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APPROXIMATION ON CORDIAL GRAPHIC TOPOLOGICAL SPACE

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ABSTRACT. The basic notions of CG-lower and CG-upper approximation in cordial topological space are introduced, which are the core concept of this paper and some of it's properties are studied. Furthermore, we have investigated some results, examples and counter examples are provided by using graphs.

1. INTRODUCTION

In practical life events need some sort of approximation to fit mathematical models. In 1982 Z. Pawlak [4], Introduced Rough set theory to handle Vagueness, imprecision and uncertainty in data analysis. There exists generalization of pawlak approximation space used by general topological structure. Pawlak's definitions for lower and upper approximations were introduced with the reference of equivalence relation [2]. Pawlak and Skowron [5,6] derived so many properties of the lower and upper approximations. H.M. Abu-Donia [2] discovered generalization of classical rough membership function of Pawlak rough sets and he concluded that generalized rough membership function can be used to analyze which decision should be made according to a conditional attribute in decision information system in 2013. In 2017, Y.Y. Yousif and S.S. Obaid [10] initiated Supra Approximation spaces using mixed degree systems in graph theory and they introduced two topological spaces, namely o-space

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and i-space. Sufficient conditions were discussed by Soon-Mo-Jung [8] that the intersection of subsets to be open. Some properties of interior and closure in general topology, were discussed in 2019 by Soon-Mo-Jung and Doyun Nam [9]. In this paper our approach is based on upper and lower approximation in cordial graphic topological space.

2. Preliminaries

The brief summary of definitions are given below.

Definition 2.1. *[1] Let $A \subseteq X$, then the upper approximation (resp.the lower approximation) of A is given by,*

$$\overline{R}A = \{x \in X : R_x \cap A \neq \emptyset\}$$
$$\underline{R}A = \{x \in X : R_x \subseteq A\},$$

where $R_x \subseteq X$ to denote the equivalence class containing $x \in X$ and X/R to denote the set of all elementary set of R.

Definition 2.2. *[3] A mapping $f : V(G) \to \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

The induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Let us denote $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 2.3. *[3] A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called cordial if it admits labeling.*

Definition 2.4. *[3] A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \to \{0,1\}$ defined by $f^*(uv) = |f(u) + f(v)|(mod 2)$ is called sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called sum cordial if it admits sum cordial labeling.*

Definition 2.5. *Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and with out isolated vertex. Define $S_{E(0G)}$ and $S_{E(1G)}$ as follows. $S_{E(0G)} = \{I_{e(0)} | e \in E\}$ and $S_{E(1G)} = \{I_{e(1)} | e \in E\}$ such that $I_{e(0)}$ and $I_{e(1)}$ is the incidence vertices having label 0 and 1 respectively. Since G has no isolated vertex,*

$S_{E(0G)} \cup S_{E(1G)}$ forms a subbasis for a topology τ_{CI} on V is called cordial incidence topology of G and it is denoted by (V, τ_{CI}).

Definition 2.6. *Let $G = (V(G), E(G))$ be a sum cordial graph and admits cordial incidence topology τ_{CI} induced by V and H be the subgraph of G, then the interior and closure of H has the following form,*

$$int_{CI}[V(H)] = \cup\{U \in \tau_{CI} | U \subseteq V(H)\}\,,$$
$$cl_{CI}[V(H)] = \cap\{U \in \tau_{CI}^{c} | V(H) \subseteq U\}\,.$$

Definition 2.7. *[7] Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and with out isolated vertex. Define S_{0G} and S_{1G} as follows. $S_{0G} = \{A_{v(0)} | v \in V\}$ and $S_{1G} = \{A_{v(1)} | v \in V\}$ such that $A_{v(0)}$ and $A_{v(1)}$ is the set of all vertices adjacent to v of G having label 0 and 1 respectively. Since G has no isolated vertex, $S_{0G} \cup S_{1G}$ forms a subbasis for a topology τ_{CG} on V is called cordial graphic topology of G and it is denoted by (V, τ_{CG}).*

Theorem 2.8. *[7] Suppose that $G = (V, E)$ is a sum cordial then the graph G admits the cordial Alexandoff space.*

Theorem 2.9. *[7] Let $G = (V, E)$ be a sum cordial graph which admits cordial graphic topology (V, τ_{CG}). If $u \in \mathscr{U}_{CG}$ then $\overline{A_v} \subseteq \overline{A_u}$ for every vertex u, v having label 0 or 1 in V, where $\mathscr{U}_{CG}$ is the intersection of all open sets containing x.*

3. CG-lower and CG-upper Approximation

Definition 3.1. *Let $G = (V(G), E(G))$ be a sum cordial and admits cordial graphic topology τ_{CG} induced by V and G_1 be the subgraph of G, then the interior and closure of G_1 has the following form,*

$$int_{CG}[V(G_1)] = \bigcup\{U \in \tau_{CG} | U \subseteq V(G_1)\}\,,$$
$$cl_{CG}[V(G_1)] = \bigcap\{U \in \tau_{CG}^{c} | V(G_1) \subseteq U\}\,.$$

Example 3.2. *Let us consider the sum cordial graph with $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$. From the fig. 1, we have $A_{v_1}(0) = \{v_2, v_3\}$, $A_{v_2}(1) = \{v_1, v_3, v_4\}$, $A_{v_3}(0) = \{v_1, v_2\}$, $A_{v_4}(1) = \{v_2\}$, $S_{0G} = \{\{v_2, v_3\}, \{v_1, v_2\}\}$ and $S_{1G} = \{\{v_1, v_3, v_4\}, \{v_2\}\}$.*

Thus $S_{0G} \cup S_{1G} = \{\{v_2, v_3\}, \{v_1, v_2\}, \{v_1, v_3, v_4\}, \{v_2\}\}$,

$$\tau_{CG} = \{V, \emptyset, \{v_2, v_3\}, \{v_1, v_2\}, \{v_1, v_3, v_4\}, \{v_2\}, \{v_1, v_2, v_3\}, \{v_3\}, \{v_1, v_3\}, \{v_1\}\}.$$

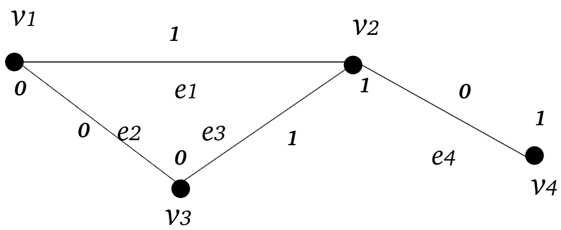

FIGURE 1

$$\tau_{CG}^{c} = \{V, \emptyset, \{v_1, v_4\}, \{v_3, v_4\}, \{v_2\}, \{v_1, v_3, v_4\},$$
$$\{v_2, v_3, v_4\}, \{v_4\}, \{v_1, v_2, v_4\}, \{v_2, v_4\}\}.$$

Now let us consider the subgraph G_1 of G as follows,

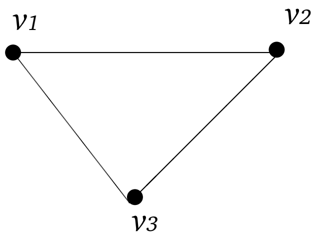

FIGURE 2

From fig. 2 we have, $V(G_1) = \{v_1, v_2, v_3\}$, $int_{CG}[V(G_1)] = \{v_1, v_2, v_3\}$ and $cl_{CG}[V(G_1)] = \{v_1, v_2, v_3, v_4\}$.

Definition 3.3. *Let $G = (V(G), E(G))$ be a sum cordial and admits cordial graphic topology τ_{CG} induced by V and G_1 be the subgraph of G, then the boundary of G_1 is defined by,*

$$b_{CG}[V(G_1)] = cl_{CG}(V(G_1)) - int_{CG}(V(G_1))\,.$$

Definition 3.4. *Let $G = (V(G), E(G))$ be approximation space and τ_{CG} be the cordial graphic topology induced by V and let G_1 be the subgraph of G, then CG-lower (resp. CG-upper) approximation of G_1 is defined by*

$$L_{CG}[V(G_1)] = int_{CG}[V(G_1)] \quad \text{and} \quad U_{CG}[V(G_1)] = cl_{CG}[V(G_1)].$$

Definition 3.5. *Let $G = (V(G), E(G))$ be approximation space and τ_{CG} be the cordial graphic topology induced by V and let G_1 be the subgraph of G, then CG-boundary region of G_1 is defined by,*

$$B_{CG}[V(G_1)] = U_{CG}[V(G_1)] - L_{CG}[V(G_1)]\,.$$

4. Properties of CG-lower and CG-upper Approximation

Theorem 4.1. *Let G_1 be the subgraph of $G = (V(G), E(G))$, where G be a sum cordial graph which admits cordial graphic topology τ_{CG} then,*

(1) $V(G) - L_{CG}[V(G_1)] = U_{CG}[V(G) - V(G_1)]$,
(2) $V(G) - U_{CG}[V(G_1)] = L_{CG}[V(G) - V(G_1)]$.

Proof.

(1) $V(G) - L_{CG}[V(G_1)] = V(G) - \bigcup\{U \in \tau_{CG} | U \subseteq V(G_1)\}$

$$= \bigcap_{U\in\tau_{CG}, U\subseteq V(G_1)} V(G) - U$$

$$= \bigcap_{W\in\tau^c_{CG}, V(G)-V(G_1)\subseteq W} W$$

$$= \bigcap\{W \in \tau^c_{CG} | V(G) - V(G_1) \subseteq W\}$$

$$V(G) - L_{CG}[V(G_1)] = U_{CG}[V(G) - V(G_1)]$$

(2) $V(G) - U_{CG}[V(G_1)] = V(G) - \bigcap_{W\in\tau^c_{CG}, V(G_1)\subseteq W} W$

$$= \bigcup_{W\in\tau^c_{CG}, V(G_1)\subseteq W} V(G) - W$$

$$= \bigcup_{U\in\tau_{CG}, U\subseteq V(G)-V(G_1)} U$$

$$= \bigcup\{U \in \tau_{CG} | U \subseteq V(G) - V(G_1\}$$

$$V(G) - U_{CG}[V(G_1)] = L_{CG}[V(G) - V(G_1)]$$

□

Theorem 4.2. *Let G_1 and G_2 be two subgraphs of G containing atleast two vertices, where G be a sum cordial graph which admits cordial graphic topology. If $V(G_1)$ and $V(G_1)$ satisfy the following conditions,*

(1) $V(G_1) \cap V(G_2) \cap B_{CG}[V(G_2)] = V(\emptyset)$,
(2) $V(G_1) \in \tau_{CG}$,

then $V(G_1) \cap V(G_2) \in \tau_{CG}$.

Proof. Since $V(G_1) \cap V(G_2) = V(\emptyset)$ or $V(G_1) \cap V(G_2) = V(G)$
Let us assume that,
$V(G_1) \cap V(G_2) \neq V(\emptyset)$ and $V(G_1) \cap V(G_2) \neq V(G)$.
$\Rightarrow v_1 \in V(G_1) \cap V(G_2)$
$\Rightarrow v_1 \notin B_{CG}[V(G_2)]$ (by using the first condition(1))
Since, $v_1 \in V(G_2) \Rightarrow v_1 \in L_{CG}[V(G_2)]$
$\Rightarrow V(G_1) \cap V(G_2) \subseteq V(G_1) \cap L_{CG}[V(G_2)]$
or
$V(G_1) \cap V(G_2) = V(G_1) \cap L_{CG}[V(G_2)]$,
by using the second condition (2)
$V(G_1) \cap V(G_2) = V(G_1) \cap L_{CG}[V(G_2)]$, is open. □

Theorem 4.3. *If G_1 and G_2 be two subgraphs of G, where G is a sum cordial graph which admits cordial graphic topology then,*

$$L_{CG}[V(G_1)] - U_{CG}[V(G_2)] = L_{CG}[V(G_1) - V(G_2)].$$

In addition, the following two conditions are equivalent:

(1) $U_{CG}[V(G_1)] - L_{CG}[V(G_2)] = U_{CG}[V(G_1) - V(G_2)]$,
(2) $L_{CG}[(V(G) - V(G_1)) \cup V(G_2)] = L_{CG}[V(G) - V(G_1)] \cup L_{CG}[V(G_2)]$.

Proof. $L_{CG}[V(G_1)] - U_{CG}[V(G_2)]$
$= L_{CG}[V(G_1)] \cap 9V(G) - U_{CG}[V(G_2)]0$
$= L_{CG}[V(G_1)] \cap L_{CG}[V(G) - V(G_2)]$ (by Theorem 4.1 (2))
$= L_{CG}[V(G_1) \cap V(G) - V(G_2)]$,
Therefore, $L_{CG}[V(G_1)] - U_{CG}[V(G_2)] = L_{CG}[V(G_1) - V(G_2)]$

(1) Let us asume that $(2) \Rightarrow (1)$
$V(G) - U_{CG}[V(G_1) - V(G_2)]$
$= V(G) - U_{CG}[V(G_1) \cap V(G) - V(G_2)]$
$= L_{CG}[V(G) - (V(G_1) \cap V(G) - V(G_2))]$
$= L_{CG}[(V(G) - V(G_1)) \cup V(G_2)]$
$= L_{CG}[V(G) - V(G_1)] \cup L_{CG}[V(G_2)]$
$= (V(G) - U_{CG}[V(G_1)]) \cup L_{CG}[V(G_2)]$
$= (V(G) - U_{CG}[V(G_1)]) \cup (V(G) - (V(G) - L_{CG}[V(G_2)]))$
$= V(G) - (U_{CG}[V(G_1)] \cap (V(G) - L_{CG}[V(G_2)]))$
$V(G) - U_{CG}[V(G_1) - V(G_2)] = V(G) - (U_{CG}[V(G_1)] - L_{CG}[V(G_2)])$
$\Rightarrow U_{CG}[V(G_1)] - L_{CG}[V(G_2)] = U_{CG}[V(G_1) - V(G_2)]$

(2) Let $(1) \Rightarrow (2)$. Assume that,

$$
\begin{aligned}
&\Rightarrow \quad U_{CG}[V(G_1)] - L_{CG}[V(G_2)] = U_{CG}[V(G_1) - V(G_2)] \\
&\Rightarrow \quad V(G) - (U_{CG}[V(G_1)] - L_{CG}[V(G_2)]) \\
&= \quad V(G) - U_{CG}[V(G_1) - V(G_2)] \,. \qquad (4.1)
\end{aligned}
$$

Now,

$$
\begin{aligned}
V(G) \quad - \quad & (U_{CG}[V(G_1)] - L_{CG}[V_{G_2}]) \\
= \quad & V(G) - (U_{CG}[V(G_1)] \cap (V(G) - L_{CG}[V(G_2)])) \\
= \quad & (V(G) - U_{CG}[V(G_1)]) \cup L_{CG}[V(G_2)] \\
= \quad & L_{CG}[V(G) - V(G_1)] \cup L_{CG}[V(G_2)] \,, \qquad (4.2)
\end{aligned}
$$

and

$$
\begin{aligned}
V(G) \quad - \quad & U_{CG}[V(G_1) - V(G_2)] = L_{CG}[V(G) - (V(G_1) - V(G_2))] \\
= \quad & L_{CG}[V(G) - (V(G_1) \cap (V(G) - V(G_2)))] \\
= \quad & L_{CG}[(V(G) - V(G_1)) \cup V(G_2)] \,. \qquad (4.3)
\end{aligned}
$$

From (4.1), (4.2) and (4.3) we have:

$$L_{CG}[(V(G) - V(G_1)) \cup V(G_2)] = L_{CG}[V(G) - V(G_1)] \cup L_{CG}[V(G_2)] \,.$$

□

Theorem 4.4. *Let G_1 and G_2 be two subgraphs of G containing atleast two vertices, where G be a sum cordial graph which admits cordial graphic topology. If $V(G_1)$ and $V(G_1)$ satisfy the following conditions:*

(1) $L_{CG}[(V(G) - V(G_1)) \cup V(G_2)] = L_{CG}[V(G) - V(G_1)] \cup L_{CG}[V(G_2)]$
(2) $V(G_1) \in \tau_{CG}$, *and*
(3) $U_{CG}[V(G_1)] - V(G_2) \in \tau_{CG}^c$, *then* $V(G_1) \cap V(G_2) \in \tau_{CG}$

Proof. Since $V(G_1) \cap V(G_2) = V(\emptyset)$ or $V(G_1) \cap V(G_2) = V(G)$. Let us assume that $V(G_1) \cap V(G_2) \neq V(\emptyset)$ or $V(G_1) \cap V(G_2) \neq V(G)$ and $V(G_1) \cap V(G_2) \notin \tau_{CG}$. Let v_1 be the vertex with the condition:

$$v_1 \in B_{CG}[V(G_1) \cap V(G_2)] \quad \text{and} \quad v_1 \in V(G_1) \cap V(G_2) \,. \qquad (4.4)$$

If $v_1 \in B_{CG}[V(G_1)]$, then $v_1 \notin V(G_1)$, since $V(G_1) \in \tau_{CG}$, by condition (2), which is contradictions to $v_1 \in V(G_1) \cap V(G_2)$. Hence $v_1 \notin B_{CG}[V(G_1)]$.

Since $v_1 \in B_{CG}[V(G_1) \cap V(G_2)] \subseteq B_{CG}[V(G_1)] \cap B_{CG}[V(G_2)]$,

$$\Rightarrow v_1 \in B_{CG}[V(G_2)]\,. \tag{4.5}$$

Now,

$$\begin{aligned} V(G_1) \cap B_{CG}[V(G_2)] &= V(G_1) \cap (U_{CG}[V(G_2)] \cap U_{CG}[V(G) - V(G_2)]) \\ &= V(G_1) \cap (U_{CG}[V(G_2)] \cap V(G) - L_{CG}[V(G_2)]) \\ &= U_{CG}[V(G_2)] \cap (V(G_1) - L_{CG}[V(G_2)]) \qquad (4.6) \\ &\subseteq V(G_2) \cap (V(G_1) - L_{CG}[V(G_2)])\,. \end{aligned}$$

From (4.4), (4.5) and (4.6) we have:

$$\begin{aligned} v_1 \in V(G_1) \cap B_{CG}[V(G_2)] &\subseteq V(G_1) - L_{CG}[V(G_2)] \\ &\subseteq U_{CG}[V(G_1)] - L_{CG}[V(G_2)]\,. \qquad (4.7) \end{aligned}$$

From above Theorem 4.1, the condition (1) and (4.7) we have:

$$\begin{aligned} v_1 \in U_{CG}[V(G_1)] &- L_{CG}[V(G_2)] \\ &= U_{CG}[V(G_1) - V(G_2)] \\ &\Rightarrow v_1 \in U_{CG}[V(G_1 - V(G_2))]\,. \end{aligned}$$

Since, $U_{CG}[V(G_1 - V(G_2))] \subseteq U_{CG}[V(G_1)] - V(G_2)$,

$$\Rightarrow v_1 \in U_{CG}[V(G_1)] - V(G_2)\,. \tag{4.8}$$

Since from (4.4), $v_1 \in V(G_2) \Rightarrow v_1 \notin U_{CG}[V(G_1)] - V(G_2)$, which is contrary to (4.8), therefore, $V(G_1) \cap V(G_2)$ should be belongs to τ_{CG}. ☐

Theorem 4.5. *Let subgraphs G_1 and G_2 be the mutually disjoints of G, that is $U_{CG}[V(G_1)] \cap V(G_2) = V(G_1) \cap U_{CG}[V(G_2)] = V(\emptyset)$. If $V(G_1) \cup V(G_2) \in \tau_{CG}$, then $V(G_1) \in \tau_{CG}$ and $V(G_2) \in \tau_{CG}$.*

Proof. Since G_1 and G_2 are mutually disjoints of G
$\Rightarrow V(G_1) \subset V(G) - U_{CG}[V(G_2)]$
Since, $V(G_1) \cup V(G_2)$ and $V(G) - U_{CG}[V(G_2)]$ are in τ_{CG}.
$\Rightarrow (V(G_1) \cup V(G_2)) \cap (V(G) - U_{CG}[V(G_2)])$
$= (V(G_1) \cap V(G) - U_{CG}[V(G_2)]) \cup (V(G_2) \cap V(G) - U_{CG}[V(G_2)])$
$= V(G_1) \cup V(\emptyset)$

$= V(G_1) \in \tau_{CG}$.

Similarly, $V(G_1) \cup V(G_2)$ and $V(G) - U_{CG}[V(G_2)]$ are in τ_{CG}

$\Rightarrow (V(G_1) \cup V(G_2)) \cap (V(G) - U_{CG}[V(G_1)])$

$= (V(G_1) \cap V(G) - U_{CG}[V(G_1)]) \cup (V(G_2) \cap V(G) - U_{CG}[V(G_1)]))$

$= V(\emptyset) \cup V(G_2)$

$= V(G_2) \in \tau_{CG}$. □

Theorem 4.6. *For subgraphs G_1 and G_2 of sum cordial graphs G which admits cordial graphic topology τ_{CG}, the following two statements are equivalent,*

(1) $U_{CG}[V(G_1) \cap V(G_2)] = U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]$

(2) $B_{CG}[V(G_1) \cap V(G_2)] = (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

Proof. (1) Let us assume that, $(2) \Rightarrow (1)$

$U_{CG}[V(G_1) \cap V(G_2)] = L_{CG}[V(G_1) \cap V(G_2)] \cup B_{CG}[V(G_1) \cap V(G_2)]$

$= (L_{CG}[V(G_1)] \cap L_{CG}[V(G_2)]) \cup (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= (L_{CG}[V(G_1)] \cap L_{CG}[V(G_2)]) \cup (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (L_{CG}[V(G_1)] \cap L_{CG}[V(G_2)]) \cup (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= (L_{CG}[V(G_1)] \cap L_{CG}[V(G_2)] \cup B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= (L_{CG}[V(G_1)] \cup B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (L_{CG}[V(G_2)] \cup B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= (U_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_2)] \cup B_{CG}[V(G_1)]) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) \cup (U_{CG}[V(G_1)] \cap B_{CG}[V(G_1)]) \cup (B_{CG}[V(G_2)] \cap U_{CG}[V(G_2)]) \cup (B_{CG}[V(G_2)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1]) \cap U_{CG}[V(G_2)])$

$= (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)] \cup B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)] \cup (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)])) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)])$

$= U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]$.

(2) Assume that, $(1) \Rightarrow (2)$

$B_{CG}[V(G_1) \cap V(G_2)] = U_{CG}[V(G_1) \cap V(G_2)] \cap U_{CG}[V(G) - (V(G_1) \cap V(G_2))]$

$= (U_{CG}[V(G_1)] \cap U_{CG}[V(G_1)]) \cap V(G) - L_{CG}[V(G_1) \cap V(G_2)]$

(by Theorem 4.1 (1))

$$= (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) \cap (V(G) - (L_{CG}[V(G_1)] \cap L_{CG}[V(G_2)]))$$
$$= (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) \cap ((V(G) - L_{CG}[V(G_1)])) \cup (V(G) - L_{CG}[V(G_2)])$$
$$= ((U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) \cap (V(G) - L_{CG}[V(G_1)])) \cup ((U_{CG}[V(G_1) \cap V(G_2)]) \cap (V(G) - L_{CG}[V(G_2)]))$$
$$= ((U_{CG}[V(G_1)] - L_{CG}[V(G_1)]) \cap U_{CG}[V(G_2)]) \cup (U_{CG}[V(G_1)] \cap (U_{CG}[V(G_2)] - L_{CG}[V(G_2)]))$$
$$= (B_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) \cup (U_{CG}[V(G_1)] \cap B_{CG}[V(G_2)])$$
$$= (B_{CG}[V(G_1)] \cup U_{CG}[V(G_2)]) \cap (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cup U_{CG}[V(G_1)]) \cap (U_{CG}[V(G_2)] \cup B_{CG}[V(G_2)])$$
$$= U_{CG}[V(G_1)] \cap (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cup U_{CG}[V(G_2)]) \cap U_{CG}[V(G_2)]$$
$$= (B_{CG}[V(G_1)] \cup B_{CG}[V(G_2)]) \cap (U_{CG}[V(G_1)] \cap U_{CG}[V(G_2)]) .$$
□

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