters on the velocity are seen, from where it is observed that an increase in the magnetic parameter leads to a decrease in the boundary layer thickness.
(3) The effect of suction parameter on the velocity is similar to that of magnetic parameter.
(4) It is further noted that diffusive waves occur in the hydromagnetic system.

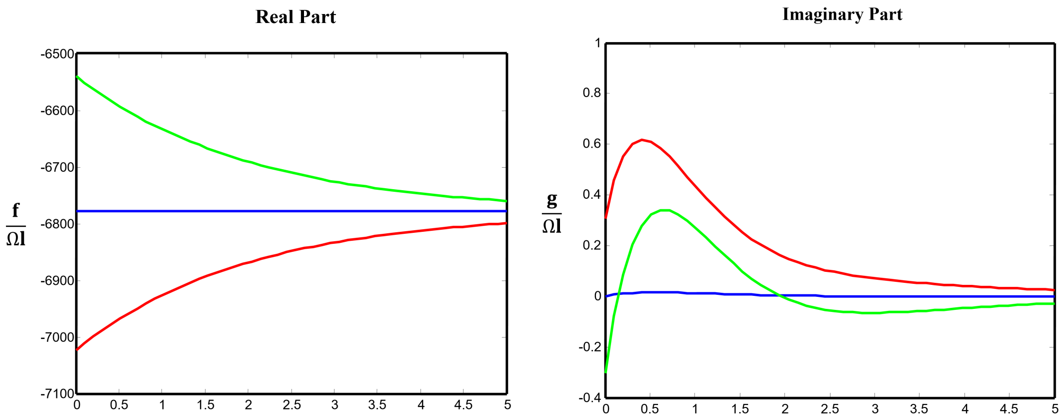

FIGURE 9. The effect of magnetic field when $w_0 = [1\ 2\ 3\ 4\ 5];\ \alpha = 1;\ t = 1;\ U = 0.08;\ n = 1;\ B_0 = 0;\ \rho = 2.5;\ \Omega = 1$

(5) It is confirmed that for large times the starting solution tends to the steady state solution.

REFERENCES

[1] S. ISLAM, T. HARRON, M. ELAHI, M. ULLAH, A.M. SIDDIQUI: *Steady and unsteady exact inverse solutions for the flow of a viscous fluid*, Int. J. Phys. Sci., **6** (2011), 6234–6240.

[2] H.V. ERSOY: *MHD flow of a second order /grade fluid due to non-coaxial rotation of a disk and the fluid at infinity*, Math. Comp. Appl., **15** (2010), 354–363.

[3] H.A. ATTIA: *Steady flow over a rotating disk in porous medium with heat transfer*, Nonlinear analysis: modelling and Control, **14** (2009), 21–26.

[4] H.V. Ersoy: *Unsteady flow of a Maxwell fluid induced by non-coaxial rotation of a disk and the fluid at infinity*, Int. J. Appl. Mech. Eng., **10** (2005), 159–165.

[5] T. HAYAT, S. ASGHAT, A.M. SIDDIQUI, T. HAROON: *Unsteady MHD flow due to non-coaxial rotations of a porous disks and a fluid at infinity*, Acta Mech., **151** (2001), 127–134.

[6] M.E. ERDOGAN: *Unsteady flow between any two eccentric rotating disks executing non – trosional oscillations*, Int. J. Non – Linear Mech., **35** (2000), 691–699.

[7] K.R. RAJAGOPAL: *On the flow of a simple fluid in a orthogonal rheometer*, Arch. Rat. Mech. Anal., **79** (1982), 39–47.

[8] M. Balaram, B. Luthra: *A Numerical Study of Rotationally Symmetric Flow of Second-Order Fluid*, J. Appl. Mech. Sep., **40**(3) (1973), 685-687.

[9] R. BERKER: *A new solution of the Navier Stokes equation for the motion of a fluid contained between two parallel plates rotting about the same axis.*, Arch. Mech. Stos. **31**(2) (1979), 265–280.

[10] R. BERKER: *An exact solution of the Navier-Stokes equation: The vortex with curvilinear axis*, Int. J. Engng. Sci. **20**(2) (1982), 217–230.

[11] R. BERKER: *Integration des equations du movement d'un fluide visquex incompressible*, Handbook of Fluid Dynamics VIII/3, 87. Springer, Berline, 1963.

[12] U.T. BODEWADT: *Die Drehstromunguber Festem Grund*, Z. Angew. Math. Mech., **20** (1940), 241—253.

[13] W.G. COCHRAN: *The flow due to a rotating disk*, P. Camb. Philos. Soc., **30** (1934), 365–375.

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