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RESEARCH ARTICLE

Disturbance Observer-Based Robust Control Design for Uncertain Periodic Piecewise Time-Varying Systems With Disturbances

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ABSTRACT With the aid of disturbance observer strategy, this article aims to investigate the disturbance rejection and stabilization problems for periodic piecewise time-varying systems that are subject to time-varying delays, parameter uncertainties, nonlinear perturbations and exogenous disturbances. To be more specific, the periodic piecewise time-varying systems are built by segmenting the fundamental period of periodic systems into a limited number of subintervals. Further, the disturbances engendered from an exogenous system are estimated by deploying the disturbance observer and subsequently, on the premise of disturbance that is esimated, a robust controller protocol is constructed for the considered system. Moreover, by bridging the time-varying periodic piecewise Lyapunov-Krasovskii functional with a matrix polynomial lemma, a set of adequate criteria is framed, which confirms the asymptotic stability of the system that is being addressed. Subsequently, on the premise of established criteria, the design of periodic piecewise gain matrices of devised controller and configured observer are presented. Eventually, the importance and potential of the presented theoretical concepts are evidenced through offering a numerical illustration with the simulation results.

INDEX TERMS Periodic piecewise time-varying systems, disturbance observer, uncertainity, nonlinear perturbations, time-varying delay.

I. INTRODUCTION

In recent years, the study on periodic systems have been a hot research topic and stimulating significant trends due to its utility in many distinct sectors namely, rotor-bearing systems, spacecraft attitude control, vehicle suspension, vibration system, wind turbine system and so on [1], [2], [3], [4], [5]. Further, the floquet theory is applied to the analysis and synthesis of periodic systems which acts as a catalyst for periodicity-related research on a variety of topics in both discrete and continuous time domains. However, solving continuous-time periodic systems are more difficult than the discrete-time periodic systems owing to their closed form solutions [6], [7], [8]. In order to confront this difficulty, the periodic piecewise systems (PPSs) has been introduced, which aids in simplifying the analysis of the system and making dynamical results more accurate [9], [10]. To be more specific, PPSs are defined by partitioning the specified fundamental period of continuous-time periodic systems into several subintervals, where performance within each subinterval is governed by a related subsystem. Additionally, it is worth mentioning that the subsystems either can be time-invariant or time-varying. Recently, many results have been reported in time-invariant subsystems [11], [12], however, under certain circumstances, PPSs with time-invariant subsystems may lack the required dynamic behavior of approximated continuous-time periodic systems. Bearing this in mind and also from a practical standpoint, it is more

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pertinent to characterize the periodic piecewise subsystems in a time-varying form. As a consequence of this, the investigation on dynamical behavior of PPSs that contain time-varying subsystems become an important research topic over the past few years [13], [14], [15]. For instance, the researchers in [13] probed the stabilization issue for PPSs through resilient control. Even if it has a considerable number of applications in the real world, there are still many qualitative studies need to be investigated, which is one of the prompting factor of the presented study.

On another research front, it is difficult to precisely simulate a plant in engineering applications since it is often exposed to unpredictable factors. Moreover, the causes for the emergence of modeling flaws, also known as parametric uncertainties, include a high input, versatile modes, time fluctuations and so on. Specifically, the presence of these sorts of unpredictable factors in the environment has the potential to undermine the system's performance and it may result in instability. In this light, taking into account the parametric uncertainties in the system model is fundamental one in both theoretical and practical aspects. Over the last several years, scholars have focused a significant amount of emphasis on the robustness analysis of a wide variety of uncertain dynamical systems (see [16], [17], [18], [19], [20], [21] and references therein). On the other hand, due to the widespread application, the modern control systems comes up with the inescapable generation of nonlinearities. As a direct result, the presence of nonlinearities will bring about certain unfavorable changes to the dynamics of the system. Therefore, the high interest has been paid by the research communities on investigation of systems with nonlinear perturbations and has resulted in the reporting of a large number of works to maintain the stability of a variety of dynamical systems despite the existence of nonlinearities [22], [23], [24]. For instance, in [24], the tracking control issue has been investigated for periodic piecewise time-varying systems in the face of nonlinear perturbations and external disturbances. Nevertheless, in regard to PPSs with time-varying subsystems, the conjunction of time-varying periodic features with the factors such as parameter uncertainty and nonlinearity will lead to dilemma. Thereby, bearing in mind the aforementioned facts, in this study, the parameter uncertainties and nonlinear perturbations have been simultaneously taken into account in the undertaken system.

The effect of time delays are a major characteristic in many practical systems, namely, network systems, aircraft systems, nuclear reactors, chemical engineering systems and so on. Moreover, ignoring delays may cause design faults and inaccurate analysis results, which will affect the stability of a system. As a result, in recent years, the stabilization problem for time-delayed systems has been received more attention [25], [26], [27], [28], [29]. For instance, the authors in [26] studied the adaptive fuzzy control for switched nonlinear time-varying delay systems with prescribed performance and unmodeled dynamics. Liu et al. [27] devised the PD control for continuous-time systems with the presence of time-varying delay. The authors in [29] investigated discrete nonlinear systems subject to mixed delays through sliding mode control. Moreover, while considering the PPSs, only a few amount of research has been published that takes into account time delays in the system [30]. From the standpoint of broad practical application and looking at the results of these groundbreaking studies, it is vital to continue the analysis and synthesis of periodic piecewise time-varying systems with time-varying delays.

Besides the foregoing, it is vital to acknowledge that disturbances often prevail in real situations, which might lead to insufficient system performance and, in worst cases, could cause instability. To tackle this issue, various disturbance attenuation and rejection techniques has been developed by the researchers [31], [32], [33], [34], [35], [36]. In particular, the disturbance observer approach is one of the most often used disturbance rejection techniques for dealing with disturbances because of its ease of use, adaptability and effectiveness in doing so. To be precise, the disturbance observer framework first builds an observer to estimate the disturbance, wherein the construction of disturbance observers is analogous to the state observers, and it has significant implications in both theoretical and real-world applications. Thereby, as a result of the intrinsic potential, it has been applied to a broad range of dynamical systems and a substantial body of literature has been generated in this respect [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47]. In particular, an interesting disturbance rejection and adaptive command filtered control approach for uncertain nonlinear systems with parametric uncertainties, mismatched and matched disturbances has been investigated in [44] by utilizing disturbance observer strategy. When it comes to periodic piecewise time-varying systems, however, the idea of a disturbance observer needs to be looked into to enhance its practicability. Consequently, it is worth noting that the stabilization problem for periodic piecewise time-varying systems in the event of disturbances in the control channel is much more difficult and intricate. This is owing to the assertion that there is indeed a trade-off between the desired stabilization and the disturbance rejection. In view of this, in this work, we focus extensively on the construction of the disturbance observer-based robust controller to achieve significant stabilization performance of the system despite the existence of disturbances. Further, this work's most salient features can be summed up in the following specific ways:

- The issues of disturbance rejection and stabilization for uncertain periodic piecewise time-varying systems (UPPTVSs) with time-varying delays, nonlinear perturbations and disturbances are scrutinised through the disturbance observer-based robust control.
- In detail, the disturbance occurring in the control path is assumed to be engendered from the exogenous system with unknown frequency and phase.
- In order to get rid of the negative impacts that disturbances have on the control route, a disturbance rejection solution has been devised by taking use of the

advantages proffered by the disturbance observer. To be more specific, a periodic piecewise disturbance observer setup is designed in order to estimate the disturbances which aroused from the exogenous system.

- After which, the disturbance observer-based robust controller is erected by putting together the output information from the configured disturbance observer and the state feedback control.
- In the context of linear matrix inequalities (LMIs), a new set of sufficient criteria for validating the expected outcomes is then developed utilizing the Lyapunov-Krasovskii functional and a matrix polynomial lemma. Furthermore, the planned controller and framed observer gain values are determined using the developed relations.
- Final analysis includes a numerical example to demonstrate the applicability of theoretical outcomes and the usefulness of controller as configured.

This paper is organised as follows: The model description of UPPTVSs and some preliminaries are given in Section II; The stability requirements are specified in Section III; Simulation verification is provided in Section IV; In end, the conclusion is given Section V.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. I represents the identity matrix. $||.||$ signifies the Euclidean vector norm of a matrix. The representation $\mathcal{J} > 0$ delineates that the matrix $\mathcal{J}$ is positive-definite. The $\{\mathcal{J}\}^T$ indicates the transpose of matrix $\mathcal{J}$. $sym(\mathcal{J})$ signifies the $sym(\mathcal{J}) = \mathcal{J} + \mathcal{J}^T$.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. SYSTEM DESCRIPTION

In this work, we focus on a class of uncertain periodic systems in the face of time-varying delays and exogenous disturbances, whose system characteristics can be expressed in the following format:

$$\begin{cases} \dot{\mu}(t) = (\mathcal{G}(t) + \Delta\mathcal{G}(t))\mu(t) + (\mathcal{G}_\varrho(t) + \Delta\mathcal{G}_\varrho(t)) \\ \qquad \mu(t-\varrho(t)) + g(t,\mu(t)) + \mathcal{H}(t)(u(t) + \delta(t)), \\ z(t) = \mathcal{I}(t)\mu(t), \\ \mu(t_0) = \varsigma(t_0), \quad \forall t_0 \in [-\varrho, 0], \end{cases} \tag{1}$$

where $\mu(t) \in \mathbb{R}^n$ stands for the system state; $z(t) \in \mathbb{R}^m$ denotes the output vector; $u(t) \in \mathbb{R}^p$ portrays the control input vector; $\delta(t) \in \mathbb{R}^p$ describes the disturbance that occurs in the input channel; $\varrho(t)$ indicates the time-varying delay that met the set-up $0 \leq \varrho(t) \leq \varrho$, $\dot{\varrho}(t) \leq \eta < 1$ with known constants $\varrho > 0$ and $\eta > 0$; $g(t, \mu(t)) \in \mathbb{R}^n$ denotes the state-dependent nonlinear perturbations; $\mu(t_0)$ designates the state vector's initial condition; $\mathcal{G}(t)$, $\mathcal{G}_\varrho(t)$, $\mathcal{H}(t)$ and $\mathcal{I}(t)$ are the periodic matrices having the fundamental period T_n with the configuration $\mathcal{G}(t) = \mathcal{G}(t+T_n)$, $\mathcal{G}_\varrho(t) = \mathcal{G}_\varrho(t+T_n)$, $\mathcal{H}(t) = \mathcal{H}(t+T_n)$ and $\mathcal{I}(t) = \mathcal{I}(t+T_n)$, $\forall t > 0$.

Following this, the each fundamental period is partitioned into S subintervals, that is, $[\ell T_n + t_{i-1}, \ell T_n + t_i)$ $\ell = 0, 1, \ldots,$ $i \in \mathrm{N} \triangleq \{1, 2, \ldots, S\}$ with $T_i = t_i - t_{i-1}$, $t_0 = 0$, $t_S = T_n$ and $\sum_{i=1}^{S} T_i = T_n$. By using preceding segmentation as a starting point, the uncertain periodic systems (1) can be reformed as UPPTVSs, having the ensuing format:

$$\begin{cases} \dot{\mu}(t) = (\mathcal{G}_i(t) + \Delta\mathcal{G}_i(t))\mu(t) + (\mathcal{G}_{\varrho i}(t) + \Delta\mathcal{G}_{\varrho i}(t)) \\ \qquad \mu(t-\varrho(t)) + g_i(t,\mu(t)) + \mathcal{H}_i(t)(u(t) + \delta(t)), \\ z(t) = \mathcal{I}_i(t)\mu(t), \\ \mu(t_0) = \varsigma(t_0), \quad \forall t_0 \in [-\varrho, 0], \end{cases} \tag{2}$$

where $\mathcal{G}_i(t) = \mathcal{G}_i + \xi_i(t)\bar{\mathcal{G}}_i$, $\mathcal{G}_{\varrho i}(t) = \mathcal{G}_{\varrho i} + \xi_i(t)\bar{\mathcal{G}}_{\varrho i}$, $\mathcal{H}_i(t) = \mathcal{H}_i + \xi_i(t)\bar{\mathcal{H}}_i$ and $\mathcal{I}_i(t) = \mathcal{I}_i + \xi_i(t)\bar{\mathcal{I}}_i$ with $\bar{\mathcal{G}}_i = \mathcal{G}_{i+1} - \mathcal{G}_i$, $\bar{\mathcal{G}}_{\varrho i} = \mathcal{G}_{\varrho(i+1)} - \mathcal{G}_{\varrho i}$, $\bar{\mathcal{H}}_i = \mathcal{H}_{i+1} - \mathcal{H}_i$ and $\bar{\mathcal{I}}_i = \mathcal{I}_{i+1} - \mathcal{I}_i$, $\xi_i(t) = \frac{t - \ell T_n - t_{i-1}}{T_i} \in [0, 1)$, $\forall t \in [\ell T_n + t_{i-1}, \ell T_n + t_i)$, $i \in \mathrm{N}$; $\mathcal{G}_i$, $\mathcal{G}_{\varrho i}$, $\mathcal{H}_i$ and $\mathcal{I}_i$ are the known matrices of the i^{th} subsystem with adequate dimensions; $\Delta\mathcal{G}_i(t)$ and $\Delta\mathcal{G}_{\varrho i}(t)$ represent the parameter uncertainty matrices having the layout $[\Delta\mathcal{G}_i(t) \;\; \Delta\mathcal{G}_{\varrho i}(t)] = \mathrm{U}_i\Theta_i(t)[\mathrm{E}_{Ai} \;\; \mathrm{E}_{Bi}]$, here U_i, E_{Ai}, E_{Bi} represents the real matrices and $\Theta_i(t)$ defines the unknown function which met the requirement $\Theta_i^T(t)\Theta_i(t) \leq I$. For all $t \geq 0$, nonlinear perturbations $g_i(t, \mu(t))$ are assumed to satisfy $g_i(t, 0) = 0$ and consist of S portions across every period, satisfying the following relation:

$$||g_i(t,\mu(t))|| \leq \rho_i ||\mathfrak{O}_i \mu(t)||, \tag{3}$$

where $\rho_i > 0$ and $\mathfrak{O}_i \in \mathbb{R}^{n \times n}$ are scalar and constant matrix, respectively.

B. DISTURBANCE OBSERVER DESIGN

In this study, it is considered that the disturbance $\delta(t)$ is generated by a family of exogenous system, whose dynamics are represented in the following periodic piecewise format:

$$\begin{cases} \dot{\varsigma}(t) = \mathfrak{A}_i(t)\varsigma(t), \\ \delta(t) = \mathfrak{B}_i(t)\varsigma(t), \end{cases} \tag{4}$$

where $\varsigma(t) \in \mathcal{R}^v$ represents the state vector of the exogenous system; $\mathfrak{A}_i(t) = \mathfrak{A}_i + \xi_i(t)\bar{\mathfrak{A}}_i$, $\bar{\mathfrak{A}}_i = \mathfrak{A}_{i+1} - \mathfrak{A}_i$ and $\mathfrak{B}_i(t) = \mathfrak{B}_i + \xi_i(t)\bar{\mathfrak{B}}_i$, $\bar{\mathfrak{B}}_i = \mathfrak{B}_{i+1} - \mathfrak{B}_i$ are the known periodic piecewise time-varying matrices.

Following that, in order to estimate the exogenous disturbances that occurs inside the control signal path, it is necessary to design a disturbance observer. In this context, the following disturbance observer system is built in the periodic piecewise format:

$$\begin{cases} \dot{\lambda}(t) = \left[\mathfrak{A}_i(t) + \mathcal{V}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)\right](\lambda(t) - \mathcal{V}_i\mu(t)) \\ \qquad +\mathcal{V}_i[\mathcal{G}_i(t)\mu(t) + \mathcal{G}_{\varrho i}(t)\mu(t-\varrho(t) + \mathcal{H}_i(t)u(t) \\ \qquad +g_i(t,\mu(t))], \\ \hat{\varsigma}(t) = \lambda(t) - \mathcal{V}_i\mu(t), \\ \hat{\delta}(t) = \mathfrak{B}_i(t)\hat{\varsigma}(t), \end{cases} \tag{5}$$

where $\mathcal{V}_i$ is the observer gain matrix, which will be determined later; $\lambda(t)$ is the disturbance observer's state; $\hat{\varsigma}(t)$ and $\hat{\delta}(t)$ mean the estimation of $\varsigma(t)$ and $\delta(t)$, respectively.

C. DESIGN OF DISTURBANCE OBSERVER-BASED ROBUST CONTROL

Moreover, in order to achieve the requisite system's stability (2), it is vital to design a robust control protocol that tackles the disturbances occurring in the system. As a result, the control framework for this study is constructed in the following manner by combining the output of the disturbance observer with the state feedback robust control:

$$u(t) = \mathcal{K}_i(t)\mu(t) - \hat{\delta}(t), \tag{6}$$

where $\mathcal{K}_i(t)$ symbolizes the controller gain matrix, which will be calculated in the latter segment. In a further, by defining the error of disturbance estimation as $\varpi(t) = \varsigma(t) - \hat{\varsigma}(t)$ and substituting the framed controller (6) in (2), we have the underlying closed-loop UPPTVSs and error system:

$$\begin{aligned}\dot{\mu}(t) = &(\mathcal{G}_i(t) + \Delta\mathcal{G}_i(t))\mu(t) + (\mathcal{G}_{\varrho i}(t) + \Delta\mathcal{G}_{\varrho i}(t))\\ &\times\mu(t - \varrho(t)) + \mathcal{H}_i(t)\mathcal{K}_i(t)\mu(t) + \mathcal{H}_i(t)\mathfrak{B}_i(t)\varpi(t)\\ &+ g_i(t, \mu(t)),\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{\varpi}(t) = &[\mathfrak{A}_i(t) + \mathcal{V}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)]\varpi(t) + \mathcal{V}_i\Delta\mathcal{G}_i(t)\mu(t)\\ &+ \mathcal{V}_i\Delta\mathcal{G}_{\varrho i}(t)\mu(t - \varrho(t)).\end{aligned} \tag{8}$$

Remark 1: In [21], the exponential stabilization issue for uncertain PPSs is investigated by designing robust time-weighted guaranteed cost control. Further, the authors in [24] studied the output tracking control issue for periodic piecewise time-varying system subject to nonlinear perturbations and external disturbances, wherein the external disturbances are attenuated by employing the H_∞ performance. Moreover, the disturbance rejection problem is investigated in [40] for the Markovian jump systems, wherein to effectively estimate and reject the footprints of matched disturbances, a disturbance observer-based control is framed. It is noted that the dynamical character of PPSs have been conducted with disturbance attenuation approaches, wherein the results show that the impacts of disturbances are mitigated up to certain level. These studies served as inspiration for implementing a disturbance rejection strategy in UPPTVSs. In more detail, the footprints of disturbances are completely eliminated with the aid of disturbance observer. Further, different from the disturbance observer that is implemented in existing literature, in this study the disturbance observer is constructed with a periodic piecewise character in order to reject disturbances. This is one of the major improvements that have been made to the system that is being considered. Apart from this, the nonlinear perturbation and parameter uncertainty are simultaneously considered for the addressed system while achieving the foremost intention of this study, since both factors have significant amount of effect on the considered system. Considering these facts, in this study, the disturbance rejection and stabilization issues are investigated for the periodic piecewise time-varying systems with parameter uncertainty, nonlinear perturbations and time-varying delay.

D. PRELIMINARIES

Ahead of proceeding to the next section, it is necessary to consider the accompanying lemmas that are required for the upcoming examinations.

Lemma 1 [25]: For known matrix $\mathfrak{L} > 0$, the inequality $\int_\alpha^\beta \varphi^T(s)\mathfrak{L}\varphi(s)ds \geq \frac{1}{\beta-\alpha}\varsigma^T\mathfrak{E}\varsigma$ obeys for all continuously differentiable functions $\varphi : [\alpha, \beta] \to R^m$, where $\mathfrak{E} = \begin{bmatrix} 4\mathfrak{L} & -\frac{6\mathfrak{L}}{\beta-\alpha} \\ -(\frac{6\mathfrak{L}}{\beta-\alpha})^T & \frac{12\mathfrak{L}}{(\beta-\alpha)^2}\end{bmatrix}$ and $\varsigma^T = \begin{bmatrix}\int_\alpha^\beta \varphi^T(s)ds & \int_\alpha^\beta\int_s^\beta \varphi^T(u)duds\end{bmatrix}$.

Lemma 2 [43]: For appropriate dimensioned matrices $\mathfrak{M}$ and $\mathfrak{N}$, the inequality $\mathfrak{M}^T\mathfrak{N} + \mathfrak{N}^T\mathfrak{M} \leq \delta\mathfrak{M}^T\mathfrak{M} + \delta^{-1}\mathfrak{N}^T\mathfrak{N}$ holds for any constant $\delta > 0$.

Lemma 3 [9]: Let $f(\omega_1, \omega_2, \ldots, \omega_m)$ be a bounded matrix polynomial function described as $f(\omega_1, \omega_2, \ldots, \omega_m) = \sigma_0 + \omega_1\sigma_1 + \omega_1\omega_2\sigma_2 + \ldots + \left(\prod_{k=1}^{m}\omega_k\right)\sigma_m$, where $m \in \mathbb{Z}^+$, σ_j $(j = 0, 1, \ldots m)$ are real symmetric matrices and ω_k $(k = 0, 1, 2, \ldots m)$ are variables with $\omega_k \in [0, 1]$. If $\sum_{k=0}^{r}\sigma_k < 0$ $(r = 0, 1, \ldots m)$, then the matrix polynomial function $f(\omega_1, \omega_2, \ldots \omega_m) < 0$.

Remark 2: In [45], to handle the implication of disturbances in the flexible air-breathing hypersonic vehicles, a nonlinear disturbance observer (NDO) is constructed, wherein the designed NDO estimates the disturbance and considered system states. Moreover, for the purpose of rejecting disturbance, a class of new NDO for uncertain dynamical systems is configured in [46]. In detail, the framed NDO is based on the tracking differentiators and it also estimates the uncertain type of disturbances. In a similar vein, the authors in [47] constructed the tracking differentiators-based NDO for the robust back stepping control of a flexible air-breathing hypersonic vehicle, wherein the tracking differentiator is planned based on hyperbolic sine function to handle the difficulties in back stepping control. In distinction to these observers, the disturbance observer in this work is structured in a manner that has a periodic piecewise feature. Further, we have incorporated the observer gain matrix into the disturbance observer configuration, which aids in effective disturbance estimation. Added to this, constructed disturbance observer in this work makes use of the knowledge regarding the states of the UPPTVSs (2). Moreover, the modelled disturbance observer estimates the disturbance in order to get rid of the imprints left behind by disturbances. Besides, the design of disturbance observer is more difficult by the fact that the exogenous system is built in a periodic piecewise pattern.

III. MAIN RESULTS

The primary objective of this segment is to derive the adequate stability conditions in terms of linear matrix inequalities for the closed-loop system (7) and the error system (8) by making use of the Lyapunov stability theory and the matrix polynomial lemma. Moreover, this part is comprised of two primary theorems. The first of which is presented by assuming the gain values of observer and controller are known. After that, the acquired results in the first theorem are expanded to the circumstance with unknown gains matrices. Also, the relation for determining the planned gain matrices of controller and observer will be provided in context of a result to acquired linear matrix inequalities.

Theorem 1: Assume that positive scalars η, ϱ, ρ_i, υ_i, ζ_b, $(b = 1, 2, 3, 4)$, T_i and gain matrices $\mathcal{K}_i(t)$, $\mathcal{V}_i$ are known. The closed-loop UPPTVSs (7) and the configured error system (8) are asymptotically stable, if the matrices $\mathcal{J}_i(t) > 0$, $\mathrm{Q}_i > 0$, $\mathrm{R}_1 > 0$, $\mathrm{R}_2 > 0$ and $\mathrm{R}_3 > 0$ $(i \in \mathrm{N})$ exist, such that the below relation holds:

$$[\Psi]_{16\times 16} < 0, \tag{9}$$

where $\Psi_{1,1} = sym\{\mathcal{J}_i(t)\mathcal{G}_i(t) + \mathcal{J}_i(t)\mathcal{H}_i(t)\mathcal{K}_i(t)\} + \frac{\bar{\mathcal{J}}_i}{T_i} + \mathrm{R}_1 + \mathrm{R}_2 + \varrho^2\mathrm{R}_3$, $\Psi_{1,2} = \mathcal{J}_i(t)\mathcal{G}_{\varrho i}(t)$, $\Psi_{1,6} = \mathcal{J}_i(t)\mathcal{H}_i(t)\mathfrak{B}_i(t)$, $\Psi_{1,7} = \mathcal{J}_i(t)$, $\Psi_{1,8} = \upsilon_i\mathcal{J}_i(t)\mathfrak{D}_i^T$, $\Psi_{1,9} = \mathcal{J}_i(t)\mathrm{U}_i$, $\Psi_{1,10} = \mathcal{J}_i(t)\mathrm{U}_i$, $\Psi_{1,11} = \mathrm{E}_{Ai}^T$, $\Psi_{1,12} = \mathrm{E}_{Ai}^T$, $\Psi_{2,2} = -(1-\eta)\mathrm{R}_1$, $\Psi_{2,15} = \mathrm{E}_{Bi}^T$, $\Psi_{2,16} = \mathrm{E}_{Bi}^T$, $\Psi_{3,3} = -\mathrm{R}_2$, $\Psi_{4,4} = -4\mathrm{R}_3$, $\Psi_{4,5} = \frac{6}{\varrho}\mathrm{R}_3$, $\Psi_{5,5} = -\frac{12}{\varrho^2}\mathrm{R}_3$, $\Psi_{6,6} = sym\{\mathrm{Q}_i\mathfrak{A}_i(t) + \mathrm{Q}_i\mathcal{V}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)\}$, $\Psi_{6,13} = \Psi_{6,14} = \mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i$, $\Psi_{7,7} = -\upsilon_i I$, $\Psi_{8,8} = \frac{-\upsilon_i}{\rho_i^2}I$, $\Psi_{9,9} = \frac{-1}{\zeta_1}I$, $\Psi_{10,10} = \frac{-1}{\zeta_2}I$, $\Psi_{11,11} = -\zeta_1 I$, $\Psi_{12,12} = -\zeta_3 I$, $\Psi_{13,13} = \frac{-1}{\zeta_3}I$, $\Psi_{14,14} = \frac{-1}{\zeta_4}I$, $\Psi_{15,15} = -\zeta_2 I$ and $\Psi_{16,16} = -\zeta_4 I$.

Proof: The Lyapunov-Krasovskii functional of the ensuing format is constructed with the aim of demonstrating this theorem:

$$\begin{aligned}\mathfrak{V}(t) &= \mu^T(t)\mathcal{J}_i(t)\mu(t) + \varpi^T(t)\mathrm{Q}_i\varpi(t)\\&\quad + \int_{t-\varrho(t)}^{t} \mu^T(v)\mathrm{R}_1\mu(v)dv + \int_{t-\varrho}^{t} \mu^T(v)\mathrm{R}_2\mu(v)dv\\&\quad + \varrho\int_{t-\varrho}^{t}\int_{s}^{t} \mu^T(v)\mathrm{R}_3\mu(v)dvds,\end{aligned} \tag{10}$$

where $\mathcal{J}_i(t) = \mathcal{J}_i + \xi_i(t)\bar{\mathcal{J}}_i$ with $\bar{\mathcal{J}}_i = \mathcal{J}_{i+1} - \mathcal{J}_i$, $\mathcal{J}_{S+1} = \mathcal{J}_1$ and $\mathcal{J}_i > 0$ is suitable dimensioned real matrix.

Subsequently, by computing the derivative of $(V)(t)$ along the trajectories of the closed-loop UPPTVSs (7) and the error systems (8), we obtain,

$$\begin{aligned}\dot{\mathfrak{V}}(t) &\leq sym\{\mu^T(t)\mathcal{J}_i(t)[(\mathcal{G}_i(t) + \Delta\mathcal{G}_i(t))\mu(t) + (\mathcal{G}_{\varrho i}(t)\\&\quad + \Delta\mathcal{G}_{\varrho i}(t))\mu(t-\varrho(t)) + \mathcal{H}_i(t)\mathcal{K}_i(t)\mu(t)\\&\quad \times\mathcal{H}_i(t)\mathfrak{B}_i(t)\varpi(t)]\} + \mu^T(t)\left(\frac{\bar{\mathcal{J}}_i}{T_i}\right)\mu(t)\\&\quad + sym\{\varpi^T(t)\mathrm{Q}_i[\mathfrak{A}_i(t) + \mathcal{V}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)]\varpi(t)\\&\quad + \mathcal{V}_i\Delta\mathcal{G}_i(t)\mu(t) + \mathcal{V}_i\Delta\mathcal{G}_{\varrho i}(t)\mu(t-\varrho(t))\}\\&\quad + \mu^T(t)[\mathrm{R}_1 + \mathrm{R}_2]\mu(t)\text{-}(1-\eta)\mu^T(t-\varrho(t))\mathrm{R}_1\\&\quad \times\mu(t-\varrho(t)) - \mu^T(t-\varrho)\mathrm{R}_2\mu(t-\varrho)\\&\quad + \varrho\mu^T(t)\mathrm{R}_3\mu(t) - \varrho^2\int_{t-\varrho}^{t} \mu^T(s)\mathrm{R}_3\mu(s)ds.\end{aligned} \tag{11}$$

Following that, by using Lemma 1, the integral term in the preceding connection is reformulated as stated below:

$$-\varrho\int_{t-\varrho}^{t} \mu^T(s)\mathrm{R}_3\mu(s)ds \leq \hat{\pi}^T(t)\begin{bmatrix}-4\mathrm{R}_3 & \frac{6\mathrm{R}_3}{\varrho}\\ (\frac{6\mathrm{R}_3}{\varrho})^T & -\frac{12\mathrm{R}_3}{\varrho^2}\end{bmatrix}\hat{\pi}(t), \tag{12}$$

where $\hat{\pi}^T(t) = [\int_{t-\varrho}^{t} \mu^T(s)ds \quad \int_{t-\varrho}^{t}\int_{s}^{t} \mu^T(v)dvds]$.

Further, by using the Lemma 2, for some positive scalars ζ_1, ζ_2, ζ_3 and ζ_4, the terms involving uncertainty matrices in (11) can be rephrased as follows:

$$\begin{aligned}&\mu^T(t)(\mathcal{J}_i(t)\mathrm{U}_i\Theta_i(t)\mathrm{E}_{Ai} + \mathrm{E}_{Ai}^T\Theta_i^T(t)\mathrm{U}_i^T\mathcal{J}_i(t))\mu(t)\\&\quad \leq \zeta_1\mu^T(t)\mathcal{J}_i(t)\mathrm{U}_i\mathrm{U}_i^T\mathcal{J}_i(t)\mu(t) + \zeta_1^{-1}\mu^T(t)\mathrm{E}_{Ai}^T\mathrm{E}_{Ai}\mu(t),\end{aligned} \tag{13}$$

$$\begin{aligned}&\mu^T(t)\mathcal{J}_i(t)\mathrm{U}_i\Theta_i(t)\mathrm{E}_{Bi}\mu(t-\varrho(t)) + \mu^T(t-\varrho(t))\mathrm{E}_{Bi}^T\\&\quad\times\Theta_i^T(t)\mathrm{U}_i^T\mathcal{J}_i(t)\mu(t) \leq \zeta_2\mu^T(t)\mathcal{J}_i(t)\mathrm{U}_i\mathrm{U}_i^T\mathcal{J}_i(t)\mu(t)\\&\quad + \zeta_2^{-1}\mu^T(t-\varrho(t))\mathrm{E}_{Bi}^T\mathrm{E}_{Bi}\mu(t-\varrho(t)),\end{aligned} \tag{14}$$

$$\begin{aligned}&\varpi^T(t)\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\Theta_i(t)\mathrm{E}_{Ai}\mu(t) + \mu^T(t)\mathrm{E}_{Ai}^T\Theta_i^T(t)\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i\\&\mu(t) \leq \zeta_3\varpi^T(t)\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i\varpi(t) + \zeta_3^{-1}\mu^T(t)\mathrm{E}_{Ai}^T\mathrm{E}_{Ai}\mu(t)\end{aligned} \tag{15}$$

$$\begin{aligned}&\varpi^T(t)\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\Theta_i(t)\mathrm{E}_{Ai}\mu(t-\varrho(t)) + \mu^T(t-\varrho(t))\mathrm{E}_{Ai}^T\\&\quad\times\Theta_i^T(t)\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i\varpi(t)\\&\quad\leq \zeta_4\varpi^T(t)\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i\varpi(t)\\&\quad + \zeta_4^{-1}\mu^T(t-\varrho(t))\mathrm{E}_{Bi}^T\mathrm{E}_{Bi}\mu(t-\varrho(t)).\end{aligned} \tag{16}$$

Consequently, on the basis of relation (3), for some scalar $\upsilon_i > 0$, we obtain

$$\upsilon_i\rho_i^2\mu^T(t)\mathfrak{D}_i^T\mathfrak{D}_i\mu(t) - \upsilon_i g_i^T(t,\mu(t))g_i(t,\mu(t)) \leq 0. \tag{17}$$

Moreover, by combining the relations (11)-(17), we acquire the subsequent expression:

$$(\dot{V})(t) \leq \chi^T(t)[\Upsilon]_{7\times 7}\chi(t), \tag{18}$$

where $\chi^T(t) = [\mu^T(t) \;\; \mu^T(t-\varrho(t)) \;\; \mu^T(t-\varrho) \;\; \int_{t-\varrho}^{t} \mu^T(s)ds \;\; \int_{t-\varrho}^{t}\int_{s}^{t} \mu^T(v)dvds \;\; \varpi^T(t) \;\; g_i^T(t,\mu(t))]$, $\Upsilon_{1,1} = sym\{\mathcal{J}_i(t)\mathcal{G}_i(t) + \mathcal{J}_i(t)\mathcal{H}_i(t)\mathcal{K}_i(t)\} + \frac{\bar{\mathcal{J}}_i}{T_i} + \mathrm{R}_1 + \mathrm{R}_2 + \varrho^2\mathrm{R}_3 + \zeta_1\mathcal{J}_i(t)\mathrm{U}_i\mathrm{U}_i^T\mathcal{J}_i(t) + \zeta_2\mathcal{J}_i(t)\mathrm{U}_i\mathrm{U}_i^T\mathcal{J}_i(t) + \zeta_1^{-1}\mathrm{E}_{Ai}^T\mathrm{E}_{Ai} + \zeta_3^{-1}\mathrm{E}_{Ai}^T\mathrm{E}_{Ai} + \upsilon_i\rho_i^2\mathcal{J}_i(t)\mathfrak{D}_i^T\mathfrak{D}_i\mathcal{J}_i(t)$, $\Upsilon_{1,2} = \mathcal{J}_i(t)\mathcal{G}_{\varrho i}(t)$, $\Upsilon_{1,6} = \mathcal{J}_i(t)\mathcal{H}_i(t)\mathfrak{B}_i(t)$, $\Upsilon_{1,7} = -\upsilon\mathcal{J}_i(t)$, $\Upsilon_{2,2} = -(1-\eta)\mathrm{R}_1 + \zeta_2^{-1}\mathrm{E}_{Bi}^T\mathrm{E}_{Bi} + \zeta_4^{-1}\mathrm{E}_{Bi}^T\mathrm{E}_{Bi}$, $\Upsilon_{3,3} = -\mathrm{R}_2$, $\Upsilon_{4,4} = -4\mathrm{R}_3$, $\Upsilon_{4,5} = \frac{6}{\varrho}\mathrm{R}_3$, $\Upsilon_{5,5} = -\frac{12}{\varrho^2}\mathrm{R}_3$, $\Upsilon_{6,6} = sym\{\mathrm{Q}_i\mathfrak{A}_i(t)\} + sym\{\mathrm{Q}_i\mathcal{V}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)\} + \zeta_3\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i + \zeta_4\mathrm{Q}_i\mathcal{V}_i\mathrm{U}_i\mathrm{U}_i^T\mathcal{V}_i^T\mathrm{Q}_i$ and $\Upsilon_{7,7} = -\upsilon_i I$.

Following that, by using Lemma 2 to the matrix $[\Upsilon]_{7\times 7}$, we get the matrix $[\Psi]_{16\times 16}$, which is defined in the theorem statement. As a result, if the relation (9) holds then it is evident that $\dot{\mathfrak{V}}(t) < 0$. Then, on the basis of Lyapunov stability theory, the closed-loop UPPTVSs (7) and error system (8) are asymptotically stable, which completes the proof of the theorem.

In what follows, the findings gained in Theorem 1 will be stretched to the setting of unknown gain matrices for the controller and observer in the following theorem. Also, the convex optimization approach is employed to derive the gain matrix relations.

Theorem 2: The systems (7) and (8) are asymptotically stable for prescribed positive scalars η, ϱ, ρ_i, υ_i, ζ_b, $(b = 1, 2, 3, 4)$, γ and T_i, if there exist positive definite matrices $\mathcal{J}_i > 0, \mathrm{Q}_i > 0$ and appropriate dimensioned matrices $\mathrm{X}_i, \mathrm{Y}_i, \mathrm{Z}_i, \mathrm{R}_a, (i \in \mathrm{N}; a = 1, 2, 3)$ satisfying the following conditions:

$$\begin{aligned} \Omega_{ia} < 0, \ \ \Omega_{ia} + \Omega_{ib} < 0, \quad \Omega_{ia} + \Omega_{ib} + \Omega_{ic} < 0, \\ \Omega_{ia} + \Omega_{ib} + \Omega_{ic} + \Omega_{id} < 0, \end{aligned} \tag{19}$$

$$\begin{aligned} \mathfrak{S}_{ia} < 0, \ \ \mathfrak{S}_{ia} + \mathfrak{S}_{ib} < 0, \ \ \mathfrak{S}_{ia} + \mathfrak{S}_{ib} + \mathfrak{S}_{ic} < 0, \\ \mathcal{J}_{S+1} = \mathcal{J}_1, \ \ \mathrm{Y}_{S+1} = \mathrm{Y}_1, \end{aligned} \tag{20}$$

where $\Omega_{ia}^{1,1} = sym\{\mathcal{J}_i\mathcal{G}_i + \mathcal{H}_i\mathrm{Y}_i\} + \frac{\bar{\mathcal{J}}_i}{T_i} + \mathrm{R}_1 + \mathrm{R}_2 + \varrho^2\mathrm{R}_3$, $\Omega_{ia}^{1,2} = \mathcal{J}_i\mathcal{G}_{\varrho i}$, $\Omega_{ia}^{1,6} = \mathcal{H}_i\mathrm{X}_i\mathfrak{B}_i$, $\Omega_{ia}^{1,7} = \mathcal{J}_i$, $\Omega_{ia}^{1,8} = \upsilon_i\mathfrak{D}_i^T$, $\Omega_{ia}^{1,9} = \mathcal{J}_i\mathrm{U}_i$, $\Omega_{ia}^{1,10} = \mathcal{J}_i\mathrm{U}_i$, $\Omega_{ia}^{1,11} = \mathrm{E}_{Ai}^T$, $\Omega_{ia}^{1,12} = \mathrm{E}_{Ai}^T$, $\Omega_{ia}^{2,2} = -(1-\eta)\mathrm{R}_1$, $\Omega_{ia}^{2,15} = \mathrm{E}_{Bi}^T$, $\Omega_{ia}^{2,16} = \mathrm{E}_{Bi}^T$, $\Omega_{ia}^{3,3} = -\mathrm{R}_2$, $\Omega_{ia}^{4,4} = -4\mathrm{R}_3$, $\Omega_{ia}^{4,5} = \frac{6}{\varrho}\mathrm{R}_3$, $\Omega_{ia}^{5,5} = -\frac{12}{\varrho^2}\mathrm{R}_3$, $\Omega_{ia}^{6,6} = sym\{\mathrm{Q}_i\mathfrak{A}_i + \mathrm{Z}_i\mathcal{H}_i\mathfrak{B}_i\}$, $\Omega_{ia}^{6,13} = \mathrm{Z}_i\mathrm{U}_i$, $\Omega_{ia}^{6,14} = \mathrm{Z}_i\mathrm{U}_i$, $\Omega_{ia}^{7,7} = -\upsilon_iI$, $\Omega_{ia}^{8,8} = \frac{-\upsilon_i}{\rho_i^2}I$, $\Omega_{ia}^{9,9} = \frac{-1}{\zeta_1}I$, $\Omega_{ia}^{10,10} = \frac{-1}{\zeta_2}I$, $\Omega_{ia}^{11,11} = -\zeta_1I$, $\Omega_{ia}^{12,12} = -\zeta_3I$, $\Omega_{ia}^{13,13} = \frac{-1}{\zeta_3}I$, $\Omega_{ia}^{14,14} = \frac{-1}{\zeta_4}I$, $\Omega_{ia}^{15,15} = -\zeta_2I$, $\Omega_{ia}^{16,16} = -\zeta_4I$, $\mathfrak{S}_{ia}^{1,1} = -\gamma I$, $\mathfrak{S}_{ia}^{1,2} = \mathcal{J}_i\mathcal{H}_i - \mathcal{H}_i\mathrm{X}_i$, $\mathfrak{S}_{ia}^{2,2} = -\gamma I$, $\Omega_{ib}^{1,1} = sym\left\{\bar{\mathcal{J}}_i\mathcal{G}_i + \mathcal{G}_i\bar{\mathcal{J}}_i + \bar{\mathcal{H}}_i\mathrm{Y}_i + \mathcal{H}_i\bar{\mathrm{Y}}_i\right\}$, $\Omega_{ib}^{1,2} = \bar{\mathcal{J}}_i\mathcal{G}_{\varrho i} + \mathcal{G}_{\varrho i}\bar{\mathcal{J}}_i$, $\Omega_{ib}^{1,6} = \mathcal{H}_i\bar{\mathrm{X}}_i\mathfrak{B}_i + \bar{\mathcal{H}}_i\mathrm{X}_i\mathfrak{B}_i + \mathcal{H}_i\mathrm{X}_i\bar{\mathfrak{B}}_i$, $\Omega_{ib}^{1,12} = \upsilon_i\rho^2\mathfrak{D}_i^T$, $\Omega_{ib}^{1,7} = \bar{\mathcal{J}}_i$, $\Omega_{ib}^{1,9} = \bar{\mathcal{J}}_i\mathrm{U}_i$, $\Omega_{ib}^{1,10} = \bar{\mathcal{J}}_i\mathrm{U}_i$, $\Omega_{ib}^{6,6} = sym\{\mathrm{Q}_i\bar{\mathfrak{A}}_i + \mathrm{Z}_i\bar{\mathcal{H}}_i\mathfrak{B}_i + \mathrm{Z}_i\mathcal{H}_i\bar{\mathfrak{B}}_i\}$, $\mathfrak{S}_{ib}^{1,2} = \mathcal{J}_i\bar{\mathcal{H}}_i + \bar{\mathcal{J}}_i\mathcal{H}_i - \bar{\mathcal{H}}_i\mathrm{X}_i - \mathcal{H}_i\bar{\mathrm{X}}_i$, $\Omega_{ic}^{1,1} = sym\left\{\bar{\mathcal{J}}_i\bar{\mathcal{G}}_i + \bar{\mathcal{H}}_i\bar{\mathrm{Y}}_i\right\}$, $\Omega_{ic}^{1,2} = \bar{\mathcal{J}}_i\bar{\mathcal{G}}_{\varrho i}$, $\Omega_{ic}^{1,6} = \bar{\mathcal{H}}_i\bar{\mathrm{X}}_i\mathfrak{B}_i + \mathcal{H}_i\bar{\mathrm{X}}_i\bar{\mathfrak{B}}_i + \bar{\mathcal{H}}_i\mathrm{X}_i\bar{\mathfrak{B}}_i$, $\Omega_{ic}^{6,6} = sym\left\{\mathrm{Z}_i\bar{\mathcal{H}}_i\bar{\mathfrak{B}}_i\right\}$, $\mathfrak{S}_{ic}^{1,2} = \bar{\mathcal{J}}_i\bar{\mathcal{H}}_i - \bar{\mathcal{H}}_i\bar{\mathrm{X}}_i$ and $\Omega_{id}^{1,6} = \{\bar{\mathcal{H}}_i\bar{\mathrm{X}}_i\bar{\mathfrak{B}}_i\}$.

Furthermore, the requisite gain matrices can be constructed using the connections as follows $\mathcal{K}_i(t) = \mathrm{X}_i^{-1}(t)\mathrm{Y}_i(t)$ and $\mathcal{V}_i = \mathrm{Q}_i^{-1}\mathrm{Z}_i$, where $\mathrm{X}_i(t) = \mathrm{X}_i + \xi_i(t)\bar{\mathrm{X}}_i$, $\bar{\mathrm{X}}_i = (\mathrm{X}_{i+1} - \mathrm{X}_i)$ and $\mathrm{Y}_i(t) = \mathrm{Y}_i + \xi_i(t)\bar{\mathrm{Y}}_i$, $\bar{\mathrm{Y}}_i = (\mathrm{Y}_{i+1} - \mathrm{Y}_i)$.

Proof: By using the aforementioned Lyapunov-Krasovskii functional candidate $\mathfrak{V}(t)$ (10), the outcomes of this theorem can be obtained by adopting the same approach as in the prior theorem. Further, by assuming $\mathcal{J}_i(t)\mathcal{H}_i(t) = \mathcal{H}_i(t)\mathrm{X}_i(t)$, $\mathrm{X}_i(t)\mathcal{K}_i(t) = \mathrm{Y}_i(t)$, $\mathrm{Q}_i\mathcal{V}_i = \mathrm{Z}_i$ and substituting them on the matrix Ψ, we get a matrix $[\Xi]_{16\times 16}$ and its elements are $\Xi_{1,1} = sym\{\mathcal{J}_i(t)\mathcal{G}_i(t) + \mathcal{H}_i\mathrm{Y}_i(t)\} + \frac{\bar{\mathcal{J}}_i}{T_i} + \mathrm{R}_1 + \mathrm{R}_2 + \varrho^2\mathrm{R}_3$, $\Xi_{1,2} = \mathcal{J}_i(t)\mathcal{G}_{\varrho i}(t)$, $\Xi_{1,6} = \mathcal{H}_i(t)\mathcal{J}_i(t)\mathfrak{B}_i(t)$, $\Xi_{1,7} = \mathcal{J}_i(t)$, $\Xi_{1,8} = \upsilon_i\mathfrak{D}_i^T$, $\Xi_{1,9} = \mathcal{J}_i(t)\mathrm{U}_i$, $\Xi_{1,10} = \mathcal{J}_i(t)\mathrm{U}_i$, $\Xi_{1,11} = \mathrm{E}_{Ai}^T$, $\Xi_{1,12} = \mathrm{E}_{Ai}^T$, $\Xi_{2,2} = -(1-\eta)\mathrm{R}_1$, $\Xi_{2,15} = \mathrm{E}_{Bi}^T$, $\Xi_{2,16} = \mathrm{E}_{Bi}^T$, $\Xi_{3,3} = -\mathrm{R}_2$, $\Xi_{4,4} = -4\mathrm{R}_3$, $\Xi_{4,5} = \frac{6}{\varrho}\mathrm{R}_3$, $\Xi_{5,5} = -\frac{12}{\varrho^2}\mathrm{R}_3$, $\Xi_{6,6} = sym\{\mathrm{Q}_i\mathfrak{A}_i(t) + \mathrm{Z}_i\mathcal{H}_i(t)\mathfrak{B}_i(t)\}$, $\Xi_{6,13} = \mathrm{Z}_i\mathrm{U}_i$, $\Xi_{6,14} = \mathrm{Z}_i\mathrm{U}_i$, $\Xi_{7,7} = -\upsilon_iI$, $\Xi_{8,8} = \frac{-\upsilon_i}{\rho_i^2}I$, $\Xi_{9,9} = \frac{-1}{\zeta_1}I$, $\Xi_{10,10} = \frac{-1}{\zeta_2}I$, $\Xi_{11,11} = -\zeta_1I$, $\Xi_{12,12} = -\zeta_3I$, $\Xi_{13,13} = \frac{-1}{\zeta_3}I$, $\Xi_{14,14} = \frac{-1}{\zeta_4}I$, $\Xi_{15,15} = -\zeta_2I$ and $\Xi_{16,16} = -\zeta_4I$.

Thereafter, the terms containing time-varying matrices in $[\Xi]$ can be equitably restructured in the following manner: $\Xi_{1,1} = sym\{\mathcal{J}_i\mathcal{G}_i + \mathcal{H}_i\mathrm{Y}_i\} + \frac{\bar{\mathcal{J}}_i}{T_i} + \xi_i(t)[\bar{\mathcal{J}}_i\mathcal{G}_i + \mathcal{J}_i\bar{\mathcal{G}}_i + \bar{\mathcal{H}}_i\mathrm{Y}_i + \mathcal{H}_i\bar{\mathrm{Y}}_i] + \xi_i^2(t)[\bar{\mathcal{J}}_i\bar{\mathcal{G}}_i + \bar{\mathcal{H}}_i\bar{\mathrm{Y}}_i] + \mathrm{R}_1 + \mathrm{R}_2 + \varrho^2\mathrm{R}_3$,
$\Xi_{1,2} = \mathcal{J}_i\mathcal{G}_{\varrho i} + \xi_i(t)[\bar{\mathcal{J}}_i\mathcal{G}_{\varrho i} + \mathcal{J}_i\bar{\mathcal{G}}_{\varrho i}] + \xi_i^2(t)[\bar{\mathcal{J}}_i\bar{\mathcal{G}}_{\varrho i}]$,
$\Xi_{1,6} = \mathcal{H}_i\mathrm{X}_i\mathfrak{B}_i + \xi_i(t)[\mathcal{H}_i\bar{\mathrm{X}}_i\mathfrak{B}_i + \bar{\mathcal{H}}_i\mathrm{X}_i\mathfrak{B}_i + \mathcal{H}_i\mathrm{X}_i\bar{\mathfrak{B}}_i] + \xi_i^2(t)[\bar{\mathcal{H}}_i\bar{\mathrm{X}}_i\mathfrak{B}_i + \mathcal{H}_i\bar{\mathrm{X}}_i\bar{\mathfrak{B}}_i + \bar{\mathcal{H}}_i\mathrm{X}_i\bar{\mathfrak{B}}_i] + \xi_i^3(t)[\bar{\mathcal{H}}_i\bar{\mathrm{X}}_i\bar{\mathfrak{B}}_i]$,
$\Xi_{1,7} = \mathcal{J}_i + \xi_i(t)\bar{\mathcal{J}}_i$, $\Xi_{1,9} = (\mathcal{J}_i + \xi_i(t)\bar{\mathcal{J}}_i)\mathrm{U}_i$, $\Xi_{1,10} = (\mathcal{J}_i + \xi_i(t)\bar{\mathcal{J}}_i)\mathrm{U}_i$ and $\Xi_{6,6} = sym\{(\mathrm{Q}_i\mathfrak{A}_i + \mathrm{Z}_i\mathcal{H}_i\mathfrak{B}_i) + \xi_i(t)[\mathrm{Q}_i\bar{\mathfrak{A}}_i + \mathrm{Z}_i\bar{\mathcal{H}}_i\mathfrak{B}_i + \mathrm{Z}_i\mathcal{H}_i\bar{\mathfrak{B}}_i] + \xi_i^2(t)[\mathrm{Z}_i\bar{\mathcal{H}}_i\bar{\mathfrak{B}}_i]\}$.

Thereby, under the above considerations, the matrix Ξ can be illustrated as follows:

$$\Xi = \Omega_{ia} + \xi_i(t)\Omega_{ib} + [\xi_i(t)]^2\Omega_{ic} + [\xi_i(t)]^3\Omega_{id},$$

where Ω_{ia}, Ω_{ib}, Ω_{ic} and Ω_{id} are same as defined in the theorem statement. Moreover, if the relations specified in (19) holds, then in line with Lemma 3 we acquire $\Xi < 0$, from which it is straightforward that $\dot{\mathfrak{V}}(t) < 0$. Thereby, it is guaranteed that the systems (7) and (8) are asymptotically stable.

Besides the above demonstration, the constraint $\mathcal{J}_i(t)\mathcal{H}_i(t) = \mathcal{H}_i(t)\mathrm{X}_i(t)$ is not a strict inequality and so it could not be directly solved via MATLAB LMI toolbox. To solve this problem, we reconstruct the equation $\mathcal{J}_i(t)\mathcal{H}_i(t) = \mathcal{H}_i(t)\mathrm{X}_i(t)$ as $[(\mathcal{J}_i(t)\mathcal{H}_i(t) - \mathcal{H}_i(t)\mathrm{X}_i(t))^T(\mathcal{J}_i(t)\mathcal{H}_i(t) - \mathcal{H}_i(t)\mathrm{X}_i(t))] < \gamma I$, for some scalar $\gamma > 0$. Now by applying Schur complement lemma, we get

$$\Pi = \begin{bmatrix} -\gamma I & (\mathcal{J}_i(t)\mathcal{H}_i(t) - \mathcal{H}_i(t)\mathrm{X}_i(t))^T \\ (\mathcal{J}_i(t)\mathcal{H}_i(t) - \mathcal{H}_i(t)\mathrm{X}_i(t)) & -\gamma I \end{bmatrix} < 0.$$

Subsequently, the afore-mentioned matrix Π can be stated in the following equivalent form:

$$\Pi = \mathfrak{S}_{ia} + \xi_i(t)\mathfrak{S}_{ib} + [\xi_i(t)]^2\mathfrak{S}_{ic},$$

where $\mathfrak{S}_{ia}$, $\mathfrak{S}_{ib}$ and $\mathfrak{S}_{ic}$ are same as stated in the theorem statement. Furthermore, if the constraint (20) is met, we procure the relation $\Pi < 0$ in accordance with Lemma 3, which completes the proof of this theorem.

Remark 3: It is significant to pinpoint that matched disturbances, which are a sort of external disturbance that enter into the system via the same channel as the control input path. Unlike other disturbances, the matched disturbances are tough or downright unattainable to be measured by sensors. To mitigate this problem, disturbance observers are built which can quantify matched disturbances utilising data from the controlled plants and their outputs could perhaps be incorporated into the design of the control protocol. Therewithal, it is essential to stress that the disturbance-observer approach used in this work, in contrast to the conventional disturbance-observer technique, is designed in a periodic piecewise layout so as to make it more adaptable to the system that is being examined. Thereby, due to the incorporation of a periodic piecewise disturbance observer, the disturbance can be rejected in an effective manner for enhancing the outcomes and robustness of the considered PPTVSs (2).

IV. SIMULATION VERIFICATION

Here, the simulation experiments are shown to reinforce the prominence and usefulness of the theoretical findings presented in the previous part. In order to make things simple, we look the UPPTVSs in the frame of (2) with three subsystems and their corresponding dwell time are taken as $T_1 = 1$, $T_2 = 0.5$, $T_3 = 0.5$ and the fundamental period as $T_n = 2$. Furthermore, the following are the system matrices related to the system under consideration and designed observer system.

Subsystem 1:

$$\mathcal{G}_1 = \begin{bmatrix} 2.1 & -1.7 \\ -1.5 & -1.2 \end{bmatrix}, \quad \mathcal{G}_{\varrho 1} = \begin{bmatrix} 0.9 & 0.7 \\ -0.5 & -1 \end{bmatrix},$$

$$\mathcal{H}_1 = \begin{bmatrix} -1.2 \\ 0.6 \end{bmatrix},$$

$$\mathcal{I}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; \mathrm{U}_1 = \begin{bmatrix} 0.7 \\ 1.2 \end{bmatrix}, \; \mathrm{E}_{A1} = \begin{bmatrix} 0.02 - 0.01 \end{bmatrix}, \quad \mathrm{E}_{B1} = \begin{bmatrix} 0.02 & -0.01 \end{bmatrix},$$

$$\mathfrak{D}_1 = \begin{bmatrix} 0.52 & -0.01 \\ 0.52 & -0.01 \end{bmatrix}, \quad \mathfrak{A}_1 = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

and $\mathfrak{B}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Subsystem 2:

$$\mathcal{G}_2 = \begin{bmatrix} 2.0 & -1.3 \\ -1.4 & -1.5 \end{bmatrix}, \quad \mathcal{G}_{\varrho 2} = \begin{bmatrix} 0.8 & 0.6 \\ -0.5 & -1 \end{bmatrix},$$

$$\mathcal{H}_2 = \begin{bmatrix} -1.00 \\ 0.36 \end{bmatrix},$$

$$\mathcal{I}_2 = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix}, \quad \mathrm{U}_2 = \begin{bmatrix} 1.4 \\ 0.7 \end{bmatrix}, \mathrm{E}_{A2} = \begin{bmatrix} 0.03 - 0.04 \end{bmatrix}, \mathrm{E}_{B2} = \begin{bmatrix} 0.05 & -0.03 \end{bmatrix}, \mathfrak{D}_2 = \begin{bmatrix} 0.23 & -0.14 \\ 0.23 & -0.14 \end{bmatrix},$$

$$\mathfrak{A}_2 = \begin{bmatrix} 0 & 0.48 \\ -0.48 & 0 \end{bmatrix} \quad \text{and } \mathfrak{B}_2 = \begin{bmatrix} 0.99 & 0 \end{bmatrix}.$$

Subsystem 3:

$$\mathcal{G}_3 = \begin{bmatrix} 2.0 & -1.5 \\ -1.6 & -1.3 \end{bmatrix}, \quad \mathcal{G}_{\varrho 3} = \begin{bmatrix} 0.6 & 0.5 \\ -0.4 & -2 \end{bmatrix},$$

$$\mathcal{H}_3 = \begin{bmatrix} -1.1 \\ 0.38 \end{bmatrix},$$

$$\mathcal{I}_3 = \begin{bmatrix} 0.6 \\ 0 \end{bmatrix}, \mathrm{U}_3 = \begin{bmatrix} 1.5 \\ 0.6 \end{bmatrix}, \mathrm{E}_{A3} = \begin{bmatrix} 0.02 - 0.03 \end{bmatrix}, \mathrm{E}_{B3} = \begin{bmatrix} 0.04 & -0.06 \end{bmatrix}, \quad \mathfrak{D}_3 = \begin{bmatrix} 0.02 & -0.13 \\ 0.23 & -0.14 \end{bmatrix},$$

$$\mathfrak{A}_3 = \begin{bmatrix} 0 & 0.46 \\ -0.46 & 0 \end{bmatrix} \quad \text{and } \mathfrak{B}_3 = \begin{bmatrix} 0.97 & 0 \end{bmatrix}.$$

Added to this, the diagrammatic representations of the aforementioned matrices are doodled in Fig. 1. Now, we choose the values of $\upsilon_1 = 1.6, \upsilon_2 = 2.5, \upsilon_3 = 3.4$, $\rho_1 = 1.4, \rho_2 = 1.3$ and $\rho_3 = 1.5$. The time-varying delay is considered as $\varrho(t) = 0.6 + \sin(0.2t)$ and also the derivative bound of the delay function is $\eta = 0.2$. Based on this, the feasible solution can be found by solving the established LMIs (19)-(20) in Theorem 2. From thereon, the gain matrices for controller and observer are reckoned by utilizing the specified connection in Theorem 2. Specifically, the observer gain matrices are shown below:

$$\mathcal{V}_1 = \begin{bmatrix} 0.8748 & -0.1994 \\ -0.1994 & 0.1598 \end{bmatrix},$$

$$\mathcal{V}_2 = \begin{bmatrix} 0.6672 & -0.2610 \\ -0.2610 & 0.5442 \end{bmatrix}$$

and

$$\mathcal{V}_3 = \begin{bmatrix} 0.4872 & -0.1903 \\ -0.1903 & 0.5093 \end{bmatrix}.$$

Further, a visual portrayal of the developed controller and its accompanying matrices are shown in Fig. 2.

In order to perform the simulation, nonlinear perturbation and initial condition are opted as $g_i(t, \mu(t)) = [0.08\cos(0.3\mu_1(t)) + 0.02\sin((0.3/2)\mu_1(t)); 0.03\cos\mu_1(t)]$, $\mu(t_0) = [2 \; -3]^T$.

Moreover, through the use of scalars and matrices described above, the graphs are obtained and presented in Figs. 3-11

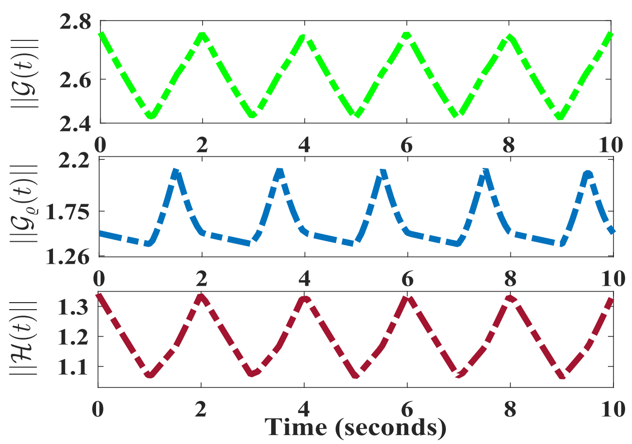

FIGURE 1. Norm Variation of the parameters $\mathcal{G}(t)$, $\mathcal{G}_{\varrho}(t)$ and $\mathcal{H}(t)$.

In detail, the addressed system's state response under the case of existence and non-existence of disturbance observer-based robust controller is pictured in Fig. 3. Therein, it indicates very clearly that the developed controller is capable of successfully accomplishing the desired goals.

Moreover, the trajectories of disturbance and its estimation are depicted in Fig. 4. Precisely, it is apparent that the

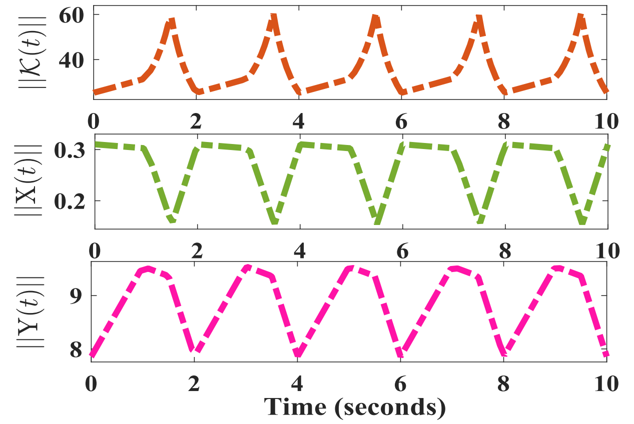

FIGURE 2. Time-history of matrices $\mathcal{K}(t)$, X(t) and Y(t).

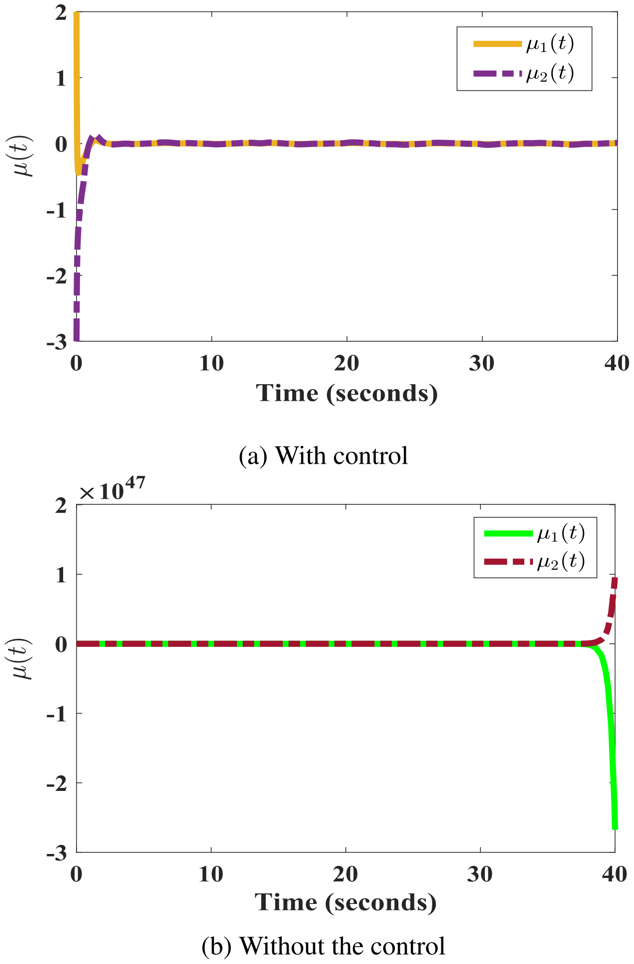

FIGURE 3. Evolution of state trajectories.

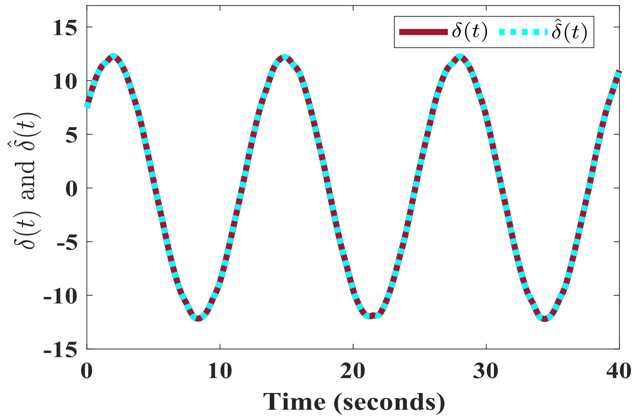

FIGURE 4. Disturbance and its estimation.

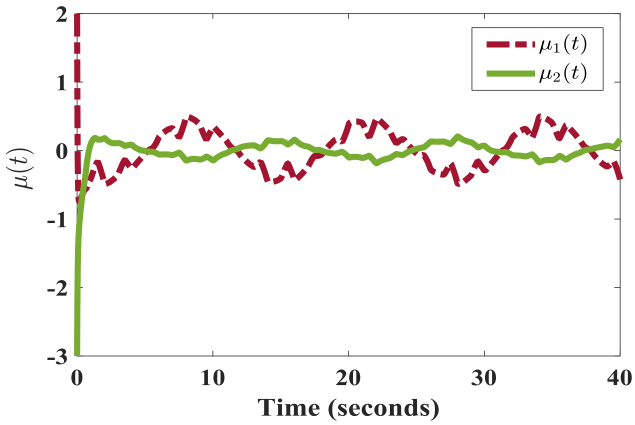

FIGURE 5. State trajectories in the absence of disturbance estimation.

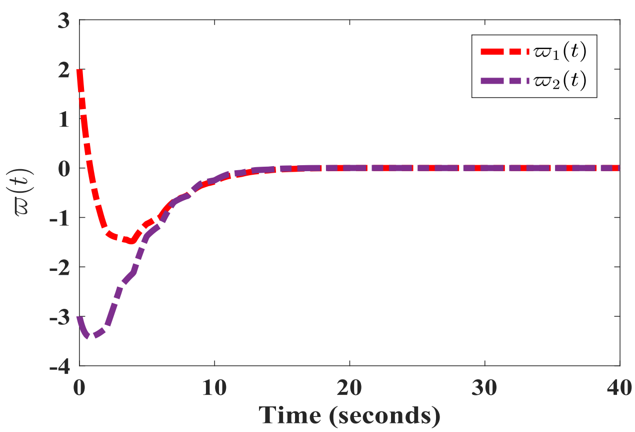

FIGURE 6. Disturbance estimation error responses.

estimated disturbance quickly estimates the actual disturbance. Alongside in Fig. 5, the states of assayed system is presented in the absence of disturbance estimation via the developed controller. From this, it is guaranteed that the periodic piecewise disturbance observer provided in this work has more advantageous for the system under consideration. Subsequently, the responses of disturbance estimation error trajectories are plotted in Fig. 6 wherein it converges to zero and it indicates that the disturbance in addressed system is estimated with high precision. Besides, the responses of devised controller is pictured in Fig. 7.

Further, with an intent to perform a comparative analysis between the criterion that are developed and extended passivity control scheme, the curves of the system and its output are,

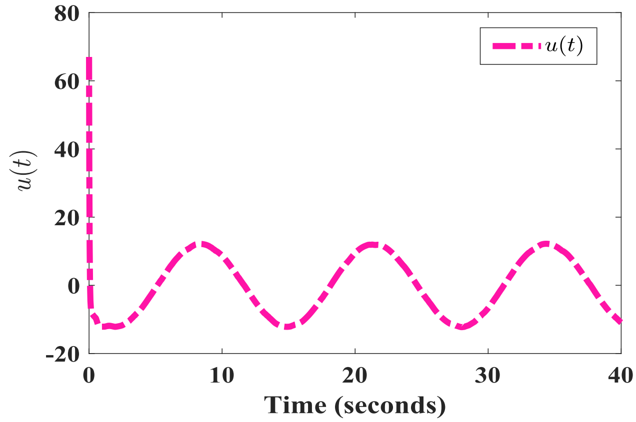

FIGURE 7. Control responses.

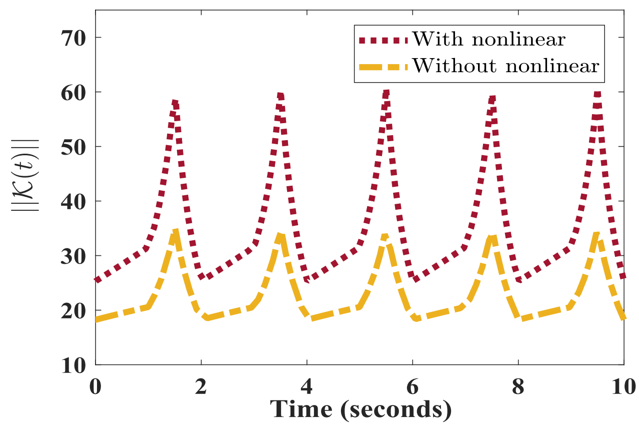

FIGURE 10. Norm Variation of gain matrix $\mathcal{K}(t)$.

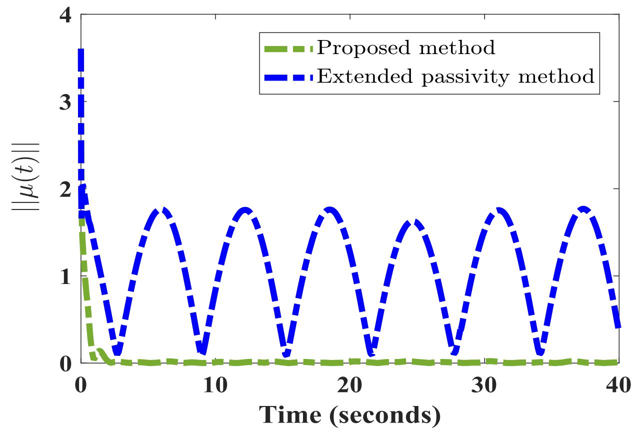

FIGURE 8. Evolution of state trajectories.

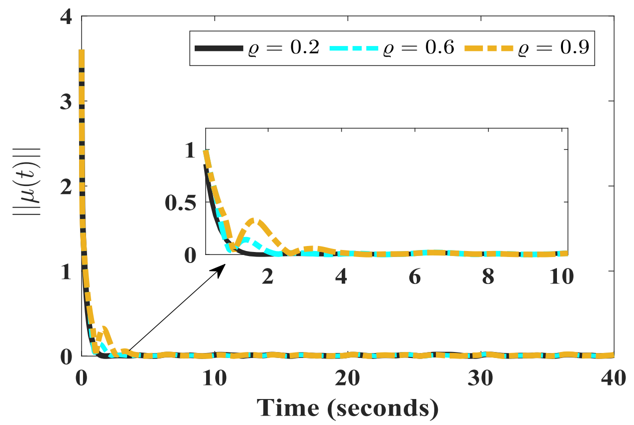

FIGURE 11. Time profile of the state trajectories for different delay bounds.

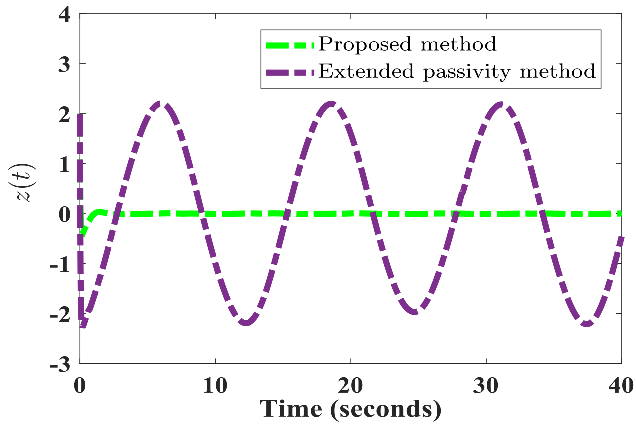

FIGURE 9. Response of output trajectories.

respectively, pictured in Figs. 8 and 9. These presented figures implies that the undertaken approach is much more efficient and superior to the existing extended passivity technique. After that in Fig. 10, the variation of controller gain matrices in the presence and exclusion of nonlinear perturbations is given. By looking at this illustration, it is clear that when compared to the case with nonlinear perturbations, the effort needed in the nonlinear perturbations-free scenario is substantially less. Moreover, the Fig. 11 provides the response of state trajectories for distinct delay bounds which makes the impact of time delay readily apparent.

Altogether, it is plainly obvious that the designed disturbance observer-based controller provides satisfactory stabilization and disturbance estimation performance simultaneously for the probed system regardless of the presence of time-varying delays, nonlinear perturbations and external disturbances.

V. CONCLUSION

In this article, the stabilization and disturbance rejection issues for UPPTVSs that are subject to time-varying delays, nonlinear perturbations and disturbances are investigated through the disturbance observer-based robust control. To put it more precisely, in order for the system model to mirror reality, uncertainty is taken into account. Further, the periodic piecewise disturbance observer is developed with the intention of estimating the disturbance that is ensued from the exogenous system. Following that, the disturbance observer-based robust controller is developed by making use of the output provided by the specified observer.

Subsequently, with the assistance of a periodic piecewise time-varying Lyapunov-Krasovskii functional, sufficient requirements in the form of LMIs are constructed, which ensures the foremost intention of this study. After that, the periodic piecewise gain matrices for controller and observer are established by solving the criteria that are stated. Conclusively, the presented simulation results reveals the usefulness of devised control scheme. Moreover, the stabilization problem of periodic piecewise time-varying systems in the presence of actuator faults and multiple disturbances is not yet investigated based on an observer-based control which simultaneously deals with the disturbances and faults, which is our future investigation in this direction.

REFERENCES

[1] B. Pang and Z.-P. Jiang, "Adaptive optimal control of linear periodic systems: An off-policy value iteration approach," *IEEE Trans. Autom. Control*, vol. 66, no. 2, pp. 888–894, Feb. 2021.

[2] B. Zhou, "On semi-global stabilization of linear periodic systems with control magnitude and energy saturations," *J. Franklin Inst.*, vol. 352, no. 5, pp. 2204–2228, May 2015.

[3] B. Pang, Z.-P. Jiang, and I. Mareels, "Reinforcement learning for adaptive optimal control of continuous-time linear periodic systems," *Automatica*, vol. 118, Aug. 2020, Art. no. 109035.

[4] V. Slyn'ko and V. Bivziuk, "Stability and stabilization of linear periodic systems: A commutator calculus approach," *Automatica*, vol. 125, Mar. 2021, Art. no. 109478.

[5] X. Xie, J. Lam, C. Fan, X. Wang, and K.-W. Kwok, "A polynomial blossoming approach to stabilization of periodic time-varying systems," *Automatica*, vol. 141, Jul. 2022, Art. no. 110305.

[6] L. Li, Y. Lu, and L. Gu, "Preview control for a class of linear discrete-time periodic systems," *Int. J. Control*, vol. 94, no. 3, pp. 823–833, Mar. 2021.

[7] N. A. Keles, C. M. Agulhari, and M. J. Lacerda, "Stability analysis and robust performance of periodic discrete-time uncertain systems via structured Lyapunov functions," *Eur. J. Control*, vol. 58, pp. 74–81, Mar. 2021.

[8] M. Sun, F. Liao, and J. Deng, "Design of an optimal preview controller for linear discrete-time periodic systems," *Trans. Inst. Meas. Control*, vol. 43, no. 12, pp. 2637–2646, Aug. 2021.

[9] L. Yang, Y. Gao, Z. Feng, and L. Wu, "Event-based output feedback control of periodic systems: A piecewise impulsive method," *Int. J. Robust Nonlinear Control*, vol. 31, pp. 7569–7586, Nov. 2021.

[10] P. Li, M. Xing, Y. Liu, and B. Zhang, "New conditions of analysis and synthesis for periodic piecewise linear systems with matrix polynomial approach," *IEEE Access*, vol. 8, pp. 52631–52640, 2020.

[11] B. Zhu, J. Lam, X. Xie, X. Song, and K. W. Kwok, "Stability and L_1-gain analysis of periodic piecewise positive systems with constant time delay," *IEEE Trans. Autom. Control*, vol. 67, pp. 2655–2662, May 2022.

[12] C. Fan, J. Lam, and X. Xie, "Fault estimation for periodic piecewise T–S fuzzy systems," *Int. J. Robust Nonlinear Control*, vol. 31, pp. 8055–8074, Nov. 2021.

[13] Y. Liu, P. Li, X. Xie, and B. Zhang, "Non-fragile control of periodic piecewise linear time-varying systems with time delay," *IET Control Theory Appl.*, vol. 13, pp. 2217–2227, Sep. 2019.

[14] X. Xie, J. Lam, and K.-W. Kwok, "A novel scheme of nonfragile controller design for periodic piecewise LTV systems," *IEEE Trans. Ind. Electron.*, vol. 67, no. 12, pp. 10766–10775, Dec. 2020.

[15] C. Fan, J. Lam, X. Xie, and X. Song, "Observer-based output reachable set synthesis for periodic piecewise time-varying systems," *Inf. Sci.*, vol. 571, pp. 246–261, Sep. 2021.

[16] Z. Liu, H. R. Karimi, and J. Yu, "Passivity-based robust sliding mode synthesis for uncertain delayed stochastic systems via state observer," *Automatica*, vol. 111, Jan. 2020, Art. no. 108596.

[17] H. Chen, C. C. Lim, and P. Shi, "Robust H_∞-based control for uncertain stochastic fuzzy switched time-delay systems via integral sliding mode strategy," *IEEE Trans. Fuzzy Syst.*, vol. 30, pp. 382–396, Feb. 2022.

[18] R. Xu, Z. Liu, and Y. Liu, "State-estimation-based adaptive sliding mode control for a class of uncertain time-delay systems: A new design," *Int. J. Syst. Sci.*, vol. 53, no. 2, pp. 375–387, Jan. 2022.

[19] X. Yang, W. Deng, and J. Yao, "Neural adaptive dynamic surface asymptotic tracking control of hydraulic manipulators with guaranteed transient performance," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Jan. 28, 2022, doi: 10.1109/TNNLS.2022.3141463.

[20] X. Cai, S. Zhong, J. Wang, and K. Shi, "Robust H_∞ control for uncertain delayed T–S fuzzy systems with stochastic packet dropouts," *Appl. Math. Comput.*, vol. 385, Nov. 2020, Art. no. 125432.

[21] X. Xie, J. Lam, and C. Fan, "Robust time-weighted guaranteed cost control of uncertain periodic piecewise linear systems," *Inf. Sci.*, vols. 460–461, pp. 238–253, Sep. 2018.

[22] Y. Wei and H. R. Karimi, "Dynamic sliding mode control for nonlinear parameter-varying systems," *Int. J. Robust Nonlinear Control*, vol. 31, pp. 8408–8419, Nov. 2021.

[23] Y. Wang, H. Pu, P. Shi, C. K. Ahn, and J. Luo, "Sliding mode control for singularly perturbed Markov jump descriptor systems with nonlinear perturbation," *Automatica*, vol. 127, May 2021, Art. no. 109515.

[24] X. Xie, J. Lam, C. Fan, X. Wang, and K.-W. Kwok, "Energy-to-peak output tracking control of actuator saturated periodic piecewise time-varying systems with nonlinear perturbations," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 4, pp. 2578–2590, Apr. 2022.

[25] M. J. Park, O. M. Kwon, J. H. Park, S. M. Lee, and E. J. Cha, "Stability of time-delay systems via Wirtinger-based double integral inequality," *Automatica*, vol. 55, pp. 204–208, May 2015.

[26] L. Shi, C. K. Ahn, and Z. Xiang, "Adaptive fuzzy control of switched nonlinear time-varying delay systems with prescribed performance and unmodeled dynamics," *Fuzzy Sets Syst.*, vol. 371, pp. 40–60, Sep. 2019.

[27] J. J. R. Liu, M. Zhang, J. Lam, B. Du, and K.-W. Kwok, "PD control of positive interval continuous-time systems with time-varying delay," *Inf. Sci.*, vol. 580, pp. 371–384, Nov. 2021.

[28] O. M. Kwon, S. H. Lee, M. J. Park, and S. M. Lee, "Augmented zero equality approach to stability for linear systems with time-varying delay," *Appl. Math. Comput.*, vol. 381, Sep. 2020, Art. no. 125329.

[29] J. Hu, H. Zhang, X. Yu, H. Liu, and D. Chen, "Design of sliding-mode-based control for nonlinear systems with mixed-delays and packet losses under uncertain missing probability," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 5, pp. 3217–3228, May 2021.

[30] X. Xie and J. Lam, "Guaranteed cost control of periodic piecewise linear time-delay systems," *Automatica*, vol. 94, pp. 274–282, Aug. 2018.

[31] H. Yu, Y. Ma, J. Liu, and L. Fu, "Extended dissipative analysis for T–S fuzzy semi-Markov jump systems with sampled-data input and actuator fault," *Nonlinear Anal., Hybrid Syst.*, vol. 40, May 2021, Art. no. 101010.

[32] L. Chen, T. Li, Y. Chen, R. Wu, and S. Ge, "Robust passivity and feedback passification of a class of uncertain fractional-order linear systems," *Int. J. Syst. Sci.*, vol. 50, pp. 1149–1162, Apr. 2019.

[33] Y. Ju, G. Cheng, and Z. Ding, "Stochastic H_∞ finite-time control for linear neutral semi-Markovian jumping systems under event-triggering scheme," *J. Franklin Inst.*, vol. 358, pp. 1529–1552, Jan. 2021.

[34] M. J. Mirzaei, E. Aslmostafa, M. Asadollahi, and M. A. Badamchizadeh, "Robust adaptive finite-time stabilization control for a class of nonlinear switched systems based on finite-time disturbance observer," *J. Franklin Inst.*, vol. 358, no. 7, pp. 3332–3352, May 2021.

[35] B. Xu, L. Zhang, and W. Ji, "Improved non-singular fast terminal sliding mode control with disturbance observer for PMSM drives," *IEEE Trans. Transp. Electrific.*, vol. 7, no. 4, pp. 2753–2762, Dec. 2021.

[36] X. Yang, Y. Ge, W. Deng, and J. Yao, "Observer-based motion axis control for hydraulic actuation systems," *Chin. J. Aeronaut.*, to be published, doi: 10.1016/j.cja.2022.10.005.

[37] L. Zhou, Z. Che, and C. Yang, "Disturbance observer-based integral sliding mode control for singularly perturbed systems with mismatched disturbances," *IEEE Access*, vol. 6, pp. 9854–9861, 2018.

[38] X. Yao, Y. Lian, and J. H. Park, "Disturbance-observer-based event-triggered control for semi-Markovian jump nonlinear systems," *Appl. Math. Comput.*, vol. 363, Dec. 2019, Art. no. 124597.

[39] T. Xu, C. Zhu, W. Qi, J. Cheng, K. Shi, and L. Sun, "Passive analysis and finite-time anti-disturbance control for semi-Markovian jump fuzzy systems with saturation and uncertainty," *Appl. Math. Comput.*, vol. 424, Jul. 2022, Art. no. 127030.

[40] X. Gao, Q. Gao, W. Qi, and Y. Kao, "Disturbance-observer-based control for time-delay Markovian jump systems subject to actuator saturation," *Trans. Inst. Meas. Control*, vol. 41, no. 3, pp. 605–614, Feb. 2019.

[41] T. Xu, X. Gao, W. Qi, and Y. Wei, "Disturbance-observer-based control for semi-Markovian jump systems with generally uncertain transition rate and saturation nonlinearity," *Appl. Math. Comput.*, vol. 362, Dec. 2019, Art. no. 124569.

[42] Y. Wei, G.-P. Liu, J. Hu, and F. Meng, "Disturbance attenuation and rejection for nonlinear uncertain switched systems subject to input saturation," *IEEE Access*, vol. 7, pp. 58475–58483, 2019.
[43] W.-H. Chen, K. Ding, and X. Lu, "Disturbance-observer-based control design for a class of uncertain systems with intermittent measurement," *J. Franklin Inst.*, vol. 354, no. 13, pp. 5266–5279, Sep. 2017.
[44] X. Yang, W. Deng, and J. Yao, "Disturbance-observer-based adaptive command filtered control for uncertain nonlinear systems," *ISA Trans.*, vol. 130, pp. 490–499, Nov. 2022.
[45] X. Bu, D. Wei, and G. He, "A robust constrained control approach for flexible air-breathing hypersonic vehicles," *Int. J. Robust Nonlinear Control*, vol. 30, pp. 2752–2776, Feb. 2020.
[46] X.-W. Bu, X.-Y. Wu, Y.-X. Chen, and R.-Y. Bai, "Design of a class of new nonlinear disturbance observers based on tracking differentiators for uncertain dynamic systems," *Int. J. Control Automat. Syst.*, vol. 13, no. 3, pp. 595–602, Mar. 2015.
[47] X. Bu, X. Wu, R. Zhang, Z. Ma, and J. Huang, "Tracking differentiator design for the robust backstepping control of a flexible air-breathing hypersonic vehicle," *J. Franklin Inst.*, vol. 352, no. 4, pp. 1739–1765, Apr. 2015.

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