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SUPRA CO BT URYSOHN SPACE IN SUPRA TOPOLOGICAL SPACES

K. KRISHNAVENI

ABSTRACT. In this article, the author interpolate prolific notion of co bT^{μ} graph and co bT^{μ} urysohn space by utilizing the concept of bT$^{\mu}$ set connected functions in supra topological spaces.

1. INTRODUCTION

Nowadays numerous imminent topologists are focused in compact space, separation axioms, connectedness, graph etc., in their research. In the year 1983, Reilly and Vamanamurthy ([9]) introduced a new concept of Clopen Relation in topological spaces. Dontchev, Ganster and Reilly ([1]) came out with a new function called regular set-connected in the year 1999. The idea behind the set-connected regular function is extended to Clopen Sets by Ekici in the year 2005 [2]. It was Mashhour et. al ([7]) who first came up with supra topology concept by studying the S*-continuous as well as the S-continuous maps. A further introduction to this concept was given by introducing S-T0, S-T1,S-T2,S-T2 spaces by discussing its relation to the T0,T1,T2,T2 topological spaces. Takashi Noiri and Sayed ([10]) in their research focused on a method of introducing supra b-continuity and b-open sets in the supra topology space in the year 2010 and also elaborated the supra b-open as well as the continuous maps as well as its interconnectivity. Ganes M.Pandya, C.Janaki and I. Arockiarani ([3]) introduced another functions as a set of connected classes named as the

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π-set connected and investigated a relations among the π-set connected function, covering properties and separation axioms. In 1999, Ramprasad Paul and P. Bhattacharyya ([8]) studied some basic properties of pre-urysohn space and discussed some properties of functions with p-θ-closed graph and pre-θ-closed graph and their relationships with the pre urysohnspace.

The author ([6]) introduced bT^{μ}-set connected functions in topological supra spaces. They also discussed the separation axioms using bT^{μ}-set connected functions in supra topological spaces. The author defined the function class under one condition that for each supra clopen there is an inverse image present (like the supra closed and open set) in the codomain is bT^{μ}-clopen(that is, bT^{μ}-open and closed)in the domain.The author investigated the fundamental properties of bT^{μ}- set connected functions.

The idea of this paper rises with a latest idea behind co-bT^{μ} graph and co-bT^{μ} urysohn space by utilizing the concept of bT^{μ}-set connected functions in supra topological spaces.

In the present research the functions (Y, σ) and (X, τ) represent supra topological space in which there is no axioms separations were assumed until they are mentioned. For X let the subset be A. The supra interior as well as the supra closure of this set by int^{μ}(A) and cl^{μ}(A) are also denoted.

2. PRELIMINARIES

Definition 2.1. *[7, 10] A μ of X subfamily is defined as a supra topology on X, if*

- (i) *X, $\phi \epsilon \mu$*
- (ii) *if $A_i \epsilon \mu$ for all iϵ J then $\cup A_i \epsilon \mu$.*

The supra topological space is defined by the pair (X, μ). In this supra open sets are called by μ and supra closed set is defined as the complimentary set of the supra open.

Definition 2.2. *[7, 10]*

- (i) *A set A supra interior is defined using the term int^{μ}(A) and $int^{\mu}(A) = \cup${B:B is defined as supra open set where A$\supseteq$ B }.*
- (ii) *A set A supra closure is defined using the term cl^{μ}(A) and $cl^{\mu}(A) = \cap${B:B is defined as supra closed set where $A \subseteq B$}.*

Definition 2.3. *[4] For a set of (X, τ) as supra subset A is defined as, bT^{μ}-closed set for bcl^{μ}(A) $\subset$ U when U is T^{μ}- open and A $\subset$ U in (X,τ).*

Definition 2.4. *[4] Assume (Y, σ) and (X, τ) be two supra topological spaces. A relation f:(X, τ) $\rightarrow$(Y, σ) is defined as bT^{μ} - continuous if in (X, τ), f^{-1}(V) is bT^{μ} - closed for each V of (Y, σ) supra closed set.*

Definition 2.5. *[5] A relation f:(X, τ) $\rightarrow$(Y, σ) is known as bT^{μ} -open map (bT^{μ}-closed) in case the image f(A) is bT^{μ} -open(bT^{μ} -closed) in (Y, σ) for every supra open (supra closed) set A in (X, τ).*

Definition 2.6. *[6] A relation f:(X,τ) $\rightarrow$(Y, σ) is known as bT^{μ}- set connected function in case f^{-1}(V) in (X,τ) is bT^{μ} clopen set for each V in (Y, σ) is supra clopen set.*

Definition 2.7. *[6] A (X,τ) supra topological space is defined as co bT^{μ} T_1 in case each distinct points pair X:x,y, bT^{μ} clopen sets exist for U having x and V associated to y in a way that x$\in$U, y$\notin$U and x$\notin$V, y$\in$V.*

Definition 2.8. *[6] A (X,τ)supra topological space is known as co bT^{μ} T_2 or co bT^{μ} Hausdorff space in case each two distinct points of (X,τ) could be separated by disarticulate bT^{μ} clopen sets.*

3. bT^{μ} CLOPEN MAP

Definition 3.1. *A function f:(X,τ) $\rightarrow$(Y, σ) can be defined as bT^{μ} clopen map if f(A) image is supra clopen in (Y, σ) for each set A of bT^{μ} clopen in (X,τ).*

Example 1. *Assume X = Y = $\{a, b, c\}$ with τ = $\{X, \phi, \{a\}, \{b\}, \{a, c\}, \{a, b\}, \{b, c\}\}$ and σ = $\{Y, \phi, \{a\}, \{b, c\}, \{a, c\}\}$. A relation f:(X,$\tau$) $\rightarrow$ (Y,σ) is defined as f(a) = a, f(b) = c, f(c) = b. The supra clopen set of (Y,σ) are $\{Y, \phi, \{a\}, \{b\}, \{b, c\}, \{a, c\}\}$ and the bT^{μ} clopen set of (X,τ) are $\{X, \phi, \{a\}, \{c\}, \{a, b\}, \{b, c\}\}$. Here f bT^{μ} clopen map.*

4. CO bT^{μ} GRAPH

Definition 4.1. *If function f:(X,τ) $\rightarrow$(Y, σ) are related, then G^{μ}(f) = $\{(x, y) : x, y \in X\}$ subset of a (X×Y, $\tau \times \sigma$) product space is defined as supra graph of f.*

Definition 4.2. *A G(f) graph of a relation f:$(X,\tau) \rightarrow (Y, \sigma)$ can be defined as co bT^μ graph, if for every (x,y)$\in$ $(X\times Y)\cap G^\mu(f)$, bT^μ clopen set U exist for X as well as V be supra clopen of Y in a way that (x,y)$\in$ U$\times$V and $(U\times V)\cap G^\mu(f) = \phi$.*

Theorem 4.1. *For function f:$(X,\tau) \rightarrow (Y, \sigma)$ is any relation with the co bT^μ graph, for every x$\in$X, f(x) $= \bigcap \{bTint^\mu\left(bTcl^\mu\left(f(U)\right)\right) : U \in bT^\mu - CO(X,x)\}$.*

Proof. Let us assume that the theorem is false. Then y$\neq$f(x) exists in a way that y$\in \bigcap \{bTint^\mu\left(bTcl^\mu\left(f(U)\right)\right) : U \in bT^\mu - CO(X,x)\}$. Which shows that, y$\in$ $bTint^\mu\left(bTcl^\mu\left(f(U)\right)\right)$ for every U$\in$ bT^μ CO(X,x). So, V$\cap$ f(U) $\neq \phi$ for every V $\in$ supra CO(Y,y). Hence, $int^\mu(cl^\mu(V))\cap$ f(U)$\supset$ V$\cap$ f(U) $\neq \phi$ contradicting the above theorem which states f is related to graph co bT^μ. □

Theorem 4.2. *Let f:$(X,\tau) \rightarrow (Y, \sigma)$ is co bT^μ graph $G^\mu(f)$. X bT^μ clopen T_1If f injective.*

Proof. Assume x, y be two distinctive points of (X,τ).Therefore, (x,f(y))$\in$ (X$\times$Y) $\cap G^\mu$(f) is achieved. By co bT^μ graph, there exist bT^μ clopen set U of X as well as the supra clopen set V in a way that (U$\times$V)$\cap G^\mu$(f) and (x,f(y)) $\in$ U$\times$V $= \phi$. Hence, f(U)$\cap$V $= \phi$ is achieved. Therefore, U$\cap f^{-1}$(V) $= \phi$. Hence, y$\notin$U which shows X bT^μ clopen T_1. □

Theorem 4.3. *Let f:$(X,\tau) \rightarrow (Y, \sigma)$ is co bT^μ graph G(f). If f surjective supra clopen function, then Y co bT^μ T_2.*

Proof. Assume that for (Y, σ), a and b be the distinct points. For f surjective, f(x) = a for some x$\in$X and (x,y)$\in$ (X$\times$Y)$\cap$G(f). With co-bT^μ graph, bT^μ-clopen present for set U of (X,τ) and supra clopen set V of (Y, σ). We know that each supra clopen set is bT^μ clopen set. Hence V is bT^μ clopen set in (Y, σ) in a way, (U$\times$V)$\cap$G(f) $= \phi$ and (x,b)$\in$U$\times$V. Hence, f(U)$\cap$V $= \phi$ is achieved. As, f supra clopen function then f(U) supra clopen in a way f(x) = a$\in$ f(U). Hence Y co bT^μ T_2. □

Theorem 4.4. *For f:$(X,\tau) \rightarrow (Y,\sigma)$ bT^μ connected functions and (Y,σ) co bT^μ Hausdorff space, then $G^\mu(f)$ is co bT^μ graph in X$\times$Y product space.*

Proof. Assume (x,y)$\in$ $(X\times$Y) G^μ(f). Then supra clopen sets V_1 and V_2 with y$\neq$f(x) of Y in a way that f(x)$\in$ V_1, y$\in$ V_2 and $V_1 \cap V_2 = \phi$. From hypothesis, there is U$\in$ bT^μ CO(X,x) for f(U)$\subset$ V_1. Hence, f(U)$\cap V_2 = \phi$ is obtained. □

5. CO bT^μ URYSOHN SPACE

Definition 5.1. *Let X is defined as supra Urysohn if for every points pair $x,y \in X$, $x \neq y$ there exist $U \in$ supra $C(x)$, $V \in$ supra $C(y)$ in a way that $cl^\mu(U) \cap cl^\mu(V)) = \phi$.*

Definition 5.2. *Let X is called co bT^μ Urysohn if for each pair $x,y \in X$, $x \neq y$ there is $U \in bT^\mu$ $CO(x)$, $V \in bT^\mu$ $CO(y)$ in a way that $bTint^\mu(bT\, cl^\mu(U)) \cap bTint^\mu(bT\, cl^\mu(V)) = \phi$.*

Theorem 5.1. *A co bT^μ Urysohn space is co bT^μ T_1.*

Proof. For X assume x and y as the two distinctive points. As X co bT^μ Urysohn space. There is $U \in bT^\mu$ CO(x), $V \in bT^\mu$ CO(y) in a way,

$$\mathrm{bTint}^\mu(bTcl^\mu(\mathrm{U})) \cap \mathrm{bTint}^\mu(bTcl^\mu(\mathrm{V})) = \phi.$$

Thos indicates that $x \notin bTint^\mu(bTcl^\mu(\mathrm{V}))$ and $y \notin bTint^\mu(bTcl^\mu(\mathrm{U}))$. Now, $\mathrm{bTint}^\mu(bTcl^\mu(\mathrm{U}))$, $\mathrm{bTint}^\mu(bTcl^\mu(V)) \in bT^\mu$ CO(X). Therefore X-$\mathrm{bTint}^\mu(bTcl^\mu(\mathrm{U}))$, X-$\mathrm{bTint}^\mu(bTcl^\mu(\mathrm{V})) \in bT^\mu$O(X) (or) bT^μC(X) in a way, $x \in X - \mathrm{bTint}^\mu(bTcl^\mu(\mathrm{V}))$ and $y \in X - \mathrm{bTint}^\mu(bTcl^\mu(\mathrm{U}))$ were $x \notin X - \mathrm{bTint}^\mu(bTcl^\mu(\mathrm{U}))$ and $y \notin X - \mathrm{bTint}^\mu(bTcl^\mu(\mathrm{V}))$. Thus X co bT^μ T_1. ☐

Theorem 5.2. *If $f:(X,\tau) \to (Y, \sigma)$ bijective bT^μ clopen map as well as X co bT^μ Urysohn then Y co bT^μ Urysohn.*

Proof. Assume $y_1 \neq y_2$ as $y_1, y_2 \in Y$. Since f bijective $f^{-1}(y_1) \neq f^{-1}(y_2)$ and $f^{-1}(y_1), f^{-1}(y_2) \in X$. The urysohn property co bT^μ of X proves an existence of sets $U \in bT^\mu CO(f^{-1}(y_1))$, $V \in bT^\mu CO(f^{-1}(y_2))$ with $bTint^\mu_X(bTcl^\mu_X(\mathrm{U})) \cap bTint^\mu_X(bTcl^\mu_X(\mathrm{V})) = \phi$. By bijective and bT^μ clopen map, $f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U}))) \in bT^\mu$ CO(Y). Again from $U \subset (bTint^\mu_X(bTcl^\mu_X(\mathrm{U})))$. It follows that

$$\begin{aligned} &f(\mathrm{U}) \subset f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U}))),\ (bTint^\mu_Y(bTcl^\mu_Y(f(\mathrm{U})))) \\ &\subset (bTint^\mu_Y(bTcl^\mu_Y(f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U})))))) = f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U}))). \end{aligned}$$

Similarly, $(bTint^\mu_Y(bTcl^\mu_Y(f(\mathrm{V})))) \subset f(bTint^\mu_X(bTcl^\mu_X(\mathrm{V}))$. Therefore by injectivity of f,

$$\begin{aligned} &bTint^\mu_Y(bTcl^\mu_Y(f(\mathrm{U}))) \cap bTint^\mu_Y(bTcl^\mu_Y(f(\mathrm{V}))) \\ &\subset f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U}))) \cap f(bTint^\mu_X(bTcl^\mu_X(\mathrm{V}))) \\ &= f(bTint^\mu_X(bTcl^\mu_X(\mathrm{U}))) \cap (bTint^\mu_X(bTcl^\mu_X(\mathrm{V}))) = \phi. \end{aligned}$$ ☐

Theorem 5.3. *If $f:(X,\tau) \to (Y, \sigma)$ bT^μ set connected and Y co bT^μ urysohn for $G^\mu(f)$ co bT^μ graph.*

Proof. For y≠f(x), assume (x,y)∈ (X×Y) - G(f). As, Y co bT^{μ} urysohn, so that V∈ bT^{μ} CO(Y,y), W∈ bT^{μ}CO(Y,f(x)) such that

$$(bTint^{\mu}_{X}(bTcl^{\mu}_{X}(\mathrm{U})))\cap(bTint^{\mu}_{X}(bTcl^{\mu}_{X}(\mathrm{V}))) = \phi.$$

Since f bT^{μ} set connected, so that U∈ bT^{μ}CO(X,x) for f(U)∈supraCO(Y). It depicts that f(U)∩V = ϕ. Hence, G$^{\mu}$(f) co bT^{μ} graph ☐

REFERENCES

[1] J. DONTCHEV, M. GANSTER, I. REILLY: *More on almost s-continuity*, Indian J. Math., **41** (1999), 139-146.

[2] E. EKICI: *Generalization of perfectly continuous, regular set-connected and clopen functions*, Acta Math. Hungar., **107**(3) (2005), 193-206.

[3] G.M. PANDYA, C. JANAKI, I. AROCKIARANI: *π-set connected functions in topological spaces*, Int. J. Contemp. Math. Sciences, **5**(36) (2010), 1789-1798.

[4] K. KRISHNAVENI, M. VIGNESHWARAN: *On bT- closed sets in supra topological spaces*, Int. J. Mat. Arc., **4**(2) (2013), 1-6.

[5] K. KRISHNAVENI, M. VIGNESHWARAN: *On bT-Closed Maps and bT-Homeomorphisms in Supra Topological Spaces*, J. Glo. Res. Mat. Arc., **1**(9) (2013), 23-26.

[6] K. KRISHNAVENI, M. VIGNESHWARANĆ: *supra bT-set connected function in supra topological Spaces*, Communications in Faculty of Sciences in Ankara university, **68**(1) (2019), 1-8.

[7] A.S. MASHHOUR, A.A. ALLAM , F.S. MOHAMOUD F.H KHEDR: *On supra topological spaces*, Indian J. Pure and App. Mat., **4**(14) (1983), 502-510.

[8] P. RAMPRASAD, P.P. BHATTACHARYYA: *On Pre-Urysohn spaces*, Bull. Malayasian Math. Soc., **22** (1999), no.223-34.

[9] I.L. REILLY, M.K. VAMANAMURTHY: *On super-continuous mapping*, Indian J. Pure and Appl. Math., **14** (1983), 767-772.

[10] O.R. SAYED, T. NOIRI: *On b-open sets and supra b-continuity on topological spaces*, Eur. J. Pure and App. Mat., **3**(2) (2010), 295-302.

DEPARTMENT OF MATHEMATICS (PG)
PSGR KRISHNAMMAL COLLEGE FOR WOMEN
COIMBATORE
TAMILNADU
INDIA
ADDRESS
Email address: `krishnavenikaliswami@gmail.com`