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J-G-BICONTINUOUS, J-G-BI-IRRESOLUTE IN COMPLEMENTED DITOPOLOGICAL TEXTURE SPACES

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ABSTRACT. The focus of this paper deals with the new notions that is to say continuity, co-continuity, bicontinuity in complemented ditopological texture spaces. The associations between these concept are based on the notion of j-g open and j-g closed sets. Some of their effective characterizations are inquired.

1. INTRODUCTION

The textures and ditopological texture spaces were placed for first time by L.M. Brown [1] in point-set setting for the appraisal of fuzzy sets. It has been proved useful as a framework to talk about the complement-free mathematical idea. A natural combination of texture space, topological space and bitopological space may considered as ditopological texture space but ditopology associates in a natural way to fuzzy topology. A generalization of fuzzy lattice is considered as a texture. The concept of ditopology is more general than general topology, fuzzy topology and bitopology. So it will be more good to generalize some distinct general (fuzzy, bi) topological ideas to the ditopological texture space. Also some basic notions are recollected, concerning textures and ditopologies and sufficient presentation to the theory and the motivation for its study are acquired from [2–5]. In [6] the concept of j-g opensets and j-g closed

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sets were introduced in ditopological texture spaces. In this paper, the concept of j-g-bicontinuity, j-g-bi-irresolute in complemented ditopological texture spaces are initiated and their properties are discussed.

2. Preliminaries

Some basic definitions of textures are listed below.

Definition 2.1. *[3] Let S be a set. Then $\psi \subseteq P(S)$ is called a texturing of S and S is said to be textured by ψ if*

(1) *$(\psi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \psi, i \in I$, the meet $\wedge_{i\in I} A_i$ and the join $\bigvee_{i\in I} A_i$ in ψ are related with the intersection and union in $P(S)$ by the equalities.*

$$\wedge_{i\in I} A_i = \bigcap_{i\in I} A_i,$$

for all I, while

$$\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i, \text{ for all finite I.}$$

(2) *ψ is completely distributive.*

(3) *ψ separates the point of S. That is, given $s_1 \neq s_2$ in S we have $L \in \psi$ with $s_1 \in L$, $s_2 \notin L$ If S is textured by ψ then (S, ψ) is called a texture space, or simply a texture.*

Definition 2.2. *[4] A mapping $\sigma : \psi \to \psi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \psi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \psi$ is called a complementation on (S, ψ) and (S, ψ, σ) is then said to be a complemented texture. For a texture (S, ψ) most properties are conveniently defined in terms of the sets,*

$$p\text{- sets } P_s = \bigcap\{A \in \psi : s \in A\},$$
$$q\text{- sets } Q_s = \bigvee\{A \in \psi : s \notin A\}.$$

Definition 2.3. *[4] A dichotomous topology on a texture (S, ψ), or ditopology, is a pair (τ, k) of subsets of ψ, where the of open sets τ satisfies*

(1) *$S, \psi \in \tau$;*

(2) *$G_1, G_2 \in \tau \Rightarrow G_1 \bigcap G_2 \in \tau$ and;*

(3) *$G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau$;*

and the set of closed sets k satisfies

(1) *$S, \psi \in k$;*

(2) $K_1, K_2 \in k \Rightarrow K_1 \bigcup K_2 \in k$;
(3) $K_i \in k, i \in I \Rightarrow \bigcap K_i \in k$.

Hence, a ditopology is essentially a topology for which there is no a priori relation between the open and closed sets.

For $A \in \psi$ define the closure clA and the interior intA of A under (τ, k) by the equalities.

$$clA = \bigcap \{K \in k : A \subseteq K\}$$

and

$$intA = \bigvee \{G \in \tau : G \subseteq A\}.$$

We refer to τ as the topology and k as the cotopology of (τ, k). If (τ, k) is a ditopology on a complemented texture (S, ψ, σ), then we say that (τ, k) is complemented if the equality $k = \sigma cl(\tau)$ is satisfied.In this study, a complemented ditopological texture space is denoted by $(S, \psi, \tau, k, \sigma)$. In this case we have $\sigma(clA) = int\sigma(A)$. We denote by $O(S, \psi, \tau, k)$ or $O(S)$,the set of open sets in ψ. Likewise $C(S, \psi, \tau, k)$ or $C(S)$ will denote the set of closed sets.

Let (S_1, ψ_1) and (S_2, ψ_2) be textures. In the following definition we consider the product texture $P(S_1) \otimes \psi_2$, and denote by $\overline{P}_{s,t}, \overline{Q}_{s,t}$, respectively the p-sets and q-sets for the product texture $(S_1 X S_2, P(S_1) \otimes \psi_2)$.

Definition 2.4. *[4] Let (S_1, ψ_1) and (S_2, ψ_2) be textures. Then*

(1) *$r \in P(S_1) \otimes \psi_2$ is called a relation from (S_1, ψ_1) to (S_2, ψ_2) if it satisfies*

$$R_1 : r \nsubseteq \overline{Q}_{s,t}, P_{s'} \Rightarrow r \nsubseteq \overline{Q}_{s',t}$$

$$R_2 : r \subseteq \overline{Q}_{s,t} \Rightarrow \exists s' \in S_1 \text{ such that } P_s \nsubseteq \overline{Q}_{s'} \text{ and } r \nsubseteq \overline{Q}_{s',t}.$$

(2) *$R \in P(S_1) \otimes \psi_2$ is called a corelation from (S_1, ψ_1) and (S_2, ψ_2) if it satisfies:*

$$CR_1 : \overline{P}_{s,t} \nsubseteq R, P_s \nsubseteq \overline{Q}_{s'} \Rightarrow \overline{P}_{s',t} \nsubseteq R.$$

$$CR_2 : \overline{P}_{s,t} \nsubseteq R \Rightarrow \exists s' \in S_1 \text{ such that } P_{s'} \nsubseteq Q_s \text{ and } \overline{P}_{s',t} \nsubseteq R.$$

(3) *A pair (r, R) where r is a relation and R a corelation from (S_1, ψ_1) to (S_2, ψ_2) is called a direlation from (S_1, ψ_1) to (S_2, ψ_2).*

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

Definition 2.5. *[4] Let (f, F) be a direlation from (S_1, ψ_1) to (S_2, ψ_2). Then (f, F) is called a difunction from (S_1, ψ_1) to (S_2, ψ_2) if it satisfies the following two conditions:*

DF_1 : *For* $s, s^{'} \in S_1, P_s \not\subseteq Q_{s^{'}} \Rightarrow \exists t \in S_2$ *such that* $f \not\subseteq \overline{Q}_{s,t}$ *and* $\overline{P}_{s^{'},t} \not\subseteq F$.

DF_2 : *For* $t, t^{'} \in S_2$ *and* $s \in S_1, f \not\subseteq \overline{Q}_{s,t}$ *and* $\overline{P}_{s,t^{'}} \not\subseteq F \Rightarrow P_{t^{'}} \not\subseteq Q_t$.

Definition 2.6. *[4] Let $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ be a difunction.*

(1) *For $A \in \psi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by,*

$$f^{\to}A = \bigcap\{Q_t : \forall s, f \not\subseteq \overline{Q}_{s,t} \Rightarrow A \subseteq Q_s\},$$
$$F^{\to}A = \bigvee\{P_t : \forall s, \overline{P}_{s,t} \not\subseteq F \Rightarrow P_s \subseteq A\}.$$

(2) *For $B \in \psi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by,*

$$f^{\leftarrow}B = \bigvee\{P_s : \forall t, f \not\subseteq \overline{Q}_{s,t} \Rightarrow P_t \subseteq B\},$$
$$F^{\leftarrow}B = \bigcap\{Q_s : \forall t, \overline{P}_{s,t} \not\subseteq F \Rightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Definition 2.7. *[2] The difunction $(f, F) : (S_1, \psi_1, \tau_1, k_1) \to (S_2, \psi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.*

Definition 2.8. *[4] Let $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition:*

SUR. For $t, t^{'} \in S_2, P_t \not\subseteq Q_{t^{'}} \Rightarrow \exists s \in S_1$ *with* $f \subseteq \overline{Q}_{s,t^{'}}$ *and* $\overline{P}_{s,t} \not\subseteq F$.

If (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$ for all $B \in \psi_2$

Definition 2.9. *[4] Let (f, F) be a difunction between the complemented textures (S_1, ψ_1, σ_1) and (S_2, ψ_2, σ_2). The complement $(f, F)^{'} = (F^{'}, f^{'})$ of the difunction (f, F) is a difunction, where $f^{'} = \bigcap\{\overline{Q}_{s,t} : \exists u, v$ with $f \subseteq \overline{Q}_{u,v,}\sigma_1(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \sigma_2(P_t)\}$ and $F^{'} = \bigvee\{\overline{P}_{s,t} : \exists u, v$ with $\overline{P}_{u,v} \not\subseteq F, P_u \not\subseteq \sigma_1(P_s)$ and $\sigma_2(Q_t) \subseteq Q_v\}$. If $(f, F) = (f, F)^{'}$ then the difunction (f, F) is called complemented.*

Definition 2.10. *[5] Let (X, τ) be generalised topological space a set $A \in \tau$ is called,*

(1) *j-open if $A \subseteq intpclA$;*
(2) *j-closed if $intpclA \subseteq A$.*

Definition 2.11. *[6] Let (S, ψ, τ, k) be a ditopological texture space. For $A \in \psi$, we define,*

(1) *The j-closure $cl_j A$ of A under (τ, k) by the equality,*

$$cl_j A = \bigcap\{B | B \in C_j(S) \text{ and } A \subseteq B\};$$

(2) *The j-interior $int_j A$ of A under (τ, k) by the equality,*

$$int_j A = \bigvee\{B | B \in O_j(S) \text{ and } B \subseteq A\}$$

Definition 2.12. *[6] Let $(S_i, \psi_i, \tau_i, k_i), i = 1, 2$ be a ditopological texture spaces and $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ a difunction.*

(1) *If $F^{\leftarrow}(G) \in O_j(S_1)$, for every $G \in O(S_2)$ this is called j- continuous.*
(2) *If $f^{\leftarrow}(K) \in C_j(S_1)$, for every $K \in C(S_2)$ this is called j-cocontinuous.*
(3) *If it is j- continuous and j-cocontinuous this is called j-bicontinuous*

Definition 2.13. *[6] Let (S, ψ, τ, k) be a ditopological texture space. A subset A of a texture ψ is said to be j-g-closed if $A \subseteq G \in O_j(S)$ then $cl(A) \subseteq G$. We denote by $jgc(S, \psi, \tau, k)$ or jgc(S), the set of j-g-closed sets in ψ.*

Definition 2.14. *[6] Let $(S, \psi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture ψ is called j-g-open if $\sigma(A)$ is j-g-closed. We denote by $jgo(S, \psi, \tau, k, \sigma)$ or jgo(S), the set of j-g-open sets in ψ.*

Theorem 2.15. *[1] For a direlation $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ the followings are equivalent.*

(1) *(f, F) is difunction.*
(2) *The following inclusions hold.*
   (a) $f^{\leftarrow}(F^{\to}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\to}(A)), \forall A \in \psi_1$.
   (b) $f^{\to}(F^{\leftarrow}(B)) \subseteq B \subseteq F^{\to}(f^{\leftarrow}(B)), \forall B \in \psi_2$.
(3) $f^{\leftarrow}B = F^{\leftarrow}B, \forall B \in \psi_2$.

Definition 2.16. *[1] Let $(S, \mathcal{S}, \sigma)$ and $(T, \mathcal{T}, \theta)$ be complemented textures and (r, R) a direlation from $(S, \mathcal{S})$ and $(T, \mathcal{T})$. Then:*

(1) *The complement r' of the relation r is the corelation. $r' = \bigcap\{\overline{Q}_{(s,t)} | \exists u, v$ with $r \not\subseteq \overline{Q}_{u,v}, \sigma(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \theta(P_t)\}$*
(2) *The complement R' of the corelation R is the relation. $R' = \bigvee\{\overline{P}_{(s,t)} | \exists u, v$ with $\overline{P}_{u,v} \not\subseteq R, (P_u) \not\subseteq \sigma(P_s)$ and $\theta(Q_t) \not\subseteq Q_v\}$*

(3) *The complement* $(r, R)^{'}$ *of the direlation* (r, R) *is the direlation.* $(r, R)^{'} = (r^{'}, R^{'})$

The direlation (r, R) *is said to be complemented if* $(r, R)^{'} = (r, R)$

Theorem 2.17. *[1] Let* $(S, \mathcal{S}, \sigma)$ *and* $(T, \mathcal{T}, \theta)$ *be complemented textures and* (r, R) *a direlation from* $(S, \mathcal{S})$ *and* $(T, \mathcal{T})$*. Then:*

(1) $(r^{'})^{\rightarrow}A = \theta(r^{\rightarrow}\sigma(A))$ *and* $(R^{'})^{\rightarrow}A = \theta(R^{\rightarrow}\sigma(A)), \forall A \in \mathcal{S}$.
(2) $(r^{'})^{\leftarrow}B = \sigma(r^{\leftarrow}\theta(B))$ *and* $(R^{'})^{\leftarrow}B = \sigma(R^{\leftarrow}\theta(B)), \forall B \in \mathcal{T}$.

3. J-G-BICONTINUOUS, J-G-BI-IRRESOLUTE

Definition 3.1. *Let* $(S, \psi, \tau, k, \sigma)$ *be a complemented ditopological texture space. For* $A \in \psi$*, we define the j-g-closure* $cl_{j-g}A$ *and the j-g-interior* $int_{j-g}A$ *of A under* (τ, k) *by the equalities,*

$$cl_{j-g}A = \bigcap\{K \in jgc(S) : A \subseteq K\}$$

and

$$int_{j-g}A = \bigcup\{G \in jgo(S) : G \subseteq A\}.$$

Definition 3.2. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-continuous if* $F^{\leftarrow}(G) \in jgo(S_1)$ *for every* $G \in O(S_2)$.

Definition 3.3. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-irresolute if* $F^{\leftarrow}(G) \in jgo(S_1)$ *for every* $G \in jgo(S_2)$.

Definition 3.4. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-cocontinuous if* $f^{\leftarrow}(G) \in jgc(S_1)$ *for every* $G \in k_2$.

Definition 3.5. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-coirresolute if* $f^{\leftarrow}(G) \in jgc(S_1)$ *for every* $G \in jgc(S_2)$.

Definition 3.6. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-bi-cocontinuous if it is j-g-continuous and j-g-cocontinuous.*

Definition 3.7. *The difunction* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *is called j-g-bi-irresolute if it is j-g-irresolute and j-g-coirresolute.*

Theorem 3.8. *Let* $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then*

(1) *Every continuous is j-g-continuous.*
(2) *Every cocontinuous is j-g-cocontinuous.*
(3) *Every j-g-irresolute is j-g-continuous.*
(4) *Every j-g-co-irresolute is j-g-cocontinuous.*

Proof.

(1) Suppose $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ be a difunction is continuous. By the definition of difunction in the continuity if $B\in\tau_2\Rightarrow F^{\leftarrow}B\in\tau_1$. From this definition clearly we get the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-continuous. Therefore every continuous is j-g-continuous.
(2) Suppose difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is co-continuous. By the definition of difunction in co-continuity if $B\in k_2\Rightarrow f^{\leftarrow}B\in k_1$. From this definition clearly we get the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-co-continuous. Therefore every co-continuous is j-g-co-continuous.
(3) Suppose that the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-irresolute. By the definition of difunction in j-g-irresolute if $F^{\leftarrow}(G)\in jgo(S_1)$ for every $G\in jgo(S_2)$. From this definition clearly we get the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-continuous. Therefore every j-g-irresolute is j-g-continuous.
(4) Suppose the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-co-irresolute. By the definition of difunction in j-g-co-irresolute if $f^{\leftarrow}(G)\in jgc(S_1)$ for every $G\in jgc(S_2)$. From this definition clearly we get the difunction $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ is j-g-co-continuous. Therefore every j-g-co-irresolute is j-g-cocontinuous.

□

Theorem 3.9. *Let $(f,F):(S_1,\psi_1,\tau_1,k_1,\sigma_1)\to(S_2,\psi_2,\tau_2,k_2,\sigma_2)$ be a difunction. Then the following are equivalent:*

(a) (f,F) *is j-g-continuous.*
(b) $int(F^{\rightarrow}(A))\subseteq(F^{\rightarrow}(int_{j-g}(A)),\forall A\in\psi_1$.
(c) $f^{\leftarrow}(int(B))\subseteq int_{j-g}(f^{\leftarrow}(B)),\forall B\in\psi_2$.

Proof.

$\underline{(a) \Rightarrow (b)}$. Let $A \in \psi_1$. Using $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$, and definition of interior, $f^{\leftarrow}(intF^{\rightarrow}(A)) \subseteq f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A$. Since in difunction, inverse image and inverse co image are equal. So $f^{\leftarrow}(intF^{\rightarrow}(A)) = F^{\leftarrow}(intF^{\rightarrow}(A)$. Then $f^{\leftarrow}(intF^{\rightarrow}(A)) \in jgo(S_1)$, by j-g-continuity. Therefore $f^{\leftarrow}(intF^{\rightarrow}(A)) \subseteq int_{j-g}A$. Then using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\rightarrow}B), \forall B \in \psi_2$

$$int(F^{\rightarrow}(A)) \subseteq F^{\rightarrow}(f^{\leftarrow}(int(F^{\rightarrow}(A)))) \subseteq F^{\rightarrow}(int_{j-g}A).$$

Therefore, $int(F^{\rightarrow}(A)) \subseteq F^{\rightarrow}(int_{j-g}(A))$.

$\underline{(b) \Rightarrow (c)}$. Let $B \in \psi_2$. Using (b) to $A = f^{\leftarrow}(B)$ and $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$, then $intB \subseteq int(F^{\rightarrow}(f^{\leftarrow}(B)) \subseteq F^{\rightarrow}(int_{j-g}f^{\leftarrow}(B))$. Also using $f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \forall A \in \psi_1$, gives

$$f^{\leftarrow}(intB) \subseteq f^{\rightarrow}F^{\leftarrow}(int_{j-g}f^{\leftarrow}(B)) \subseteq int_{j-g}f^{\leftarrow}(B).$$

Therefore, $f^{\leftarrow}(intB) \subseteq int_{j-g}f^{\leftarrow}(B)$

$\underline{(c) \Rightarrow (a)}$. Using (c) into $B \in O(S_2)$, $f^{\leftarrow}(B) = f^{\leftarrow}(intB) \subseteq int_{j-g}f^{\leftarrow}(B)$, $F^{\leftarrow}(B) = f^{\leftarrow}(B) = int_{j-g}f^{\leftarrow}(B) \in jgo(S_1)$. Therefore, (f, F) is j-g- continuous. □

Theorem 3.10. *Let $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \to (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then the following are equivalent:*

(a) *(f, F) is j-g-co-continuous.*

(b) *$(f^{\rightarrow}(cl_{j-g}(A)) \subseteq cl(f^{\rightarrow}(A)), \forall A \in \psi_1$.*

(c) *$cl_{j-g}(F^{\leftarrow}(B)) \subseteq (F^{\leftarrow}(cl(B))), \forall B \in \psi_2$.*

Proof.

$\underline{(a) \Rightarrow (b)}$. Let $A \in \psi_1$. Using $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$, and definition of closure, $F^{\leftarrow}(clf^{\rightarrow}(A)) \supseteq F^{\leftarrow}(f^{\rightarrow}(A)) \supseteq A$. Since in difunction, inverse image and inverse co image are equal. So $F^{\leftarrow}(clf^{\rightarrow}(A)) = f^{\leftarrow}(clf^{\leftarrow}(A))$. Then $F^{\leftarrow}(clf^{\rightarrow}(A)) \in jgc(S_1)$, by j-g-co-continuity. Therefore $F^{\leftarrow}(clf^{\rightarrow}(A)) \supseteq cl_{j-g}A$. Then using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$,

$$cl(f^{\rightarrow}(A)) \supseteq f^{\rightarrow}(F^{\leftarrow}(cl(f^{\rightarrow}(A)))) \supseteq f^{\rightarrow}(cl_{j-g}A).$$

Therefore, $cl(f^{\rightarrow}(A)) \supseteq f^{\rightarrow}(cl_{j-g}A)$.

$\underline{(b) \Rightarrow (c)}$. Let $B \in \psi_2$. Using (b) to $A = F^{\leftarrow}(B)$ and $F^{\rightarrow}(f^{\leftarrow}B) \supseteq B \supseteq f^{\rightarrow}(F^{\rightarrow}B), \forall B \in \psi_2$, then

$$clB \supseteq cl(f^{\rightarrow}(F^{\rightarrow}(B)) \supseteq f^{\rightarrow}(cl_{j-g}F^{\leftarrow}(B)).$$

Also using $f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \forall A \in \psi_1$ gives

$$F^{\leftarrow}(clB) \supseteq F^{\leftarrow}f^{\rightarrow}(cl_{j-g}F^{\leftarrow}(B)) \supseteq cl_{j-g}F^{\leftarrow}(B).$$

Therefore, $F^{\leftarrow}(clB) \supseteq cl_{j-g}F^{\leftarrow}(B)$.

$(c) \Rightarrow (a)$. Using (c) into $B \in C(S_2)$, $F^{\leftarrow}(B) = F^{\leftarrow}(clB) \supseteq cl_{j-g}F^{\leftarrow}(B)$,

$$f^{\leftarrow}(B) = F^{\leftarrow}(B) = cl_{j-g}F^{\leftarrow}(B) \in jgc(S_1).$$

Therefore, (f, F) is j-g- co-continuous. □

Theorem 3.11. *Let $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \to (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then the followings are equivalent:*

(a) (f, F) *is j-g-irresolute.*

(b) $int_{j-g}F^{\rightarrow}A \subseteq F^{\rightarrow}(int_{j-g}A), \forall A \in \psi_1$.

(c) $f^{\leftarrow}(int_{j-g}B) \subseteq int_{j-g}(f^{\leftarrow}B), \forall B \in \psi_2$.

Proof.

$(a) \Rightarrow (b)$. Let $A \in \psi_1$. Using $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$,then $f^{\leftarrow}(int_{j-g}F^{\rightarrow}(A)) \subseteq f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A$. $f^{\leftarrow}(int_{j-g}F^{\rightarrow}(A)) = F^{\leftarrow}(int_{j-g}F^{\rightarrow}(A)) \in jgo(S_1)$, by j-g-irresolute. So $f^{\leftarrow}(int_{j-g}F^{\rightarrow}(A)) \subseteq int_{j-g}A$ and using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$,

$$int_{j-g}(F^{\rightarrow}(A)) \subseteq F^{\rightarrow}(f^{\leftarrow}(int_{j-g}(F^{\rightarrow}(A)))) \subseteq F^{\rightarrow}(int_{j-g}A).$$

Therefore, $int_{j-g}(F^{\rightarrow}(A)) \subseteq F^{\rightarrow}(int_{j-g}(A))$.

$(b) \Rightarrow (c)$. Let $B \in \psi_2$. Using (b) to $A = F^{\rightarrow}(B)$ and using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$ gives $int_{j-g}B \subseteq int_{j-g}F^{\rightarrow}(f^{\leftarrow}B) \subseteq F^{\rightarrow}(int_{j-g}f^{\leftarrow}B)$. Therefore $f^{\leftarrow}(int_{j-g}B) \subseteq f^{\leftarrow}F^{\rightarrow}(int_{j-g}f^{\leftarrow}(B)) \subseteq int_{j-g}f^{\leftarrow}(B)$, by $f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \forall A \in \psi_1$ Therefore, $f^{\leftarrow}(int_{j-g}B) \subseteq int_{j-g}f^{\leftarrow}(B)$.

$(c) \Rightarrow (a)$. Using (c) into $B \in jgo(S_2)$ gives

$$f^{\leftarrow}(B) = f^{\leftarrow}(int_{j-g}B) \subseteq int_{j-g}f^{\leftarrow}(B).$$

So $F^{\leftarrow}(B) = f^{\leftarrow}(B) = int_{j-g}f^{\leftarrow}(B) \in jgo(S_1)$. Therefore, (f, F) is j-g-irresolute. □

Theorem 3.12. *Let $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \to (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then the followings are equivalent:*

(a) (f, F) *is j-g-co-irresolute.*

(b) $f^{\rightarrow}(cl_{j-g}A) \subseteq cl_{j-g}(f^{\rightarrow}A), \forall A \in \psi_1$.

(c) $cl_{j-g}(F^{\leftarrow}B) \subseteq F^{\leftarrow}(cl_{j-g}B), \forall B \in \psi_2$.

Proof. $(a) \Rightarrow (b)$
Let $A \in \psi_1$. Using $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)), \forall A \in \psi_1$, then $F^{\leftarrow}(cl_{j-g}f^{\rightarrow}(A)) \supseteq F^{\leftarrow}(f^{\rightarrow}(A)) \supseteq A$. $F^{\leftarrow}(cl_{j-g}f^{\rightarrow}(A)) = f^{\leftarrow}(cl_{j-g}f^{\rightarrow}(A)) \in jgc(S_1)$, by j-g-co- irresolute. So $F^{\leftarrow}(cl_{j-g}f^{\rightarrow}(A)) \supseteq cl_{j-g}A$ and using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$
$cl_{j-g}(f^{\rightarrow}(A)) \supseteq f^{\rightarrow}(F^{\leftarrow}(cl_{j-g}(f^{\rightarrow}(A)))) \supseteq f^{\rightarrow}(cl_{j-g}A)$.
Therefore, $cl_{j-g}(f^{\rightarrow}(A)) \supseteq f^{\rightarrow}(cl_{j-g}(A))$

$(b) \Rightarrow (c)$
Let $B \in \psi_2$. Using (b) to $A = f^{\rightarrow}(B)$ and using $f^{\rightarrow}(F^{\leftarrow}B) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}B), \forall B \in \psi_2$ gives $cl_{j-g}B \supseteq cl_{j-g}f^{\rightarrow}(F^{\leftarrow}B) \supseteq f^{\rightarrow}(cl_{j-g}F^{\leftarrow}B)$. Therefore $F^{\leftarrow}(cl_{j-g}B) \supseteq F^{\leftarrow}f^{\rightarrow}(cl_{j-g}F^{\leftarrow}(B)) \supseteq cl_{j-g}F^{\leftarrow}(B)$, by $f^{\leftarrow}(F^{\rightarrow}A) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}A), \forall A \in \psi_1$.
Therefore, $F^{\leftarrow}(cl_{j-g}B) \supseteq cl_{j-g}F^{\leftarrow}(B)$.

$(c) \Rightarrow (a)$
Using (c) into $B \in jgc(S_2)$ gives $F^{\leftarrow}(B) = F^{\rightarrow}(cl_{j-g}B) \supseteq cl_{j-g}F^{\leftarrow}(B)$
So $f^{\leftarrow}(B) = F^{\leftarrow}(B) = cl_{j-g}F^{\leftarrow}(B) \in jgc(S_1)$.
Therefore, (f, F) is j-g- co-irresolute. □

Theorem 3.13. *Let $(S_j, \psi_j, \tau_j, k_j, \sigma_j), j = 1, 2$ complemented ditopology and $(f, F) : (S_1, \psi_1) \rightarrow (S_2, \psi_2)$ be a complemented difunction. If (f, F) is j-g-continuous then (f, F) is j-g-co-continuous.*

Proof. Suppose $(S_1, \psi_1, \tau_1, k_1, \sigma_1)$ and $(S_2, \psi_2, \tau_2, k_2, \sigma_2)$ complemented ditopology and $(f, F) : (S_1, \psi_1) \rightarrow (S_2, \psi_2)$ be a complemented difunction. In this (f, F) is complemented. Since (f, F) is complemented, $(F', f') = (f, F)$. Using
1) $(f')^{\rightarrow}A = \sigma_2(f^{\rightarrow}\sigma_1(A))$ and $(F')^{\rightarrow}A = \sigma_2(F^{\rightarrow}\sigma_1(A)), \forall A \in \psi_1$
2) $(f')^{\leftarrow}B = \sigma_1(f^{\leftarrow}\sigma_2(B))$ and $(F')^{\leftarrow}B = \sigma_1(F^{\leftarrow}\sigma_2(B)), \forall B \in \psi_2$
We get, $\sigma_1((f')^{\leftarrow}(B)) = f^{\leftarrow}(\sigma_2(B))$ and
$\sigma_1((F')^{\leftarrow}(B)) = F^{\leftarrow}(\sigma_2(B)), \forall B \in \tau_2$
We know that the difunction $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ is called j-g-continuous, if $F^{\leftarrow}(G) \in jgo(S_1)$ for every $G \in O(S_2)$. Also the difunction $(f, F) : (S_1, \psi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \psi_2, \tau_2, k_2, \sigma_2)$ is called j-g-co-continuous, if $f^{\leftarrow}(G) \in jgc(S_1)$ for every $G \in K_2$. Using these the proof is proved. □

Theorem 3.14. *Let* $(S_j, \psi_j, \tau_j, k_j, \sigma_j), j = 1, 2$ *complemented ditopology and* $(f, F) : (S_1, \psi_1) \to (S_2, \psi_2)$ *be a complemented difunction. If* (f, F) *is j-g-irresolute then* (f, F) *is j-g-co-irresolute.*

Proof. Since (f, F) is complemented $(F^{'}, f^{'}) = (f, F)$. The proof is obvious from above theorem. □

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