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A contemporary postulates on resolvable sets and functions

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Abstract. In this article, a new class of sets namely neutrosophic resolvable sets in neutrosophic topological space have been introduced. We present the neutrosophic resolvable functions between neutrosophic topological spaces by neutrosophic resolvables sets. Also we examine the characteristics of neutrosophic resolvable sets and neutrosophic resolvable functions with the existing sets.

Keywords: Neutrosophic resolvable sets, Neutrosophic continuous functions, Neutrosophic contra continuous functions, Neutrosophic resolvable functions.

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1. Introduction

Zadeh presented the notion of fuzzy sets with membership functions in 1965 [15]. This concept is succesfully used to handle uncertainty in real life where each element has a membership function. In 1986, Attanassov proposed vague intuitionistic fuzzy sets which are characterised by the membership function and the non-member function. [2]. But in real world we have to handle the indeterminancy and incompleteness. For this purpose, Smarandache introduced the neutrosophic set theory to solve many practical problems in the real world [5]. Topology has been a vital part of research in recent years. In this context, Salama and Albowi succesfully applied the neutrosophic sets in the topological space called as neutrosophic topological space in 2012 [12].

The concept of resolvable sets in topological space was presented by Kuratowski in 1966 [6]. Maximilian Ganster gave the concept of pre-open sets and resolvable spaces in 1987 [7]. Resolvable spaces and irresolvable spaces were explored by Chandan Chakkopadhyay and Chhanda Bandyopadhyay in 1993 [4]. In 2017 neutrosophic resolvable, neutrosophic irresolvable, neutrosophic open hereditarily irresolvable spaces and maximally neutrosophic irresolvable spaces were studied by Caldas, Maximilian Ganster, Dhavaseelan, and Jafari through neutrosophic topological spaces [3]. In 2017, Thangaraj initiated to introduce the idea of resolvable sets and their functions in fuzzy topological spaces. Additionally, they discussed the conditions for fuzzy topological spaces to become a fuzzy Baire spaces obtained by fuzzy resolvable sets. [8]. Thangaraj and Lokeshwari studied irresolvable sets and open hereditarily irresolvable spaces in fuzzy topological spaces [9]. Also, they discussed resolvable sets and functions in fuzzy hyper-connected spaces [10]. In 2020, fuzzy resolvable functions were briefly inspected by Thangaraj and Senthil. [11]. These studies motivated us to ideate a new notion of sets namely neutrosophic resolvable sets in neutrosophic topological spaces. Also we define the new functions namely neutrosophic resolvable functions between neutrosophic topological spaces $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ and $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. Finally the properties of neutrosophic resolvable sets and functions are discussed by theorems and suitable examples.

Through out this paper, neutrosophic topological space [simply nts] is denoted by $\mathcal{N}(\mathcal{X}, \pi)$.

2. Preliminaries

Definition 2.1. [1] Let $\mathcal{X}$ be a nonempty fixed set. A neutrosophic set $\mathcal{P}$ is an object having the form $\mathcal{P} = \{< r, \mu_{\mathcal{P}}(r), \nu_{\mathcal{P}}(r), \omega_{\mathcal{P}}(r) >; r \in \mathcal{X}\}$ where $\mu_{\mathcal{P}}(r)$, $\nu_{\mathcal{P}}(r)$ and $\omega_{\mathcal{P}}(r)$ represent the degree of true, indeterminancy and false membership functions respectively of each element $r \in \mathcal{X}$ to the set $\mathcal{P}$.

Definition 2.2. [1] Let $\mathcal{P} = \{< r, \mu_{\mathcal{P}}(r), \nu_{\mathcal{P}}(r), \omega_{\mathcal{P}}(r) >, r \in \mathcal{X}\}$ be a neutrosophic sets, then the complement of $\mathcal{P}$ can be defined as by the following three kinds,
$C_1 : C[\mathcal{P}] = \{< r, 1_{\mathcal{N}} - \mu_{\mathcal{P}}(r), 1_{\mathcal{N}} - \nu_{\mathcal{P}}(r), 1_{\mathcal{N}} - \omega_{\mathcal{P}}(r) >; r \in \mathcal{X}\}$
$C_2 : C[\mathcal{P}] = \{< r, \omega_{\mathcal{P}}(r), \nu_{\mathcal{P}}(r), \mu_{\mathcal{P}}(r) >; r \in \mathcal{X}\}$
$C_3 : C[\mathcal{P}] = \{< r, \omega_{\mathcal{P}}(r), 1_{\mathcal{N}} - \nu_{\mathcal{P}}(r), \mu_{\mathcal{P}}(r) >; r \in \mathcal{X}\}$
Throughout this paper, the examples are derived using C_1.

proposition 2.3. *[1] For any neutrosophic set $\mathcal{P}$ in $\mathcal{N}(\mathcal{X}, \pi)$, the following conditions are hold*

(i) $0_{\mathcal{N}} \leq 0_{\mathcal{N}}, 0_{\mathcal{N}} \leq \mathcal{P}$
(ii) $1_{\mathcal{N}} \leq 1_{\mathcal{N}}, \mathcal{P} \leq 1_{\mathcal{N}}$

Definition 2.4. [12] Let $\mathcal{X}$ be a non empty set and the neutrosophic sets $\mathcal{P}$ and $\mathcal{Q}$ in the form $\mathcal{P} = \{< r, \mu_{\mathcal{P}}(r), \nu_{\mathcal{P}}(r), \omega_{\mathcal{P}}(r) >, r \in \mathcal{X}\}$ and $\mathcal{Q} = \{< r, \mu_{\mathcal{Q}}(r), \nu_{\mathcal{Q}}(r), \omega_{\mathcal{Q}}(r) >; r \in \mathcal{X}\}$, then $\mathcal{P}$ is the subset of $\mathcal{Q}$ i.e., $\mathcal{P} \subseteq \mathcal{Q}$ is defined by the following two ways:

(i) $\mathcal{P} \leq \mathcal{Q} \Leftrightarrow \mu_{\mathcal{P}}(r) \leq \mu_{\mathcal{Q}}(r), \nu_{\mathcal{P}}(r) \leq \nu_{\mathcal{Q}}(r), \omega_{\mathcal{P}}(r) \geq \omega_{\mathcal{Q}}(r); \forall r \in \mathcal{X}$
(ii) $\mathcal{P} \leq \mathcal{Q} \Leftrightarrow \mu_{\mathcal{P}}(r) \leq \mu_{\mathcal{Q}}(r), \nu_{\mathcal{P}}(r) \geq \nu_{\mathcal{Q}}(r), \omega_{\mathcal{P}}(r) \geq \omega_{\mathcal{Q}}(r); \forall r \in \mathcal{X}$

Definition 2.5. [12] Let $\mathcal{X}$ be a non-empty set and π be a collection of all neutrosophic subsets of $\mathcal{X}$. Then π is said to be neutrosophic topology on $\mathcal{X}$ if the following conditions are hold.

(i) $0_{\mathcal{N}}$, $1_{\mathcal{N}} \in \pi$

(ii) $\bigcup \mathcal{P}_i \in \pi$, $\forall\{\mathcal{P}_i; i \in \pi\} \leq \pi$

(iii) $\mathcal{P}_1 \cap \mathcal{P}_2 \in \pi$, for any $\mathcal{P}_1$, $\mathcal{P}_2 \in \pi$

Then the pair $\mathcal{N}(\mathcal{X}, \pi)$ is called as neutrosophic topological space. The elements of $\mathcal{N}(\mathcal{X}, \pi)$ are called neutrosophic open sets. A neutrosophic set is said to be neutrosophic closed if its complement is neutrosophic open.

Definition 2.6. [1] Let $\mathcal{X}$ be a *nts* and $\mathcal{P} = \{< r, \mu_{\mathcal{P}}(r), \nu_{\mathcal{P}}(r), \omega_{\mathcal{P}}(r) >; r \in \mathcal{X}\}$ be a neutrosophic set in $\mathcal{X}$. Then the neutrosophic interior and neutrosophic closure of $\mathcal{P}$ are defined as,

$$\mathcal{N}int[\mathcal{P}] = \bigcup\{H : H \text{ is a neutrosophic open set in } \mathcal{X} \text{ and } H \leq \mathcal{P}\}$$
$$\mathcal{N}cl[\mathcal{P}] = \bigcap\{K : K \text{ is a neutrosophic closed set in } \mathcal{X} \text{ and } \mathcal{P} \leq K\}$$

It follows that,

(i) $\mathcal{P}$ is a neutrosophic closed set if and only if $\mathcal{N}cl[\mathcal{P}] = \mathcal{P}$

(ii) $\mathcal{P}$ is a neutrosophic open set if and only if $\mathcal{N}int[\mathcal{P}] = \mathcal{P}$

Definition 2.7. [1] A neutrosophic subset $\mathcal{P}$ in a *nts* $\mathcal{N}(\mathcal{X}, \pi)$ is called as

(i) neutrosophic semi open set if $\mathcal{P} \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{P}]]$

(ii) neutrosophic pre open set if $\mathcal{P} \leq \mathcal{N}int[\mathcal{N}cl[\mathcal{P}]]$

(iii) neutrosophic semi pre open set if $\mathcal{P} \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}cl[\mathcal{P}]]]$

(iv) neutrosophic regular open set if $\mathcal{P} = \mathcal{N}int\mathcal{N}cl[\mathcal{P}]$

Definition 2.8. [5] A neutrosophic subset $\mathcal{P}$ in a *nts* $\mathcal{N}(\mathcal{X}, \pi)$ is called

(i) neutrosophic dense if $\mathcal{N}cl[\mathcal{P}] = 1_{\mathcal{N}}$

(ii) neutrosophic nowhere dense $\mathcal{N}int\mathcal{N}cl[\mathcal{P}] = 0_{\mathcal{N}}$

(iii) neutrosophic somewhere dense if $\mathcal{N}int[\mathcal{N}cl[\mathcal{P}]] \neq 0_{\mathcal{N}}$

Definition 2.9. [5] Let $\mathcal{N}(\mathcal{X}, \pi)$ be a *nts* and $\mathcal{P}_1$, $\mathcal{P}_2$ are neutrosophic subsets in $\mathcal{X}$, then the following conditions are hold.

(i) $\mathcal{N}int[\mathcal{P}_1] \leq \mathcal{P}_1$

(ii) $\mathcal{N}cl[\mathcal{P}_1] \geq \mathcal{P}_1$

(iii) $\mathcal{P}_1 \leq \mathcal{P}_2 \implies \mathcal{N}int[\mathcal{P}_1] \leq \mathcal{N}int[\mathcal{P}_2]$

(iv) $\mathcal{P}_1 \leq \mathcal{P}_2 \implies \mathcal{N}cl[\mathcal{P}_1] \leq \mathcal{N}cl[\mathcal{P}_2]$

(v) $\mathcal{N}int[\mathcal{N}int[\mathcal{P}_1]] = \mathcal{N}int[\mathcal{P}_1]$

(vi) $\mathcal{N}cl[\mathcal{N}cl[\mathcal{P}_1]] = \mathcal{N}cl[\mathcal{P}_1]$

(vii) $\mathcal{N}int[\mathcal{P}_1 \wedge \mathcal{P}_2] = \mathcal{N}int[\mathcal{P}_1] \wedge \mathcal{N}int[\mathcal{P}_2]$

(viii) $\mathcal{N}cl[\mathcal{P}_1 \vee \mathcal{P}_2] = \mathcal{N}cl[\mathcal{P}_1] \vee \mathcal{N}cl[\mathcal{P}_2]$

(ix) $\mathcal{N}int[0_\mathcal{N}] = 0_\mathcal{N}$

(x) $\mathcal{N}int[1_\mathcal{N}] = 1_\mathcal{N}$

(xi) $\mathcal{N}cl[0_\mathcal{N}] = 0_\mathcal{N}$

(xii) $\mathcal{N}cl[1_\mathcal{N}] = 1_\mathcal{N}$

(xiii) $\mathcal{P}_1 \leq \mathcal{P}_2 \implies C[\mathcal{P}_2] \leq C[\mathcal{P}_1]$

(xiv) $\mathcal{N}cl[\mathcal{P}_1 \wedge \mathcal{P}_2] \leq \mathcal{N}cl[\mathcal{P}_1] \wedge \mathcal{N}cl[\mathcal{P}_2]$

(xv) $\mathcal{N}int[\mathcal{P}_1 \wedge \mathcal{P}_2] \geq \mathcal{N}int[\mathcal{P}_1] \wedge \mathcal{N}int[\mathcal{P}_2]$

Definition 2.10. [14] Let $\mathcal{N}(\mathcal{X}, \pi_\mathcal{X})$ and $\mathcal{N}(\mathcal{Y}, \pi_\mathcal{Y})$ be any two neutrosophic topological space. Then the function $f : [\mathcal{X}, \pi_\mathcal{X}] \to [\mathcal{Y}, \pi_\mathcal{Y}]$ is called a

(i) Neutrosophic continuous function if $f^{-1}[\mathcal{P}]$ is neutrosophic open in $\mathcal{N}(\mathcal{X}, \pi_\mathcal{X})$, for each neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_\mathcal{Y})$

(ii) Neutrosophic contra continuous function if $f^{-1}[\mathcal{P}]$ is neutrosophic closed in $\mathcal{N}(\mathcal{X}, \pi_\mathcal{X})$, for each neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_\mathcal{Y})$.

3. Neutrosophic resolvable sets

Definition 3.1. A neutrosophic set $\mathcal{P}$ is said to be neutrosophic resolvable set in neutrosophic topological space $\mathcal{N}(\mathcal{X}, \pi)$, if $\{\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]\}$ is neutrosophic nowhere dense in $\mathcal{N}(\mathcal{X}, \pi)$ for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi)$. i.e.,$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]\} = 0_\mathcal{N}$, where $C[\mathcal{Q}] \in \pi$.

Example 3.2. Consider $\mathcal{X} = \{\mu, \nu\}$ and the neutrosophic sets $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ in $\mathcal{X}$ as follows

$\mathcal{P}_1$=$\{< \mu, 0.4, 0.3, 0.6 >, < \nu, 0.5, 0.3, 0.2 >; \mu, \nu \in \mathcal{X}\}$

$\mathcal{P}_2$=$\{< \mu, 0.3, 0.4, 0.3 >, < \nu, 0.6, 0.5, 0.2 >; \mu, \nu \in \mathcal{X}\}$

$\mathcal{P}_3$=$\{< \mu, 0.4, 0.4, 0.3 >, < \nu, 0.6, 0.5, 0.2 >; \mu, \nu \in \mathcal{X}\}$

$\mathcal{P}_4$=$\{< \mu, 0.3, 0.3, 0.6 >, < \nu, 0.5, 0.3, 0.2 >; \mu, \nu \in \mathcal{X}\}$ Then $\pi = \{0_\mathcal{N}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_4, 1_\mathcal{N}\}$ is a nts. Now, $\pi^C = \{0_\mathcal{N}, C[\mathcal{P}_1], C[\mathcal{P}_2], C[\mathcal{P}_3], C[\mathcal{P}_4], 1_\mathcal{N}\}$

Let A=$\{< \mu, 0.2, 0.1, 0.7 >, < \nu, 0.3, 0.1, 0.3 >; \mu, \nu \in \mathcal{X}\}$. Now,

$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[\mathcal{P}_1] \wedge A] \wedge \mathcal{N}cl[C[\mathcal{P}_1] \wedge C[A]]\} = 0_\mathcal{N}$

$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[\mathcal{P}_2] \wedge A] \wedge \mathcal{N}cl[C[\mathcal{P}_2] \wedge C[A]]\} = 0_\mathcal{N}$

$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[\mathcal{P}_3] \wedge A] \wedge \mathcal{N}cl[C[\mathcal{P}_3] \wedge C[A]]\} = 0_\mathcal{N}$

$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[\mathcal{P}_4] \wedge A] \wedge \mathcal{N}cl[C[\mathcal{P}_4] \wedge C[A]]\} = 0_\mathcal{N}$.

Therefore A is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$.

Example 3.3. Consider $\mathcal{X} = \{\mu\}$ and consider the neutrosophic sets $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{Q}_3$ and R in $\mathcal{X}$ as follows:
$\mathcal{Q}_1 = \{< \mu, 0.4, 0.5, 0.6 >, \mu \in \mathcal{X}\}$
$\mathcal{Q}_2 = \{< \mu, 0.3, 0.4, 0.8 >, \mu \in \mathcal{X}\}$
$\mathcal{Q}_3 = \{< \mu, 0.4, 0.5, 0.8 >, \mu \in \mathcal{X}\}$
Then $[\mathcal{X}, \pi] = \{0_{\mathcal{N}}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, 1_{\mathcal{N}}\}$ is a nts. Here $\pi^C = \{0_{\mathcal{N}}, C[\mathcal{Q}_1], C[\mathcal{Q}_2], C[\mathcal{Q}_3], 1_{\mathcal{N}}\}$. Take $R = \{< \mu, 0.6, 0.6, 0.4 >, \mu \in x\}$. Now,
$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[\mathcal{Q}_1] \wedge R] \wedge \mathcal{N}cl[C[\mathcal{Q}_1] \wedge C[R]]] = \mathcal{Q}_1 \neq 0_{\mathcal{N}}$
$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[\mathcal{Q}_2] \wedge R] \wedge \mathcal{N}cl[C[\mathcal{Q}_2] \wedge C[R]]] = \mathcal{Q}_1 \neq 0_{\mathcal{N}}$
$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[\mathcal{Q}_3] \wedge R] \wedge \mathcal{N}cl[C[\mathcal{Q}_3] \wedge C[R]]] = 0_{\mathcal{N}}$
This implies R is not a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$.

Remark 3.4. In a nts $\mathcal{N}(\mathcal{X}, \pi)$, every neutrosophic resolvable set need not be a neutrosophic open set. In example 3.2, A is a neutrosophic resolvable set but not neutrosophic open set.

proposition 3.5. *In a nts $\mathcal{N}(\mathcal{X}, \pi)$, if $\mathcal{P}$ is a neutrosophic resolvable set, then $\mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$, then for each neutrosophic closed set $\mathcal{Q}$, we have $\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]]\} = 0_{\mathcal{N}}$, Using 3.1, $\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \geq \mathcal{N}cl[[\mathcal{Q} \wedge \mathcal{P}] \wedge [\mathcal{Q} \wedge C[\mathcal{P}]] \geq \mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P} \wedge C[\mathcal{P}]]$. Now, $\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]\} \geq \mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P} \wedge C[\mathcal{P}]]$. This implies $0_{\mathcal{N}} \geq \mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P} \wedge C[\mathcal{P}]]$. Hence $\mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P} \wedge C[\mathcal{P}]] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi)$. □

proposition 3.6. *In a nts $\mathcal{N}(\mathcal{X}, \pi)$, if $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$, then $\mathcal{N}cl[\mathcal{P} \vee C[\mathcal{P}] \vee R] = 1_{\mathcal{N}}$ for each neutrosophic open set R in $\mathcal{N}(\mathcal{X}, \pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$ using proposition 3.5, we have $\mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi)$. Then $C[\mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}]] = 1_{\mathcal{N}}$. We know that $\mathcal{N}cl\mathcal{N}int[C[\mathcal{P}] \vee \mathcal{P} \vee C[\mathcal{Q}]] \leq \mathcal{N}cl[C[\mathcal{P}] \vee \mathcal{P} \vee C[\mathcal{Q}]]$. This implies $1_{\mathcal{N}} \leq \mathcal{N}cl[C[\mathcal{P}]] \vee [\mathcal{P}] \vee C[\mathcal{Q}]$ and put $C[\mathcal{Q}] = R$. Then we have $\mathcal{N}cl[\mathcal{P} \vee C[\mathcal{P}] \vee R] = 1_{\mathcal{N}}$ for each neutrosophic open set R in $\mathcal{N}(\mathcal{X}, \pi)$. □

proposition 3.7. *Let $\mathcal{N}(\mathcal{X}, \pi)$ be a nts, then the neutrosophic interior of a neutrosophic closed set is neutrosophic regular open.*

Proof. Let $\mathcal{Q}$ be a neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \pi)$ and take $R = \mathcal{N}int\mathcal{Q}$. Therefore $R \leq \mathcal{Q}$. Since $\mathcal{Q}$ is closed set in $\mathcal{N}(\mathcal{X}, \pi)$. $\mathcal{N}cl[R] \leq \mathcal{N}cl[\mathcal{Q}] = \mathcal{Q}$. This implies $\mathcal{N}int\mathcal{N}cl[R] \leq$

$\mathcal{N}int[\mathcal{Q}] = R$. We have $R \leq \mathcal{N}cl[R]$. Now $\mathcal{N}int[R] \leq \mathcal{N}int\mathcal{N}cl[R]$. This implies $R \leq \mathcal{N}int\mathcal{N}cl[R]$ gives $\mathcal{N}int\mathcal{N}cl[R] = R$. Therefore R is a neutrosophic regular open set. □

proposition 3.8. *In a nts$\mathcal{N}(\mathcal{X},\pi)$, if $\mathcal{P}$ is a neutrosophic resolvable set, then there exists a neutrosophic regular open set R in $\mathcal{N}(\mathcal{X},\pi)$ such that $R \leq \mathcal{N}cl[\mathcal{P} \vee C[\mathcal{P}]]$.*

Proof. Let $\mathcal{P}$ be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$. Using proposition 3.5, we have $\mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ for each closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X},\pi)$.$\mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] \leq \mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}} \implies \mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi) \implies \mathcal{N}int[\mathcal{P}\wedge C[\mathcal{P}]] \wedge \mathcal{N}int[\mathcal{Q}] = 0_{\mathcal{N}} \implies \mathcal{N}int\mathcal{Q} \leq C[\mathcal{N}int[\mathcal{P}\wedge C[\mathcal{P}]]] = \mathcal{N}cl[C[\mathcal{P}]\vee\mathcal{P}]$ in $\mathcal{N}(\mathcal{X},\pi)$. Since $\mathcal{Q}$ is a neutrosophic closed set. Using Proposition 3.7 , $\mathcal{N}int\mathcal{Q}$ is a neutrosophic regular open in $\mathcal{N}(\mathcal{X},\pi)$. Put $\mathcal{N}int\mathcal{Q} = R$. Hence if $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$, there exist a neutrosophic regular open set R in $\mathcal{N}(\mathcal{X},\pi)$ such that $R \leq \mathcal{N}cl[\mathcal{P} \vee C[\mathcal{P}]]$. □

proposition 3.9. *In a nts$\mathcal{N}(\mathcal{X},\pi)$ if $\mathcal{P}$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$, then $C[\mathcal{P}]$ is also a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$ then $\mathcal{N}int\mathcal{N}cl\{[\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]]\} = 0_{\mathcal{N}}$ for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X},\pi)$. For the set $C[\mathcal{P}]$, $\mathcal{N}int\mathcal{N}cl\{[\mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[C[\mathcal{P}]]]]\} = 0_{\mathcal{N}} \implies \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \wedge \mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}]] = 0_{\mathcal{N}}$. Hence $C[\mathcal{P}]$ is also a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$. □

proposition 3.10. *In a nts $\mathcal{N}(\mathcal{X},\pi)$, if $\mathcal{P}$ is a neutrosophic closed set with $\mathcal{N}int[\mathcal{P}] = 0_{\mathcal{N}}$, then $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic closed set and $\mathcal{N}int[\mathcal{P}] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi)$. For a neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X},\pi)$, we have $\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \leq \mathcal{N}cl[\mathcal{P}] \wedge \mathcal{N}cl[C[\mathcal{P}]] \leq \mathcal{P}$. Since $\mathcal{P}$ is a neutrosophic closed set in $\mathcal{N}(\mathcal{X},\pi)$. Thus $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]] \leq \mathcal{N}int\mathcal{N}cl[\mathcal{P}] = \mathcal{N}int[\mathcal{P}] = 0_{\mathcal{N}}$. Hence $\mathcal{P}$ is a resolvable set in $\mathcal{N}(\mathcal{X},\pi)$. □

proposition 3.11. *Let $\mathcal{N}(\mathcal{X},\pi)$ be the nts. If $\mathcal{P}$ is a neutrosophic open set and neutrosophic dense in $\mathcal{N}(\mathcal{X},\pi)$, then $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic open set and neutrosophic dense in $\mathcal{N}(\mathcal{X},\pi)$. Then $C[\mathcal{P}]$ is neutrosophic closed and $\mathcal{N}cl[\mathcal{P}] = 1_{\mathcal{N}}$. For a neutrosophic closed set $\mathcal{Q}$, we have $\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \leq \mathcal{N}cl[\mathcal{Q}] \wedge \mathcal{N}cl[\mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q}] \wedge \mathcal{N}cl[C[\mathcal{P}]] \implies \mathcal{Q} \wedge C[\mathcal{P}]$. Since $\mathcal{N}cl[\mathcal{P}] = 1_{\mathcal{N}}$, $\mathcal{Q}$ and $C[\mathcal{P}]$ are neutrosophic closed set in $\mathcal{N}(\mathcal{X},\pi)$. This implies $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge$

$\mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]] \leq \mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]] \leq \mathcal{N}int[\mathcal{N}cl[\mathcal{Q}] \wedge \mathcal{N}cl[C[\mathcal{P}]]]$. By computation, we have $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[\mathcal{Q} \wedge \mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[\mathcal{P}]]] = 0_{\mathcal{N}}$. Hence $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$. □

proposition 3.12. *If $\mathcal{P}$ is a neutrosophic open and neutrosophic dense in a nts $\mathcal{N}(\mathcal{X}, \pi)$, then $C[\mathcal{P}]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic open set and neutrosophic dense set in $\mathcal{N}(\mathcal{X}, \pi)$. This implies $C[\mathcal{P}]$ is a neutrosophic closed set and $\mathcal{N}cl[\mathcal{P}] = 1_{\mathcal{N}}$. Then $C[\mathcal{N}cl[\mathcal{P}]] = C[1_{\mathcal{N}}] \implies \mathcal{N}int[C[\mathcal{P}]] = 0_{\mathcal{N}}$ Using Proposition 3.10, $C[\mathcal{P}]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$. □

proposition 3.13. *Let $\mathcal{N}(\mathcal{X}, \pi)$ be a nts. If $\mathcal{P}$ is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$, then $\mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}]] \leq \mathcal{N}cl[C[\mathcal{Q}]]$ for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi)$.*

Proof. Let $\mathcal{P}$ be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$. Using Proposition 3.5 $\mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi)$. We know that, $\mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] \leq \mathcal{N}int\mathcal{N}cl[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{Q}] = 0_{\mathcal{N}}$ $\implies \mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}] \wedge \mathcal{N}int[\mathcal{Q}]] = 0_{\mathcal{N}} \implies \mathcal{N}int[\mathcal{P} \wedge C[\mathcal{P}]] \leq C[\mathcal{N}int\mathcal{Q}] = \mathcal{N}cl[C[\mathcal{Q}]]$ in $\mathcal{N}(\mathcal{X}, \pi)$. □

4. **Neutrosophic resolvable functions**

Definition 4.1. Let $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ and $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ be any two neutrosophic topological spaces. A function $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ is called as neutrosophic resolvable functions if $R^{-1}[\mathcal{P}]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ for each neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$.

Example 4.2. Let $\mathcal{X} = \{\mu, \nu, \omega\}$ and consider the neutrosophic sets as follows,
$U_1 = \{< \mu, 0.6, 0.7, 0.2 >, < \nu, 0.7, 0.8, 0.2 >, < \omega, 1.0, 0.6, 0.3 >; \mu, \nu, \omega \in \mathcal{X}\}$
$U_2 = \{< \mu, 0.5, 0.6, 0.3 >, < \nu, 0.8, 0.7, 0.4 >, < \omega, 1.0, 0.8, 0.4 >; \mu, \nu, \omega \in \mathcal{X}\}$
$U_3 = \{< \mu, 0.5, 0.6, 0.3 >, < \nu, 0.7, 0.7, 0.4 >, < \omega, 1.0, 0.6, 0.4 >; \mu, \nu, \omega \in \mathcal{X}\}$
$U_4 = \{< \mu, 0.6, 0.7, 0.2 >, < \nu, 0.8, 0.8, 0.2 >, < \omega, 1.0, 0.8, 0.3 >; \mu, \nu, \omega \in \mathcal{X}\}$
$S = \{< \mu, 0.3, 0.2, 0.8 >, < \nu, 0.1, 0.2, 0.9 >, < \omega, 0.0, 0.3, 0.8 >; \mu, \nu, \omega \in \mathcal{X}\}$
$T = \{< \mu, 0.0, 0.3, 0.8 >, < \nu, 0.3, 0.2, 0.8 >, < \omega, 0.1, 0.2, 0.4 >; \mu, \nu, \omega \in \mathcal{X}\}$
Then $\pi_{\mathcal{X}} = \{0_{\mathcal{N}}, U_1, U_2, U_3, U_4, 1_{\mathcal{N}}\}$, $\pi_{\mathcal{Y}} = \{0_{\mathcal{N}}, T, 1_{\mathcal{N}}\}$ are the nts. Therefore
$\pi_{\mathcal{X}}^C = \{0_{\mathcal{N}}, C[U_1], C[U_2], C[U_3], C[U_4], 1_{\mathcal{N}}\}$, $\pi_{\mathcal{Y}}^C = \{0_{\mathcal{N}}, C[T], 1_{\mathcal{N}}\}$. Now,
$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[U_1] \wedge S] \wedge \mathcal{N}cl[C[U_1] \wedge C[S]]] = 0_{\mathcal{N}}$, $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[U_2] \wedge S] \wedge \mathcal{N}cl[C[U_2] \wedge C[S]]] = 0_{\mathcal{N}}$, $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[U_3] \wedge S] \wedge \mathcal{N}cl[C[U_3] \wedge C[S]]] = 0_{\mathcal{N}}$, $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[U_4] \wedge S] \wedge \mathcal{N}cl[C[U_4] \wedge C[S]]] = 0_{\mathcal{N}}$.
This implies S is a neutrosophic resolvable set. Now we define a function $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to$

$\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$ by $R(\mu)=\nu$, $R(\nu)=\omega$ and $R(\omega)=\mu$. By computation $R^{-1}[T]=S$, for each neutrosophic open set T in $\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$. Thus, S is neutrosophic resolvable set. Hence R is a neutrosophic resolvable function.

Example 4.3. Consider the set $\mathcal{X}=\{\mu,\nu,\omega\}$ and the neutrosophic sets V_1, V_2, V_3, V_4 and V_5 are defined in $\mathcal{X}$ as follows:-

$$V_1 = \{<\mu, 0.0, 0.0, 0.5>, <\nu, 0.1, 0.1, 0.6>, <\omega, 0.2, 0.1, 0.5>, \mu,\nu,\omega \in \mathcal{X}\}$$
$$V_2 = \{<\mu, 1.0, 0.9, 0.0>, <\nu, 0.9, 0.8, 0.1>, <\omega, 0.8, 0.7, 0.5>, \mu,\nu,\omega \in \mathcal{X}\}$$
$$V_3 = \{<\mu, 0.0, 0.0, 0.4>, <\nu, 0.1, 0.1, 0.5>, <\omega, 0.1, 0.1, 0.4>, \mu,\nu,\omega \in \mathcal{X}\}$$
$$V_4 = \{<\mu, 0.2, 0.2, 0.4>, <\nu, 0.3, 0.3, 0.5>, <\omega, 0.3, 0.2, 0.5>, \mu,\nu,\omega \in \mathcal{X}\}$$
$$V_5 = \{<\mu, 0.3, 0.2, 0.5>, <\nu, 0.2, 0.2, 0.5>, <\omega, 0.3, 0.3, 0.5>, \mu,\nu,\omega \in \mathcal{X}\}$$

Then $\pi_{\mathcal{X}}=\{0_{\mathcal{N}},V_1,V_2,1_{\mathcal{N}}\}$, $\pi_{\mathcal{Y}}=\{0_{\mathcal{N}},V_5,1_{\mathcal{N}}\}$ are two neutrosophic topological spaces. Now we define a function $R:\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$ by $R(\mu)=\nu$, $R(\nu)=\mu$ and $R(\omega)=\mu$. Now $\implies \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_1]\wedge V_3]\wedge\mathcal{N}cl[C[V_1]\wedge C[V_3]]]=0_{\mathcal{N}} \implies \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_2]\wedge V_3]\wedge \mathcal{N}cl[C[V_2]\wedge C[V_3]]]=0_{\mathcal{N}}$. This implies V_3 is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. But $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_1]\wedge V_4]]\wedge\mathcal{N}cl[C[V_1]\wedge C[V_4]]=V_1\neq 0_{\mathcal{N}}$. Therefore V_4 is not a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. By computation, we have $R^{-1}[V_5]=V_4\neq V_3$ for a non empty neutrosophic open set V_5 in $\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$. Therefore the function $R:\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$ is not a neutrosophic resolvable function. Since V_4 is not a neutrosophic resolvable set.

proposition 4.4. *If a function $R:\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$ is a neutrosophic resolvable function, then for any neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$*

(a) *$\mathcal{N}int\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]=0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi)$ for each neutrosophic closed set $\mathcal{Q}$, where $C[\mathcal{Q}]\in\pi_{\mathcal{X}}$.*

(b) *For the neutrosophic closed set $\mathcal{Q}$, $\mathcal{N}int[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]=0_{\mathcal{N}}$.*

Proof. (a) Let $R:\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$ be the neutrosophic resolvable function. Then for the neutrosophic open set $0_{\mathcal{N}}\neq\mathcal{P}$ in $\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$, there exist the neutrosophic resolvable set $R^{-1}[\mathcal{P}]$ in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. Using the definition of resolvable set, we have $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}]]\wedge \mathcal{N}cl[\mathcal{Q}\wedge C[R^{-1}[\mathcal{P}]]]]=0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. $\mathcal{N}cl[[\mathcal{Q}\wedge R^{-1}[\mathcal{P}]]\wedge[\mathcal{Q}\wedge C[R^{-1}[\mathcal{P}]]]]\leq\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}]]\wedge\mathcal{N}cl[\mathcal{Q}\wedge C[R^{-1}[\mathcal{P}]]] \implies \mathcal{N}int\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}]\wedge C[R^{-1}[\mathcal{P}]]]=0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. Therefore $\mathcal{Q}\wedge R^{-1}[\mathcal{P}]\wedge C[R^{-1}[\mathcal{P}]]$ is the neutrosophic nowhere dense set in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$.
(b) Using (a), we have $\mathcal{N}int\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]=0_{\mathcal{N}}$, in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$ for the neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y},\pi_{\mathcal{Y}})$. Now, $\mathcal{N}int[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]\leq\mathcal{N}int\mathcal{N}cl[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]=0_{\mathcal{N}}$ $\implies \mathcal{N}int[\mathcal{Q}\wedge R^{-1}[\mathcal{P}\wedge C[\mathcal{P}]]]=0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X},\pi_{\mathcal{X}})$. □

proposition 4.5. *If a function* $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ *is the neutrosophic resolvable function then* $\mathcal{N}int[R^{-1}[\mathcal{P} \wedge C[\mathcal{P}]]] \leq \mathcal{N}cl[C[\mathcal{Q}]]$ *for the neutrosophic open set* $\mathcal{P}$ *in* $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$*, and* $\mathcal{Q}$ *is the neutrosophic closed set in* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$*.*

Proof. Let us take a neutrosophic resolvable function $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. Then for any neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ using the proposition 4.4 (b), we have $\mathcal{N}int[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = 0_{\mathcal{N}}$, here $\mathcal{Q}$ is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Now, $\mathcal{N}int[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = \mathcal{N}int[\mathcal{Q}] \wedge \mathcal{N}int[R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = 0_{\mathcal{N}} \implies \mathcal{N}int[R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] \leq C[\mathcal{N}int[\mathcal{Q}]] = \mathcal{N}cl[C[\mathcal{Q}]]$. □

proposition 4.6. *If a function* $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ *is the neutrosophic resolvable function from the neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ *to nts* $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$*, then for any neutrosophic open set* $\mathcal{P}$ *in* $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$*.*

(a) *there exist a regular open set* S *in* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ *such that* $\mathcal{N}cl[R^{-1}[\mathcal{P} \vee C[\mathcal{P}]]] \geq S$*.*
(b) $\mathcal{N}int\mathcal{N}cl[\mathcal{P} \vee C[\mathcal{P}]] \neq 0_{\mathcal{N}}$ *in* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$*.*

Proof. (a) Let R be a neutrosophic resolvable function from *nts* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ into *nts* $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. Then for any neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$, using the Proposition 4.4 (b), we have $\mathcal{N}int[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$, where $\mathcal{Q}$ is the neutrosophic closed set. This implies $\mathcal{N}int[\mathcal{Q}] \wedge \mathcal{N}int[R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = 0_{\mathcal{N}} \implies \mathcal{N}int[\mathcal{Q}] \leq C[\mathcal{N}int[R^{-1}[\mathcal{P}]] \wedge C[R^{-1}[\mathcal{P}]]] = \mathcal{N}cl[C[R^{-1}[\mathcal{P}]] \vee R^{-1}[\mathcal{P}]]$. Since $\mathcal{Q}$ is the neutrosophic closed set. Using 3.7, $\mathcal{N}int[\mathcal{Q}]$ is the neutrosophic regular open set in $\mathcal{N}(\mathcal{X}, \pi)$. Put $R = \mathcal{N}int[\mathcal{Q}]$. Then $\mathcal{N}cl[R^{-1}[C[\mathcal{P}] \wedge \mathcal{P}]] \geq R$, for a neutrosophic regular open set R in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$.
(b) Since every neutrosophic regular open set is neutrosophic open in a *nts* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Therefore the neutrosophic regular open set S is neutrosophic open in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Using (a) $S \leq \mathcal{N}cl[R^{-1}[[C[\mathcal{P}]] \vee [\mathcal{P}]]] \neq 0_{\mathcal{N}} \implies \mathcal{N}intS = S \leq \mathcal{N}int\mathcal{N}cl[R^{-1}[C[\mathcal{P}] \vee [\mathcal{P}]]] \neq 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. □

proposition 4.7. *Let* $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ *be a neutrosophic resolvable function, then for any neutrosophic open set* $\mathcal{P}$ *in* $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$*, there exists a neutrosophic regular closed set* R *in* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ *such that* $S \geq \mathcal{N}int[R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]]$*.*

Proof. Consider $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ be a neutrosophic resolvable function. Using the proposition 4.5, for any neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. $\mathcal{N}cl[C[\mathcal{Q}]] \geq \mathcal{N}int[R^{-1}[\mathcal{P} \wedge C[\mathcal{P}]]]$, here $\mathcal{Q}$ is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Therefore $C[\mathcal{Q}]$ is the neutrosophic open set. Put $S = C[\mathcal{Q}]$. This implies, $\mathcal{N}cl[S]$ is the neutrosophic regular closed set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Hence $S \geq \mathcal{N}int[R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]]$. □

proposition 4.8. *If $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ is the neutrosophic resolvable function, then $\mathcal{N}cl[R^{-1}[\mathcal{P} \vee C[\mathcal{P}]] \vee S] = 1_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ for the neutrosophic open set $\mathcal{P}$ in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ and $S \in \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ be the neutrosophic resolvable function. By Proposition 4.5 $\mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]] = 0_{\mathcal{N}}$ for the neutrosophic open set $\mathcal{P}$ and neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Then $C[\mathcal{N}int\mathcal{N}cl[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge C[R^{-1}[\mathcal{P}]]]] = 1_{\mathcal{N}}$ $\Longrightarrow \mathcal{N}cl\mathcal{N}int[C[\mathcal{Q}] \vee C[R^{-1}[\mathcal{P}]] \vee R^{-1}[\mathcal{P}]] = 1_{\mathcal{N}}$. Since $C[C[\mathcal{P}]] = \mathcal{P}$. Now, $\mathcal{N}cl\mathcal{N}int[C[\mathcal{Q}] \vee C[R^{-1}[\mathcal{P}]] \vee R^{-1}[\mathcal{P}]] \leq \mathcal{N}cl[C[\mathcal{Q}] \vee C[R^{-1}[\mathcal{P}]] \vee R^{-1}[\mathcal{P}]] \Longrightarrow \mathcal{N}cl[C[\mathcal{Q}] \vee R^{-1}[\mathcal{P} \vee C[\mathcal{P}]]] = 1_{\mathcal{N}}$. Put $S = C[\mathcal{Q}]$ then we have $\mathcal{N}cl[S \vee R^{-1}[\mathcal{P} \vee C[\mathcal{P}]]] = 1_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ where S is the neutrosophic open set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. □

proposition 4.9. *If the function $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ is the neutrosophic resolvable function and $\mathcal{P}$ is the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$, then $C[R^{-1}[\mathcal{P}]]$ is also neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$.*

Proof. Let $\mathcal{P}$ be a neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ and R be a neutrosophic resolvable function from a *nts* $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ into a $nts[\mathcal{Y}, \pi_{\mathcal{Y}}]$. This implies $R^{-1}[\mathcal{P}]$ is the neutrosophic resolvable set $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$, using Proposition 3.9 $C[R^{-1}[\mathcal{P}]]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. □

proposition 4.10. *Let $R_1 : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ be the neutrosophic resolvable function and $R_2 : \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}}) \to \mathcal{N}(\mathcal{Z}, \pi_{\mathcal{Z}})$ be the neutrosophic continuous function, then $R_2 \circ R_1 : [\mathcal{X}, \pi_{\mathcal{X}}] \to [\mathcal{Z}, \pi_{\mathcal{Z}}]$ is the neutrosophic resolvable function.*

Proof. Let $0_{\mathcal{N}} \neq \mathcal{P}$ be a neutrosophic open set in $\mathcal{N}(\mathcal{Z}, \pi_{\mathcal{Z}})$. Since R_2 is the neutrosophic continuous function from $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ into $\mathcal{N}(\mathcal{Z}, \pi_{\mathcal{Z}})$. This implies $R^{-1}[\mathcal{P}]$ is the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. Since R_1 is the neutrosophic resolvable set from $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. Therefore $R_1^{-1}[R_2^{-1}[\mathcal{P}]]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Thus $[R_2 \circ R_1]^{-1}[\mathcal{P}]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$, for the neutrosophic resolvable function from $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Z}, \pi_{\mathcal{Z}})$. □

proposition 4.11. *If $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ is the neutrosophic contra continuous function from the nts $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$ into the nts $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$, and if $\mathcal{N}int[\mathcal{Q}] = 0_{\mathcal{N}}$, for each neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$, then $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}}))$ is the neutrosophic resolvable function.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ be the neutrosophic contra continuous function. Take $\mathcal{P}$ be the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$. This implies $R^{-1}[\mathcal{P}]$ is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Using the hypothesis, $\mathcal{N}int[R^{-1}[\mathcal{P}]] = 0_{\mathcal{N}}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. For the neutrosophic closed set $\mathcal{Q}$ in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. $\mathcal{N}cl[\mathcal{Q} \wedge [R^{-1}][\mathcal{P}]] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[R^{-1}[\mathcal{P}]]] \leq \mathcal{N}cl[R^{-1}[\mathcal{P}]] \wedge \mathcal{N}cl[C[R^{-1}[\mathcal{P}]]]= \mathcal{N}cl[R^{-1}[\mathcal{P}]] \wedge C[\mathcal{N}int[R^{-1}[\mathcal{P}]]]= R^{-1}[\mathcal{P}]$. Since $R^{-1}[\mathcal{P}]$ is neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}})$. Therefore $\mathcal{N}cl[\mathcal{Q} \wedge R^{-1}[\mathcal{P}]] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[R^{-1}[\mathcal{P}]]] \leq R^{-1}[\mathcal{P}]$. Now, $\mathcal{N}int\mathcal{N}cl\{[\mathcal{N}cl[\mathcal{Q} \wedge R^{-1}[\mathcal{P}] \wedge \mathcal{N}cl[\mathcal{Q} \wedge C[R^{-1}[\mathcal{P}]]]\} \leq \mathcal{N}int\mathcal{N}cl[R^{-1}[\mathcal{P}]] = \mathcal{N}int[R^{-1}[\mathcal{P}]] = 0_{\mathcal{N}}$. This implies $R^{-1}[\mathcal{P}]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \pi)$. Hence $R : \mathcal{N}(\mathcal{X}, \pi_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \pi_{\mathcal{Y}})$ is the neuttosophic resolvable function. □

5. Conclusion

Neutrosophic resolvable sets and neutrosphic resolvable functions were introduced in this article. The characteristics of such sets are closely examined and studied to arrive at a solution. As a result, the concept of neutrosophic resolvable sets has been generalized according to areas of research.

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