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ROOT QUAD MEAN LABELING FOR SOME SPECIAL GRAPHS

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ABSTRACT. A graph G $= (p, q)$ is a Root Quad Mean graph if there is an injective function f from the vertices of G to $1, 2, \dots, p^2 - 1$ such that when each edge uv is labeled with $\frac{\sqrt{f(u)^4 + f(v)^4}}{2}$ then the resultant edges are distinct. In this paper we proved Comb graph, Ladder graph , Slanting ladder graph, Gear graph, Helm graph and Heger graph are Root Quad Mean Graphs.

1. INTRODUCTION

A graph is made up of vertices which are connected by edges. In this paper a concise summary of definitions and other information is given aiming to maintain compactness. In 1967, Alexander Rosa introduced the concept of labeling. Graph labeling is the assignment of labels, normally represented by integers, to edges and vertices of a graph. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [1].

Somasundaram and Ponraj [2] have introduced the notion of Mean Labeling of graphs. Sandhya, Somasundaram and Anusa have introduced Root Square Mean Labeling [3]. Gowri and Vembarasi have introduced Root cube Mean Labeling[4]. Inspired by the above works we introduced new labeling called Root Quad Mean Labeling. In this paper we probe the Root Quad Mean Labeling of

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Comb graph, Ladder graph, Slanting ladder graph, Gear graph, Helm graph and a new graph is found.

2. Basic Definitions

Definition 2.1. *A graph G = (V,E) is a set of all vertices and edges in which each edge is associated by a pair of two vertices.*

Definition 2.2. *The graph obtained by attaching a single pendent edge to each vertex of a path is called comb.*

Definition 2.3. *A ladder L_n is the graph obtained from two paths u_1, u_2, ..., u_n and v_1, v_2, ..., v_n by joining each u_i with v_i, $1 \leq i \leq n$.*

Definition 2.4. *A slanting ladder SL_n is the graph obtained from two paths u_1,u_2, ..., u_n and v_1, v_2, ..., v_n by joining each u_i with v_{i+1}, $1 \leq i \leq n-1$ [5].*

Definition 2.5. *Gear graph G_p is a graph obtained from wheel by adding a vertex between each pair of adjacent vertices of rim of the cycle.*

Definition 2.6. *The Helm H_p is a graph obtained by joining pendant vertices to each rim vertex of the Wheel.*

Definition 2.7. *Heger graph M_p is graph obtained from gear graph G_p by attaching pendant edge to each vertex of the rim of G_p which is not connected to centre vertex of G_p.*

Definition 2.8. *A graph G with p vertices and q edges is a Mean graph if there is an injective function f from the vertices of G to $0, 1, 2, \dots, q$ such that when each edge uv is labeled with $\dfrac{f(u)+f(v)}{2}$ if f(u)+f(v) is even and $\dfrac{f(u)+f(v)+1}{2}$ if f(u)+f(v) is odd then the resulting edges are distinct.*

Definition 2.9. *A graph $G = (p, q)$ is a Root Quad Mean graph if there is an injective function f from the vertices of G to $1, 2, \dots, p^2$ - 1 such that when each edge uv is labeled with $\dfrac{\sqrt{f(u)^4+f(v)^4}}{2}$ then the resultant edges are distinct.*

3. Main Results

Theorem 3.1. *Every Comb graph is a root quad mean graph.*

Proof. Let G be a comb graph, p the number of vertices on the graph G, $V(G)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_{\frac{p}{2}}$ be the upper set of vertices, $v_1, v_2, \ldots, v_{\frac{p}{2}}$ be the lower set of vertices, and let edges be defined as

$$E(G) = \{u_i u_{i+1}/1 \leq i \leq \frac{p-2}{2}\} \cup \{u_i v_i/1 \leq i \leq \frac{p}{2}\}.$$

Here $V(G) = p$ and $E(G) = p - 1$.

Now define a function $f : V(G) \to \{1, 2, \ldots p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq \frac{p}{2}$, $f(v_i) = pi - 2$ for $1 \leq i \leq \frac{p}{2}$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(G)$ are all distinct. The edge sets are

$$E_1 = \{u_i u_{i+1}/1 \leq i \leq \frac{p-2}{2}\},$$
$$E_2 = \{u_i v_i/1 \leq i \leq \frac{p}{2}\}.$$

Thus the resultant edges are distinct. Hence every comb graph G is a root quad mean graph. □

Theorem 3.2. *Every Ladder graph L_n is a root quad mean graph.*

Proof. Let L_n be a ladder graph. Let $V(L_n)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_n$ be the upper set of vertices, $v_1, v_2, \ldots, v_n$ be the lower set of vertices. Let edges be defined as

$$E(L_n) = \{u_i u_{i+1}/1 \leq i \leq n-1\} \cup \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{u_i v_i/1 \leq i \leq n\}.$$

Let p the number of vertices on the graph L_n. Here $V(L_n) = 2n = p$, $E(L_n) = 3n - 2$.

Now define a function $f : V(L_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq n$, $f(v_i) = pi - 2$ for $1 \leq i \leq n$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(L_n)$ are all distinct. The edge sets are:

$$E_1 = \{u_i u_{i+1}/1 \leq i \leq n-1\},$$
$$E_2 = \{v_i v_{i+1}/1 \leq i \leq n-1\},$$
$$E_3 = \{u_i v_i/1 \leq i \leq n\}.$$

Thus the resultant edges are distinct. Hence every ladder graph L_n is a root quad mean graph. □

Theorem 3.3. *Every Slanting ladder graph SL_n is a root quad mean graph.*

Proof. Let SL_n be a slanting ladder graph. Let $V(SL_n)$ be the vertices of the graph such that $u_1, u_2, \ldots, u_n$ be the upper set of vertices, $v_1, v_2, \ldots, v_n$ be the lower set of vertices. Let edges be defined as

$$E(SL_n) = \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\} \\ \cup \{u_iv_{i+1}/1 \leq i \leq n-1\}.$$

Let p the number of vertices on the graph SL_n. Here $V(SL_n) = 2n = p$, $E(SL_n) = 3(n-1)$. Now define a function $f : V(SL_n) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(u_i) = pi - 1$ for $1 \leq i \leq n$, $f(v_i) = pi - 2$ for $1 \leq i \leq n$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(SL_n)$ are all distinct. The edge sets are

$$E_1 = \{u_iu_{i+1}/1 \leq i \leq n-1\},$$
$$E_2 = \{v_iv_{i+1}/1 \leq i \leq n-1\},$$
$$E_3 = \{u_iv_{i+1}/1 \leq i \leq n-1\}.$$

Thus the resultant edges are distinct. Hence every slanting ladder graph SL_n is a root quad mean graph. □

Theorem 3.4. *Every Gear graph G_p is a root quad mean graph.*

Proof. Let G$_p$ be a gear graph. Let $V(G_p)$ be the vertices of the graph such that v_0 be the centre vertex and $v_1, v_2, \ldots, v_p$ be the vertices on the rim. Let edges be defined as

$$E(G_p) = \{v_iv_{i+1}/1 \leq i \leq p-1\} \cup \{v_pv_1\} \cup \{v_0v_{2i-1}/1 \leq i \leq \frac{p}{2}\}.$$

Here $V(G_p) = p + 1$, $E(G_p) = \frac{3p}{2}$. Now define a function $f : V(G_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(v_0) = p$, $f(v_i) = pi - 1$ for $1 \leq i \leq p$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(G_p)$ are all distinct. The edge sets are

$$E_1 = \{v_i v_{i+1}/1 \le i \le p-1\},$$
$$E_2 = \{v_p v_1\},$$
$$E_3 = \{v_0 v_{2i-1}/1 \le i \le \frac{p}{2}\}.$$

Thus the resultant edges are distinct. Hence every Gear graph G_p is a root quad mean graph. □

Theorem 3.5. *Every Helm graph H_p is a root quad mean graph.*

Proof. Let H_p be a helm graph. Let p the number of vertices on the rim. Let $V(H_p)$ be the vertices of the graph such that v_0 be the centre vertex and $v_1, v_2, \ldots, v_p$ be the vertices on the rim and $u_1, u_2, \ldots, u_p$ be the pendant vertices. Let edges be defined as

$$E(H_p) = \{v_i v_{i+1}/1 \le i \le p-1\} \cup \{v_p v_1\} \cup \{v_0 v_i/1 \le i \le p\} \\ \cup \{v_i u_i/1 \le i \le p\}.$$

Here $V(H_p) = 2p+1$, $E(H_p) = 3p$. Now define a function $f : V(H_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(v_0) = 2p$, $f(v_i) = pi - 1$ for $1 \le i \le p$, $f(u_i) = pi - 2$ for $1 \le i \le p$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(H_p)$ are all distinct. The edge sets are

$$E_1 = \{v_i v_{i+1}/1 \le i \le p-1\},$$
$$E_2 = \{v_p v_1\},$$
$$E_3 = \{v_0 v_i/1 \le i \le p\},$$
$$E_4 = \{v_i u_i/1 \le i \le p\}.$$

Thus the resultant edges are distinct. Hence every helm graph H_p is a root quad mean graph. □

Theorem 3.6. *Every Heger graph M_p is a root quad mean graph.*

Proof. Let M_p be a heger graph. Let $V(M_p)$ be the vertices of the graph such that v_0 be the centre vertex and $v_1, v_2, \ldots, v_p$ be the vertices on the rim and

$u_1, u_2, \ldots, u_{\frac{p}{2}}$ be the pendant vertices. Let edges be defined as

$$E(M_p) = \{v_i v_{i+1}/1 \leq i \leq p-1\} \cup \{v_p v_1\} \cup \{v_0 v_{2i-1}/1 \leq i \leq \frac{p}{2}\}$$
$$\cup \{v_{2i} u_i/1 \leq i \leq \frac{p}{2}\}.$$

Here $V(M_p) = \frac{3p+2}{2}$, $E(M_p) = 2p$. Now define a function $f : V(M_p) \to \{1, 2, \ldots, p^2 - 1\}$ by $f(v_0) = p$, $f(v_i) = pi - 1$ for $1 \leq i \leq p$, $f(u_i) = 2(pi-1)$ for $1 \leq i \leq \frac{p}{2}$. Then the induced edge labeling $f : E \to \mathbb{N}$ defined by $f(e_i) = \lfloor \frac{\sqrt{f(u)^4 + f(v)^4}}{2} \rfloor$, $e_i \in E(M_p)$ are all distinct. The edge sets are

$$E_1 = \{v_i v_{i+1}/1 \leq i \leq p-1\},$$
$$E_2 = \{v_p v_1\},$$
$$E_3 = \{v_0 v_{2i-1}/1 \leq i \leq \frac{p}{2}\},$$
$$E_4 = \{v_{2i} u_i/1 \leq i \leq \frac{p}{2}\}.$$

Thus the resultant edges are distinct. Hence every heger graph M_p is a root quad mean graph. □

REFERENCES

[1] J. A. GALLIAN: *A dynamic survey of labeling*, The Electronics Journal of Combinatorics, **17** (2014), 233-242.

[2] S. SOMASUNDARAM , R. PONRAJ: *Mean labeling of graphs*, Natl. Acad, Sci. Let., **26** (2003), 210-213.

[3] S. SANDHYA, S. SOMASUNDARAM, S. ANUSA: *Root Square Mean labeling of graphs*, International Journal of Contemporary Mathematical Sciences, **9**(14) (2014), 667-676.

[4] R. GOWRI, G. VEMBARASI: *Root Cube Mean labeling of graphs*, International Journal of Engineering Science, Advanced Computing and Bio-Technology, **8**(4) (2017), 248-255.

[5] P. SUMATHI , A. RATHI: *Quotient Labeling of Some Ladder Graphs*, American Journal of Engineering Research, E-ISSN: 2320-0847, p-ISSN:2320-0936.

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