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A NOTE ON CONNECTEDNESS IN TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce two new types of connectedness namely j-connectedness and $\frac{1}{2}$j-connectedness in a topological space. Also, we discuss some of their basic properties and analyze the characterization using theorems.

1. INTRODUCTION

Connectedness is one of the most important topological property. In 1975, Pipetone and Russo introduced semiconnectedness [6] in a topological space. Based on the sets of preopen , α open, β open, the concepts of preconnectedness [7], α connectedness [6] and β connectedness [3] were introduced. In 1982, Mashhour et.al [4] introduced preopen sets and pre continuous function in topological space.

In 2005, the concept of (α, β) semi-connectedness [2] was introduced by Ennis Rosas, Carlos Carpintere and Jose Sanabria. In 2015, Tapi, Bhagyashri Deole introduced semiconnectedness and preconnectedness in Biclosure spaces [8]. The new concepts of half b-connectedness in topological space was introduced by T.Noiri and Shyamapada Modak in 2016 [5]. In 2017, Tyagi, Sumit Singh and Manoj Bhardwaj introduced P_β connectedness in topological space [9]. I. Arokiarani and D. Sasikala introduced a new type of set namely j-open sets in 2011, [1].

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In this paper, we define and examine the notions of j-separated and j-connected sets with the help of j-open sets. Also we introduce the stronger form of j-connectedness namely $\frac{1}{2}$j- connectedness. Here we discuss some of the properties using theorems.

2. Preliminaries

Definition 2.1. *[1] A subset A in a topological space (X, τ) is said to be j-open if $A \subseteq int(pcl(A))$. The complement of j-open set is j-closed.*

Definition 2.2. *[5] Two subsets A and B of a topological space X are said to be half separated if and only if $A \cap cl(B) = \emptyset$ or $cl(A) \cap B = \emptyset$.*

Definition 2.3. *[9] A preopen subset A of a topological space X is said to be P_β-open if for each $x \in A$ there exists a β-closed set F such that $x \in F \subseteq A$ that is preopen set A is expressed as a union of β-closed sets.*

Definition 2.4. *[9] Non-empty subsets A and B of a topological space X is said to be P_β-connected if $A \cap P_\beta cl(B) = \emptyset = P_\beta cl(A) \cap B$.*

Definition 2.5. *[9] A subset S of a topological space X is said to be P_β-connected if S is not the union of two P_β-separated sets in X.*

Theorem 2.1. *[5] Let A and B be two non-empty sets in a space X. The following statements hold:*

(i) *If A and B are half b-separated and $A_1 \subseteq A$ and $B_1 \subseteq B$, then A_1, B_1 are also half b-separated.*
(ii) *If $A \cap B = \emptyset$ and one of A and B is b-closed or b-open, then A and B are half b-separated.*
(iii) *If one of A and B is b-closed or b-open and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G are half b-separated.*

Definition 2.6. *[5] Two subsets P and Q in a space X are said to be cl-cl separated if and only if $cl(P) \cap cl(Q) = \emptyset$.*

Definition 2.7. *[1] A function $f : X \to Y$ is said to be*

(i) *j-continuous if the inverse image of each open set in Y is j-open in X.*
(ii) *j-irresolute if for each point $x \in X$ and each j-open set V of Y containing $f(x)$, there exist a j-open set U of X containing x such that $f(U) \subset V$*

(iii) *j-closed if the image of each closed set in X is j-closed in Y.*

3. J-CONNECTEDNESS

Definition 3.1. *Two non-empty subsets P and Q of a topological space* (X, τ) *is said to be j-separated if and only if* $P \cap jcl(Q) = jcl(P) \cap Q = \emptyset$.

Definition 3.2. *A topological space* (X, τ) *is said to be j-connected if X cannot be expressed as a union of two non-empty j-separated sets in X.*

Theorem 3.1. *A topological space X is j-connected if and only if the only subsets of X that are both j-open and j-closed in X are the null set and X itself.*

Proof. Let P be a non-empty proper subset of X which is both j-open and j-closed in X. Then there exists a sets $U = P$ and $V = X - P$ which forms a j-separation of X. Conversely, assume that if U and V forms a j-separation of X and $X = U \cup V$. This implies U is non-empty and different from X. Since $U \cap V = U \cap (jcl(V)) = jcl(U) \cap V = \emptyset$. Hence both sets are j-open and j-closed. □

Remark 3.1. *Every two j-separated sets are always disjoint since* $P \cap Q \subseteq P \cap jcl(Q) = \emptyset$. *The converse of the above theorem may not be true as shown by the following example.*

Example 1. *Let* $X = \{p, q, r, s\}$, $\tau = \{\emptyset, X, \{p\}, \{s\}, \{p, s\}, \{q, r\}, \{p, q, r\}, \{q, r, s\}\}$. *Here the subsets* $\{r\}$ *and* $\{q, s\}$ *are disjoint sets but not j-separated. Since* $\{r\} \cap jcl\{q, s\} = \{r\} \cap \{q, r, s\} \neq \emptyset$.

Theorem 3.2. *Two subsets P and Q of X are j-separated if and only if there exists a two j-open sets U and V such that* $P \subset U$, $Q \subset V$ *and* $P \cap V = \emptyset$, $Q \cap U = \emptyset$.

Proof. Let P and Q be j-separated sets and $V = X - jcl(P)$, $U = X - jcl(Q)$. Then U and V are j-open sets in X such that $P \subset U$ and $Q \subset V$. Also $P \cap V = \emptyset$, $Q \cap U = \emptyset$. Conversely, suppose U and $V \in jO(X)$ such that $P \subset U$, $Q \subset V$ and $P \cap V = \emptyset$, $Q \cap U = \emptyset$. Since $X - U$ and $X - V$ are j-closed then $jcl(P) \subset X - V \subset X - Q$ and $jcl(Q) \subset X - U \subset X - P$. Therefore, $jcl(P) \cap Q = \emptyset$ and $jcl(Q) \cap P = \emptyset$. Hence P and Q are j-separated. □

Theorem 3.3. *Let P and Q be two non-empty subset in a space X. Then the following statements hold:*

(i) *If $P \cap Q = \emptyset$ such that each of the sets P and Q are both j-closed(j-open), then P and Q are j-separated.*
(ii) *Suppose P and Q are j-separated sets, $P_1 \subseteq P$ and $Q_1 \subseteq Q$, then P_1 and Q_1 are also j-separated sets.*
(iii) *If each of these sets P and Q are both j-closed(j-open) and if $R = P \cap (X - Q)$ and $S = Q \cap (X - P)$, then R and S are j-separated.*

Proof. (i) Since P and Q are both j-open(j-closed) and $P \cap Q = \emptyset$, then $P = jcl(P)$ and $Q = jcl(Q)$. This implies $P \cap jcl(Q) = \emptyset$ and $Q \cap jcl(P) = \emptyset$. Hence P and Q are j-separated.
(ii) Since $P_1 \subseteq P$, then $jcl(P_1) \subset jcl(P)$. We show that $P_1 \cap jcl(Q) = jcl(P_1) \cap Q_1 = \emptyset$. Since P and Q are j-separated, then $P \cap jcl(Q) = \emptyset$. This implies $P_1 \cap jcl(Q) = \emptyset$ and $P_1 \cap jcl(Q_1) = \emptyset$. Similarly, $Q \cap jcl(P_1) = \emptyset$. Hence P_1 and Q_1 are j-separated.
(iii) If P and Q are j-open ,then $X - P$ and $X - Q$ are j-closed. Since $R \subset X - Q$, $jcl(R) \subset jcl(X - Q) = X - Q$ and so $jcl(R) \cap Q = \emptyset$. Thus $S \cap jcl(R) = \emptyset$. Similarly $R \cap jcl(S) = \emptyset$. Hence R and S are j-separated. □

Definition 3.3. *A point $p \in X$ is called j-limit point of a set $P \subset X$ if each j-open set $U \subseteq X$ containing p contains a point of P other than x.*

Theorem 3.4. *Let P and Q be two non-empty disjoint subsets of a space X and $R = P \cup Q$. Then P and Q are j-separated if and only if P and Q are j-closed(j-open) in R.*

Proof. Let P and Q be j-separated sets. Using the definition of j-separated, P does not contains j-limit of Q. Therefore, Q contains all the j-limit points of Q. Then the limit points lie in $P \cup Q$ and also Q is j-closed in $P \cup Q$. Hence Q is j-closed in R. Similarly, P is j-closed in R. □

Theorem 3.5. *If a subset P of X is j-connected, then jcl(P) is also j-connected.*

Proof. Assume the contrary, if $jcl(P)$ is disconnected. Then there exists two non-empty j-separated sets R and S in X such that $P = R \cup S$, in consideration of $P = (R \cap P) \cup (S \cap P)$ and $jcl(R \cap P) \subset jcl(R)$ and $jcl(S \cap P) \subset jcl(S)$ and also $R \cap S = \emptyset$ which implies $jcl(R \cap P) \cap S = \emptyset$. Hence $jcl(R \cap P) \cap (S \cap Q) = \emptyset$. Equivalently, $jcl(S \cap P) \cap (R \cap S) = \emptyset$. Therefore P is j-connected. Hence P is j-connected implies $jcl(P)$ is also j-connected. □

Theorem 3.6. *Let $P \subseteq Q \cup R$ such that P be a non-empty j-connected set in X and Q, R are j-separated. Then only one of the following conditions hold:*

(i) $P \subseteq Q$ *and* $P \cap R = \emptyset$.

(ii) $P \subset R$ *and* $P \cap Q = \emptyset$.

Proof. Suppose $P \cap R = \emptyset$ implies $P \subseteq Q$. If $P \cap Q = \emptyset$, then $P \subseteq R$. Since $Q \cap R$ then both $P \cap Q = \emptyset$ and $P \cap R = \emptyset$ does not hold. Similarly, assume that $P \cap Q \neq \emptyset$ and $P \cap R \neq \emptyset$. Then, by the theorem 3.3 (ii), $P \cap Q$ and $P \cap R$ are j-separated such that $P = (P \cap Q) \cup (P \cap R)$ which contradicts the definition of j-connectedness of P. □

Remark 3.2. *In difference, connectedness of a topological space (X, τ), if a topology τ_1 on the space X is strictly finer than the another topology τ_2 on X, then j-connectedness of $(X, \tau_1) \;\not\Rightarrow\;$ j-connecteness of (X, τ_2). Also j-connectedness of $(X, \tau_2) \;\not\Rightarrow\;$ j-connectedness of (X, τ_1). This result is verified by the following example.*

Let $X = \{1, 2, \}$ with $\tau_1 = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, X\}$, $\tau_2 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\}$. Then $\tau_2 \subset \tau_1$. In (X, τ_1), PO(X)=JO(X). In (X, τ_2), $PO(X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, X\} = JO(X)$. But in (X, τ_2), X cannot be expressed as the union of two j-separated sets in X. Therefore, (X, τ_2) is j-connected as (X, τ_1) is not j-connected.

Theorem 3.7. *Let X be a topological space and $X = P \cup Q$ be a j-separation of X. If Y is a j-connected subset of X, then Y is completely contained in either P or Q.*

Proof. Let $X = P \cup Q$ be a j-separation of X. Suppose Y intersecting both P and Q, then Y can be denoted by $Y = (P \cap Y) \cup (Q \cap Y)$. It denotes j-separation of Y. This is a contradiction. Therefore, Y is completely contained in either P or Q. □

Theorem 3.8. *Let P and Q be two non-empty subsets of X. If P and Q are j-connected and not j-separated in X, then $P \cup Q$ is j-connected.*

Proof. Assume $P \cup Q$ is not j-connected. Then there exist j-separated sets R and S such that $P \cup Q = R \cup S$. This implies $P \subset R \cup S$. Therefore, $P \subset R$ or $P \subset S$. Similarly, $Q \subset R \cup S$ implies $Q \subset R$ or $Q \subset S$. Suppose $P \subset R$ and $Q \subset R$ implies $P \cup Q \subset R$ and $S = \emptyset$. This is a contradiction. Therefore, $P \subset R$ and $Q \subset S$ or $P \subset S$ and $Q \subset R$. In the first case, $jcl(P) \cap Q \subset jcl(R) \cap S = \emptyset$ and

$jcl(Q) \cap P \subset jcl(S) \cap R = \emptyset$. Similarly, we have this result for the second case. This implies P and Q are j-separated in X. It contradicts our assumption. Hence $P \cup Q$ is j-connected. □

Theorem 3.9. *If $\{G_\sigma \backslash \sigma \in \tau\}$ is a non-empty family of j-connected subset of a topological space X such that $\bigcap \lim_{\sigma\in\tau} G_\sigma \neq \emptyset$ then $\bigcup \lim_{\sigma\in\tau} G_\sigma \neq \emptyset$ is j-connected.*

Proof. Assume that $H = \bigcup \lim_{\sigma\in\tau} G_\sigma$ and H is not j-connected. Then $H = R \cup S$, where R and S are j-separated sets in X. Since V $\bigcap \lim_{\sigma\in\tau} G_\sigma \neq \emptyset$. Now we take point x in $\bigcap \lim_{\sigma\in\tau} G_\sigma$. Therefore, $x \in \bigcup \lim_{\sigma\in\tau} G_\sigma = H$. Since $H = R \cup S$ implies $x \in R$ or $x \in S$. Suppose that $x \in R$. Since $x \in G_\sigma$ for each $\sigma \in \tau$. Therefore, G_σ and R intersect for each $\sigma \in \tau$. Using the theorem 3.8, $G_\sigma \subset R$ or $G_\sigma \subset S$. Since R and S are disjoint, $G_\sigma \subset R$ for all $\sigma \in \tau$ and hence $H \subset R$. Therefore we have $S = \emptyset$. This is a contradiction to our assumption. Hence $H = \bigcup \lim_{\sigma\in\tau} G_\sigma \neq \emptyset$ is j-connected. □

Definition 3.4. *Let X be a topological space $x \in X$. The j-component of X containing x is the union of all j-connected subsets of X containing x.*

Definition 3.5. *A topological space X is called as locally j-connected at $x \in X$ if for each j-neighbourhood U containing x, there is a j-connected neighborhood V of x contained in U i.e. $x \in V \subseteq U$. The space X is locally j-connected if it is locally j-connected at each of its points.*

Theorem 3.10. *A space X is locally j-connected if and only if for each j-open set U of X, each j-component of U is j-open in X.*

Proof. Suppose that X is locally j-connected. Let U be j-open in X. Let C be the j-component of U. If we take a point x in C, we select a neighborhood V of x such that $V \subset U$. Since V is j-connected, this implies V entirely contained in the j-component C of U. Hence C is j-open in X. Conversely, assume that $U \subseteq X$ be a j-open and $x \in U$. By our hypothesis, the j-component V of U containing x is j-open. Hence X is locally j-connected in X. □

4. $\frac{1}{2}$J-CONNECTEDNESS

Definition 4.1. *Two subsets P and Q in a space X are said to be $\frac{1}{2}$j-separated if and only if $P \cap jcl(Q) = \emptyset$ or $jcl(P) \cap Q = \emptyset$.*

Definition 4.2. *A subset P of a space X is said to be $\frac{1}{2}$j-connected(resp. cl-cl connected) if P is not the union of two non-empty half j-separated sets(resp. cl-cl separated) sets in X.*

From the above definitions, we have the following implications:

$$\boxed{\text{cl-cl separated}} \Rightarrow \boxed{\text{separated}}$$
$$\Downarrow$$
$$\boxed{\tfrac{1}{2}\text{j-separated}} \Leftarrow \boxed{\text{j-separated}}$$

The converse of the above implications need not be true as shown in the following examples.

Example 2. *Let $X = \{p, q, r, s\}$ with a topology $\tau = \{\emptyset, X, \{p\}, \{p, q\}\}$, $\tau^c = \{\emptyset, X, \{q, r, s\}, \{r, s\}\}$. The j-open sets are $\emptyset, X, \{p\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}$. The j-closed sets are $\emptyset, X, \{q, r, s\}, \{r, s\}, \{q, s\}, \{q, r\}, \{s\}, \{r\}, \{q\}$. Here $\{p\}$ and $\{q, r, s\}$ are $\frac{1}{2}$j-separated sets as $\{p\} \cap jcl\{q, r, s\} = \emptyset$ but $jcl\{p\} \cap \{q, r, s\} \neq \emptyset$. Therefore the two sets $\{p\}$ and $\{q, r, s\}$ are not j-separated.*
Since $jcl(P) \subset cl(P)$ for every subset P of X, every cl-cl separated set is $\frac{1}{2}$j-separated. But the converse may not be true as shown by the example 2. The sets $\{p\}$ and $\{q, r, s\}$ are $\frac{1}{2}$j-separated. But $cl\{p\} \cap cl\{q, r, s\} \neq \emptyset$. Therefore the sets $\{p\}$ and $\{q, r, s\}$ are not cl-cl separated.

Theorem 4.1. *A topological space (X, τ) is $\frac{1}{2}$j-connected if and only if it cannot be expressed as the union of disjoint non-empty j-open set and a non-empty j-closed set.*

Proof. Let X be a $\frac{1}{2}$j-connected space. Suppose that $X = P \cup Q$, where $P \cap Q = \emptyset$. Also P be a non-empty j-open set and Q be a non-empty closed set in X. Then $P \cap jcl(Q) = \emptyset$ since Q is j-closed set in X. Therefore P and Q are $\frac{1}{2}$j-separated. Hence X is not a $\frac{1}{2}$j-connected space. This is a contradiction.

Conversely, suppose that X is not a $\frac{1}{2}$j-connected space, then there exist non-empty $\frac{1}{2}$j-separated sets R and S such that $X = R \cup S$. Let $R \cap jcl(S) = \emptyset$. Set $P = X - jcl(S)$ and $Q = X - P$. Then $P \cup Q = X$ and $P \cap Q = \emptyset$. P and Q are non-empty j-open set and j-closed set respectively. Hence X can be expressed as the disjoint union of non-empty j-open set and non-empty j-closed set. Similar argument is used for the another case $jcl(R) \cap S = \emptyset$. □

Theorem 4.2. *Let X be a topological space if P is a $\frac{1}{2}$j-connected subset of X and R, S are the $\frac{1}{2}$j-separated subsets of X with $P \subset R \cup S$ then either $P \subset R$ or $P \subset S$.*

Proof. Let P be a $\frac{1}{2}$j-connected set. Take $P \subset R \cup S$. Since R and S are $\frac{1}{2}$j-separated, $jcl(R) \cap S = \emptyset$ or $S \cap jcl(R) = \emptyset$. Consider $S \cap jcl(R) = \emptyset$. Therefore, we set $P = (P \cap R) \cup (P \cap S)$, then $(P \cap S) \cap jcl(P \cap R) \subset S \cap jcl(R) = \emptyset$. Suppose $P \cap R$ and $P \cap S$ are non-empty sets. Then P is not $\frac{1}{2}$j-connected. This is a contradiction. Hence either $P \cap R = \emptyset$ or $P \cap S = \emptyset$ which implies $P \subset R$ or $p \subset S$. Similar argument is used for another case $jcl(S) \cap R = \emptyset$. □

Theorem 4.3. *In a topological space (X, τ), j-irresolute image of a $\frac{1}{2}$j-connected space is $\frac{1}{2}$j-connected.*

Proof. Let X be a $\frac{1}{2}$j-connected space and $f : X \to Y$ be a j-irresolute function. Suppose that we take $f(x)$ is not a $\frac{1}{2}$j-connected subset of Y such that $f(x) = R \cup S$. Since R and S are $\frac{1}{2}$j-separated i.e. $jcl(R) \cap S = \emptyset$ or $R \cap jcl(S) = \emptyset$. Since a function f is irresolute, therefore we have $jcl(f^{-1}(R)) \cap f^{-1}(Q) \subset f^{-1}(jcl(R)) \cap f^{-1}(S) = f^{-1}(jcl(R) \cap (S)) = \emptyset$ or $f^{-1}(R) \cap jcl(f^{-1}(S) \subset f^{-1}(R) \cap f^{-1}(jcl(Q)) = f^{-1}(R \cap jcl(S)) = \emptyset$. But $R \neq \emptyset$, there exist a point $r \in X$ such that $f(r) \in R$ and hence $f^{-1}(R) \neq \emptyset$. Equivalently, we have $f^{-1}(S) \neq \emptyset$. Therefore, $f^{-1}(R)$ and $f^{-1}(S)$ are non-empty $\frac{1}{2}$j-separated sets such that $X = f^{-1}(R) \cup f^{-1}(S)$ which implies X is not a $\frac{1}{2}$j-connected space. This is a contradiction to our assumption that $f(x)$ is not a $\frac{1}{2}$j-connected subset of Y. Hence $f(x)$ is a $\frac{1}{2}$j-connected space. □

Theorem 4.4. *In a topological space (X, τ), the continuous image of a $\frac{1}{2}$j-connected space is $\frac{1}{2}$j-connected.*

Proof. Let $f : X \to Y$ be a continuous function and X be $\frac{1}{2}$j-connected space. Suppose that $f(X)$ is not $\frac{1}{2}$j-connected subset of Y. Then there exists $\frac{1}{2}$j-separated sets R and S in Y such that $f(X) = R \cup S$. Since R and S are $\frac{1}{2}$j-separated, Therefore $jcl(R) \cap S = \emptyset$ or $R \cap jcl(S) = \emptyset$. Since f is j-continuous, $jcl(f^{-1}(R) \cap f^{-1}(S)) \subset f^{-1}(jcl(R) \cap f^{-1}(S)) = f^{-1}(jcl(R) \cap S) = \emptyset$ or $f^{-1}(R) \cap jcl(f^{-1}(S)) \subset f^{-1}(R) \cap f^{-1}(jcl(S)) = f^{-1}(R \cap jcl(S)) = \emptyset$. Since $R \neq S$, Then there exist a point $r \in X$ such that $f(r) \in R$ and hence $f^{-1}(R) \neq \emptyset$. Similarly, $f^{-1}(S) \neq \emptyset$. This implies $f^{-1}(R)$ and $f^{-1}(S)$ are $\frac{1}{2}$j-separated sets such that $X = f^{-1}(R) \cup f^{-1}(S)$. Therefore, X is not a $\frac{1}{2}$j-connected space. This is a contradiction to the fact that X is $\frac{1}{2}$j-connected space. Hence $f(X)$ is $\frac{1}{2}$j-connected in Y. □

Lemma 4.1. *Let $f : X \to Y$ be a j-continuous function. Then $jcl(f^{-1}(S) \subseteq f^{-1}(cl(S))$ for each $S \subseteq Y$.*

Theorem 4.5. *If $f : X \to Y$ be a j-continuous function and τ is $\frac{1}{2}$j-connected set in a space X, then $f(T)$ is cl-cl connected in Y.*

Proof. Suppose that $f(T)$ is not cl-cl connected in Y. There exists two non-empty cl-cl separated sets R and S of Y such that $f(T) = R \cup S$. Let us take a set $C = T \cap f^{-1}(R)$ and $D = T \cap f^{-1}(S)$. Since $f(T) \cap R \neq \emptyset$ then $T \cap f^{-1}(R) \neq \emptyset$ and also $C \neq \emptyset$. Similarly, $D \neq \emptyset$. Now we have $C \cup D = (T \cap f^{-1}(R)) \cup (T \cap f^{-1}(S)) = T \cap (f^{-1}(R) \cup f^{-1}(S)) = T \cap f^{-1}(R \cup S) = T \cap f^{-1}(f(T)) = T$. Since f is continuous, by lemma 4.1, $C \cap cl(D) \subset f^{-1}(R) \cap cl(f^{-1}(Q)) \subset f^{-1}(cl(R)) \cap f^{-1}(cl(S)) = f^{-1}(cl(R) \cap cl(S)) = \emptyset$. This is a contradiction to our assumption that T is $\frac{1}{2}$j-connected. Hence $f(T)$ cl-cl connected in Y. □

Theorem 4.6. *If P is $\frac{1}{2}$j-connected then $jcl(P)$ is also $\frac{1}{2}$j-connected.*

Proof. Suppose that $jcl(P)$ is not $\frac{1}{2}$j-connected. Then it can be expressed as a union of two $\frac{1}{2}$j-separated sets R and S in X. Since $P = (R \cap P) \cup (S \cap P)$ and $jcl(R \cap P) \cap S = \emptyset, jcl(R \cap P) \cap (S \cap P) = \emptyset$. This implies P is not $\frac{1}{2}$j-connected, contradiction. Hence $jcl(P)$ is $\frac{1}{2}$j-connected. □

Theorem 4.7. *If $f : X \to Y$ is bijective j-closed function and T is $\frac{1}{2}$j-connected in Y, then $f^{-1}(T)$ is cl-cl connected in X.*

Proof. Let $f : X \to Y$ be a j-closed bijective i.,e one-one and onto, then $f^{-1} : Y \to X$ is a continuous bijection. Since T is $\frac{1}{2}$j-connected in Y, by theorem 4.5, $f^{-1}(T)$ is cl-cl connected in X. □

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