CHAPTER 1

Chapter 1

Introduction

Rene Thom, a French Mathematician once stated Topology as a mathematical discipline that allows the passage from local to global. An important feature of topology is that it allows the possibility of making qualitative predictions when quantitative ones are impossible. Another important concept of topology is connectedness. In literature, several notions of connectedness in topological spaces exist. A stronger form of connectedness is known as hyperconnected spaces. Several types of research have been carried out to investigate the basic properties of hyperconnected spaces.

The concept of the neutrosophic set is used to solve real world practical problems with incompleteness and indeterminacy. These sets are successfully applied in topological space as neutrosophic topology. Neutrosophic topology is a rapidly evolving branch of topology. This concept of topology is carried out to investigate the various properties of neutrosophic sets in different fields.

A general mathematical tool for dealing with uncertainty and vagueness is soft set theory. Soft sets are considered a mathematical tool for dealing with uncertainties. The application of soft set theory is diverse into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability, etc. Along with this, a new model has been generalized with a combination of fuzzy sets and soft sets as a soft expert system.

Based on β- open[11] and α-open[12], the concepts of β-connectedness[49] , α-connectedness and α_β connectedness[105], S_α- connectedness[106] were introduced, respectively. The essential features of connectedness in topological spaces were studied by many authors. In 1975, Arhangelskie and Richard gave the remarkable notion of connectedness and disconnectedness in topological spaces briefly[16]. In 2006, Duszynski provided some concepts of weak connectedness in topological spaces[37]. In 2015, Abd El. Latif and Rodyna studied further properties of fuzzy soft pre connected spaces in fuzzy topological spaces[1].Here after many researchers discussed their views on various forms of connectedness in different fields[87]. Also they focussed on finding several types disconnectedness such as basically disconnected, extremally disconnected, perfectly disconnected and totally disconnected in topological spaces. Of all these areas extremally disconnectedness has peculiar applications in topological spaces. Majority of researchers have examined this area of disconnectedness[22][95]. In 1967, Dona discusses the characterization of disconnected spaces among Hausdorff spaces [35]. In 2013, Majid Mirmiran found the various equivalent statements of extremally disconnected spaces[62]. Sanjay Mishra introduced $\alpha - \tau$ disconnectedness [83] and investigate the relationship between $\alpha - \tau$ disconnected and $\alpha - \tau$ connected sets in 2015. Recently, Researchers examined various properties of disconnected spaces in different topological spaces.

In 1968, Levine introduced D - space as every nonempty open set of X is dense in X [58]. In 1970, Steen and Seebach paid some attention to hyperconnectedness in topological spaces [94]. Sharma determined that D-spaces are hyperconnected spaces in 1977 [91]. In 1979, Noiri initiated the concept of hyperconnected sets in topological space by semi-open sets [74]. Also, he formulated various properties of hyperconnected spaces using semi pre-open sets and pre-open sets[73],[76]. Ekici simply defined the separated sets and connected spaces in topological space in 2004 [38]. Navalagi, LellisThivagar, Rajarajeswari, and Athisiya Ponmani scrutinize $(1, 2)\alpha$-hyperconnected spaces in 2006 [71]. In 2011, Bose and Tiwari discussed ω-connectedness and hyperconnectedness [25]. In 2015, Arup Roy Choudhury, Ajoy Mukharjee, and Bose analyzed the characteristics of hyperconnectedness and extremally disconnectedness in (a) topological spaces [17]. Also, S* Hyperconnectedness was proposed by Adiya K. Hussein in the same year [2]. In 2019, Al-Saadi, HalisAygiin and Al-Omari extend the concept of hyperconnected space in soft topological spaces [7]. In 2016, Bhattacharya, Chakraborty, and Paul introduced $\gamma^\star$ hyperconnectedness in fuzzy topological spaces [24]. Al-Saadi, Al-Omari, and Noiri studied hyperconnected spaces via m-structures in 2019 [8]. In 2020, H-hyperconnected space was proposed by Zahiruddin sheriff and Ponraj [111]. In reason time Glasia, Catalan, Michael, and Baldado inaugurate the concept of $\beta^\star$ compactness and $\beta^\star$ hyperconnectedness in ideal topological spaces [46]. Recently the notion of hyperconnected spaces was studied by many authors [21],[50],[57],[63],[63],[81],[86].

In 1965, Zadeh introduced the concept of fuzzy sets and fuzzy logic [110]. It is an important concept in handling uncertainty in real life where each element has a membership function. In 1986, Attanassov proposed the concept of intuitionistic fuzzy sets, which is a generalization of fuzzy sets [19]. Intuitionistic fuzzy sets are characterized by the membership function and non-membership function with each element, whereas in real life we need to handle the incompleteness and indeterminacy. In this context, Smarandache applied neutrosophic set theory to solve real-world practical problems. Smarandache's neutrosophic set theory focused on medical, engineering fields, social science, etc. Neutrosophic sets are characterized by a true membership function, indeterminant membership function and falsity membership function [42],[43],[44],[45].

In 2012, Salama and Alblowi defined neutrosophic topological space by using neutrosophic sets [84]. In 2020, Murad Aran and Jafari inaugurated Neutrosophic μ-topological spaces [69]. Neutrosophic nano topological space was introduced by Sasikala and Radhamani in 2020 [85]. In 2021, Ishwarya and Bageerathi created Neutrosophic semi-connected spaces via Neutrosophic semi-open sets [48]. Jeya Puvaneswari and Bageerathi introduced neutrosophic feebly open set in neutrosophic topological spaces [51].

Dense sets plays an important role in the study of hyperconnected spaces [10],[78]. Dhavaseelan, Narmadha Devi and Jafari examined the characteristics of neutrosophic nowhere dense sets in 2018 [33], and also found some new notions and functions of neutrosophic topological spaces [34]. In 2019, Al-Shami found the properties and mappings via somewhere dense sets [9]. Resolvable sets are created by using the concepts of dense sets in various fields .

The concept of resolvable sets in topological space was presented by Kuratowski in 1966 [55]. Maximilian Ganster gave the concept of pre-open sets and resolvable spaces in 1987 [64]. Resolvable spaces and irresolvable spaces were explored by Chandan Chakkopadhyay and Chhanda Bandyopadhyay in 1993 [28]. In 2017 these spaces were studied by Caldas, Maximilian Ganster, Dhavaseelan, and Jafari through neutrosophic topological spaces [26]. In 2017, Thangaraj initiated to introduce the idea of resolvable sets and their functions in fuzzy topological spaces [101]. Thangaraj and Lokeshwari studied irresolvable sets and open hereditarily irresolvable spaces in fuzzy topological spaces [99]. Also, they discussed resolvable sets and functions in fuzzy hyperconnected spaces [100]. In 2020, fuzzy resolvable functions were briefly inspected by Thangaraj and Senthil [103].

Mathematics is used in the practical and complicated problems arising in the field of medicine. The use of probability and statistics in evaluating the efficacy of new drugs, and procedures or calculating the survival rate of people suffering from diseases and undergoing treatments is one of the most universal uses of mathematics. Every year, coronary artery disease affects millions of people around the world. This disease has been proven to be the top cause of death in both men and women, particularly in developed countries. Techniques including fuzzy sets, soft sets, probability, ambiguous sets, and rough sets have been proposed to answer such problems quickly. Several theories for dealing with such systems effectively have been proposed in recent years.

The fuzzy set theory has been generalized into soft set theory. Molodtsov coined the term ”soft set theory” to describe a new mathematical instrument for dealing with uncertainty [65]. A parameterized family of sets is referred to as a soft set. Theoretically soft sets and soft topological spaces are applied and studied in a lot of fields [6],[27].

The soft set theory [60] grew in popularity as a result of its founding and growth, and it has applications in a wide range of domains. In recent years, development in the field of the soft expert system has been taking place at a rapid pace. The accuracy of the concerned soft expert systems investigated by Chen [30] noticed that the outcomes of soft sets are not feasible. So he annihilated it with the notion of a parameter reduction algorithm.

In 2011, Xiquin Ma reduced the amount of computation in the pathway of parameter reduction of soft sets by presenting a novel normal parameter reduction algorithm [60]. In 2019, Fariha Iftikhar constructed an expert system using soft sets to identify the risk factor of a patient suffering from dengue fever which is an acute viral disease caused by female Aedes aegypti (a type of mosquito) [41].

In 1979, William B. Kannel and Daniel L Mc Gee discussed the risk factors of diabetes and Cardiovascular disease for men and women briefly [109]. The fuzzy expert system was created to help with heart disease diagnosis [66]. In the genuine sense, a generic fuzzy expert system simulates the behaviour of an expert [20],[32]. Instead of using Boolean reasoning, this expert system employs fuzzy logic. It consists of a set of membership functions and rules for reasoning with data. These systems are designed to process data numerically. Later it has been widely used to mimic the reasoning of a doctor in the lung cancer diseases [4], kidney diseases [40], and cardiac diseases [5] etc. By forecasting acceptable risk and providing reliable results, the system serves as an excellent diagnostic tool. The major conclusion is the requirement to arrive at an accurate diagnosis in a timely manner.

These studies on hyperconnectedness motivated the researcher to investigate the role of hyperconnectedness in various directions in topological spaces.

1.1 Preliminaries

Definition 1.1.1. *[13] A subset P of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be j-open if $P \subseteq int(pcl(P))$. The complement of j-open set is j-closed set.*
The family of j-open sets denoted by $JO(\mathcal{X})$ and the family of j-closed sets are denoted by $JC(\mathcal{X})$.

Definition 1.1.2. *[35] Two non-empty subsets P and Q of $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be separated if and only if $P \cap cl(Q) = \emptyset$ and $cl(P) \cap Q = \emptyset$.*

Definition 1.1.3. *[73] Two non-empty subsets P and Q of $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be half separated if and only if $P \cap cl(Q) = \emptyset$ or $cl(P) \cap Q = \emptyset$.*

Definition 1.1.4. *[72] Two subsets P and Q is a space $(\mathcal{X}, \tau_{\mathcal{X}})$ are said to be cl-cl-separated if and only if $cl(P) \cap cl(Q) = \emptyset$.*

Definition 1.1.5. *[13] Let P be any subset of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$. Then j-interior and j-closure are defined as follows,*

$$int_j(P) = \vee\{Q : Q \subseteq P, Q \textit{is a j-open set}\}$$
$$cl_j(P) = \cap\{Q : P \subseteq Q, Q \textit{ is a j-closed set}\}$$

Definition 1.1.6. *[35] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is connected if $\mathcal{X}$ cannot be the union of two separated sets.*

Definition 1.1.7. *[35] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is half connected if $\mathcal{X}$ cannot be the union of two half separated sets.*

Definition 1.1.8. *[38] Let $(\mathcal{X}, \tau_{\mathcal{X}})$ be a topological space and $x \in \mathcal{X}$. The component of $\mathcal{X}$ containing x is the union of all connected subsets of $\mathcal{X}$ containing x.*

Definition 1.1.9. *[35] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be locally connected at $\mathcal{X}$ if for every neighborhood U of $\mathcal{X}$, there is a connected neighborhood V of x contained in U.*

Definition 1.1.10. *[13] A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is said to be*

(i) j-continuous if $f^{-1}(P)$ is j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$, for each open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.

(ii) j-irresolute if for each point $x \in \mathcal{X}$ and each j-open set V of $(\mathcal{Y}, \tau_{\mathcal{Y}})$ containing $f(x)$, there exists a j-open set U of $\mathcal{X}$ containing x such that $f(U) \subset V$.

(iii) j-open if $f(P)$ is j-open in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ for each open set P in $(\mathcal{X}, \tau_{\mathcal{X}})$.

(iv) j-closed if $f(P)$ is j-closed in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ for each closed set P in $(\mathcal{X}, \tau_{\mathcal{X}})$.

Definition 1.1.11. *[18] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be disconnected if $\mathcal{X}$ can be expressed as the union of two disjoint non-empty open sets in $\mathcal{X}$ i.,e $\mathcal{X} = P \cup Q$ where P and Q are two non-empty open sets with $R_1 \cap R_2 = \emptyset$.*

Definition 1.1.12. *[62] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is called as extremally disconnected if* $\mathrm{cl}(P)$ *is open for every open sets* P *of* $(\mathcal{X}, \tau_{\mathcal{X}})$.

Definition 1.1.13. *[70] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be hyperconnected if every pair of non-empty open sets of $(\mathcal{X}, \tau_{\mathcal{X}})$ has non-empty intersection. Equivalently, if every non-empty open set is dense in $(\mathcal{X}, \tau_{\mathcal{X}})$ i.,e $cl(P) = \mathcal{X}$ for every open set P in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

Theorem 1.1.14. *[70] In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, the following are equivalent :*

1. *$(\mathcal{X}, \tau_{\mathcal{X}})$ is hyperconnected.*
2. *$scl(P) = \mathcal{X}$ for every non-empty $P \in S0(\mathcal{X})$.*

Lemma 1.1.15. *[70] A functiom $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is semiopen if and only if $f^{-1}(scl_{\mathcal{Y}}(Q)) \subset cl_{\mathcal{X}}(f^{-1}(Q))$ for every subset Q of $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Lemma 1.1.16. *[70] For a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, the following properties hold:*

(i) $\tau_{\mathcal{X}} \subset SO(\mathcal{X}) \cap PO(\mathcal{X})$.

(ii) $SO(\mathcal{X}) \cup PO(\mathcal{X}) \subset SPO(\mathcal{X})$

Lemma 1.1.17. *[70] Let P be a subset of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$. Then the following properties hold:*

(i) $scl(P) = P \cup int[cl(P)]$.

(ii) $pcl(P) = P \cup cl[int(P)]$.

(iii) $spcl(P) = P \cup int(cl[int(P)])$.

Lemma 1.1.18. *[74] A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is hyperconnected if and only if $P \cap Q \neq \emptyset$ for any non-empty sets P, $Q \in SO(\mathcal{X})$.*

Definition 1.1.19. *[42] Let $\mathcal{X}$ be a non empty set. A neutrosophic set P is an object having the form $P = \{< x, \lambda_P(x), \mu_P(x), \nu_P(x) >: x \in \mathcal{X}\}$ where $\lambda_P(x), \mu_P(x)$ and $\nu_P(x)$ represents the degree of membership function, the degree of indeterminancy and the degree of non membership function respectively of each element $x \in \mathcal{X}$ to the set P. It is simply denoted by $P =< x, \lambda_P(x), \mu_P(x), \nu_P(x) >$.*

Definition 1.1.20. *[42] Let $P =< \lambda_P(x), \mu_P(x), \nu_P(x) >$ be a neutrosophic set on $\mathcal{X}$, then the complement of the set P can be defined by the following three kinds as*
$(i)C[P] = \{x, 1 - \lambda_P(x), 1 - \mu_P(x), 1 - \nu_P(x) >: x \in \mathcal{X}\}$
$(ii)C[P] = \{x, \nu_P(x), \mu_P(x), \lambda_P(x) >: x \in \mathcal{X}\}$
$(iii)C[P] = \{x, \nu_P(x), 1 - \mu_P(x), \lambda_P(x) >: x \in \mathcal{X}\}$

Proposition 1.1.21. *[42] For any neutrosophic set P, the following conditions hold:*
(1) $0_N \subseteq P, 0_N \subseteq 0_N$
(2) $P \subseteq 1_N, 1_N \subseteq 1_N$

Definition 1.1.22. *[79] Let $\mathcal{X}$ be a non-empty set and τ be a collection of all neutrosophic subsets on $\mathcal{X}$. Then τ is said to be neutrosophic topology on $\mathcal{X}$ if the following conditions are hold.*
(i) $0_N, 1_N \in \tau$
(ii) $\cup P_i \in \tau, \forall\{P_i; i \in \tau\} \leq \tau$.
(iii) $P_1 \cap P_2 \in \tau$*, for any* $P_1, P_2 \in \tau$.

Then the pair $(\mathcal{X}, \tau)$ is called neutrosophic topological space. The elements of $[\mathcal{X}, \tau]$ are called neutrosophic open sets. A neutrosophic set is said to be neutrosophic closed if its complement is neutrosophic open.

Definition 1.1.23. *[79] Let $\mathcal{X}$ be neutrosophic topological space and $P =< x, \lambda_P(x), \mu_P(x), \nu_P(x) >$ be a neutrosophic set in $\mathcal{X}$. Then the neutrosophic closure and neutrosophic interior of P are defined by*
$\mathcal{N}cl[P] = \cap\{M : M$ *is a neutrosophic closed set in* $\mathcal{X}$ *and* $P \subseteq M\}$.
$\mathcal{N}int[P] = \cup\{N : N$ *is a neutrosophic open set in* $\mathcal{X}$ *and* $N \subseteq P\}$.

It follows that $\mathcal{N}cl[P]$ is neutrosophic closed set and $\mathcal{N}int[P]$ is a neutrosophic open set in $\mathcal{X}$.
(a) P is neutrosophic open set if and only if $P = \mathcal{N}int[P]$.
(b) P is neutrosophic closed set if and only if $P = \mathcal{N}cl[P]$.

Proposition 1.1.24. *[44] For any neutrosophic set P in $\mathcal{X}$ we have*
(a) $\mathcal{N}cl(C[P]) = C(\mathcal{N}int[P])$,
(b) $\mathcal{N}int(C[P]) = C(\mathcal{N}cl[P])$.

Definition 1.1.25. *[44],[79]* *A neutrosophic set P in a nts $\mathcal{X}$ is called*

(i) Neutrosophic semiopen set if $P \subseteq \mathcal{N}cl[\mathcal{N}int[P]]$.

(ii) Neutrosophic Preopen set if $P \subseteq \mathcal{N}int[\mathcal{N}cl[P]]$.

(iii) Neutrosophic regular open set if $P = \mathcal{N}int[\mathcal{N}cl[P]]$.

(iv) Neutrosophic j-open set if $P \subseteq \mathcal{N}int[\mathcal{N}pcl[P]]$.

Definition 1.1.26. *[33]* *A neutrosophic subset P in a neutrosophic topological space $\mathcal{X}$ is called*

(i) neutrosophic dense if $\mathcal{N}cl[P] = 1_N$

(ii) neutrosophic nowhere dense if $\mathcal{N}int[\mathcal{N}cl[P]] = 0_N$.

Proposition 1.1.27. *[48]* *Let $\mathcal{X}$ be neutrosophic topological space and P, Q be two neutrosophic subsets in $\mathcal{X}$. Then the following conditions hold:*

(i) $\mathcal{N}int[P] \subseteq P$

(ii) $P \subseteq \mathcal{N}cl[P]$

(iii) $P \subseteq Q \implies \mathcal{N}int[P] \subseteq \mathcal{N}int[Q]$

(iv) $P \subseteq Q \implies \mathcal{N}cl[P] \subseteq \mathcal{N}cl[Q]$

(v) $\mathcal{N}int[\mathcal{N}int[P]] = \mathcal{N}int[P]$

(vi) $\mathcal{N}cl[\mathcal{N}cl[P]] = \mathcal{N}cl[P]$

(vii) $\mathcal{N}int[P \cap Q] = \mathcal{N}int[P] \cap \mathcal{N}int[Q]$

(viii) $\mathcal{N}cl[P \cup Q] = \mathcal{N}cl[P] \cup \mathcal{N}cl[Q]$

(ix) $\mathcal{N}int[0_N] = 0_N$

(x) $\mathcal{N}int[1_N] = 1_N$

(xi) $\mathcal{N}cl[0_N] = 0_N$

(xii) $\mathcal{N}cl[1_N] = 1_N$

(xiii) $P \subseteq Q \implies C[Q] \subseteq C[P]$

(xiv) $\mathcal{N}cl[P \cap Q] \subseteq \mathcal{N}cl[P] \cap \mathcal{N}cl[Q]$

(xv) $\mathcal{N}int[P \cup Q] \supseteq \mathcal{N}int[P] \cup \mathcal{N}int[Q]$

Definition 1.1.28. *[34] Let $(\mathcal{X}, \tau_{\mathcal{X}})$ and $(\mathcal{Y}, \tau_{\mathcal{Y}})$ be any two neutrosophic topological space. Then the function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called a*

(i) Neutrosophic continuous function if $f^{-1}[P]$ is neutrosophic open in $(\mathcal{X}, \tau_{\mathcal{X}})$, for each neutrosophic open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.

(ii) Neutrosophic contra continuous function if $f^{-1}[P]$ is neutrosophic closed in $(\mathcal{X}, \tau_{\mathcal{X}})$, for each neutrosophic open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.

Theorem 1.1.29. *[68] If $(\mathcal{X}, \tau_{\mathcal{X}})$ is a fuzzy topological space, then:*

(i) The b-closure of a fuzzy b-open set is a fuzzy b-regular closed set.

(ii) The b-interior of a fuzzy b-closed set is a fuzzy b-regular open set.

Proposition 1.1.30. *[85] Let S be a subset of $(\mathcal{X}, \tau_{\mathcal{X}})$, then:*

(i) $int(S) \subseteq pint(S) \subseteq S \subseteq pcl(S) \subseteq S$.

(ii) $pcl(\mathcal{X}/A) = \mathcal{X}/pint(S)$

(iii) $pint(X/S) = X/pcl(S)$.

Definition 1.1.31. *[79] In a neutrosophic topological space $(\mathcal{X}, \tau_{\mathcal{X}})$,*
0_N *may be defined as:*
$(0_1) 0_N = \{< x, 0, 0, 1 >: x \in X\}$
$(0_2) 0_N = \{< x, 0, 1, 1 >: x \in X\}$
$(0_3) 0_N = \{< x, 0, 1, 0 >: x \in X\}$
$(0_4) 0_N = \{< x, 0, 0, 0 >: x \in X\}$
1_N *may be defined as:*
$(1_1) 1_N = \{< x, 1, 0, 0 >: x \in X\}$
$(1_2) 1_N = \{< x, 1, 0, 1 >: x \in X\}$
$(1_3) 1_N = \{< x, 1, 1, 0 >: x \in X\}$
$(1_4) 1_N = \{< x, 1, 1, 1 >: x \in X\}$

Definition 1.1.32. *[51] A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called as feebly continuous if for each $\emptyset \neq P$ of $(\mathcal{Y}, \tau_{\mathcal{Y}})$, $f^{-1}(P) \neq \emptyset$ implies $int[f^{-1}[P] \neq \emptyset$.*

Definition 1.1.33. *[14] A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ from a neutrosophic topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ into a nts $(\mathcal{Y}, \tau_{\mathcal{Y}})$ is called neutrosophic semi continuous if $f^{-1}[P]$ is neutrosophic semi open in $(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Definition 1.1.34. *[14] A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called neutrosophic contra continuous if $f^{-1}(P)$ is neutrosophic closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Definition 1.1.35. *[14] A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called neutrosophic almost continuous if $f^{-1}(P)$ is neutrosophic open in $(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic regular open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$*

Definition 1.1.36. *[50] Let P be the fuzzy set in a fuzzy topological space $(\mathcal{X}, \tau_{\mathcal{X}})$. Then P is called as*

(i) Fuzzy dense if there exist no fuzzy closed set Q in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P < Q < 1$, that is $cl(P) = 1$ in $(\mathcal{X}, \tau_{\mathcal{X}})$.

(ii) Fuzzy nowhere dense set if there exist no non-empty fuzzy open set Q in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $Q < cl(P)$. That is $int[cl(P)] = 0$ in $(\mathcal{X}, \tau_{\mathcal{X}})$.

Definition 1.1.37. *[92] If P is called fuzzy resolvable set in $(\mathcal{X}, \tau_{\mathcal{X}})$ if for each fuzzy closed set Q in $(\mathcal{X}, \tau_{\mathcal{X}})$ $\{cl(Q \wedge P) \wedge cl(Q \wedge (1 - P)\}$ is a fuzzy nowhere dense set in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

Theorem 1.1.38. *[91] In a fuzzy hyperconnected space $(\mathcal{X}, \tau_{\mathcal{X}})$ any fuzzy subset P of $(\mathcal{X}, \tau_{\mathcal{X}})$ is an fuzzy semiopen set if $int[P] \neq 0$ in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

Definition 1.1.39. *[92] Let $(\mathcal{X}, \tau_{\mathcal{X}})$ and $(\mathcal{Y}, \tau_{\mathcal{Y}})$ be any two fuzzy topological spaces. A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called fuzzy resolvable function if $f^{-1}(P)$ is fuzzy resolvable set in $(\mathcal{X}, \tau_{\mathcal{X}})$ for each fuzzy open set P in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Definition 1.1.40. *[65] Let $\mathcal{X}$ be an initial universe and E be a set of parameters. Let $P(\mathcal{X})$ denotes the power set of $\mathcal{X}$ and A be a non-empty subset of E. A pair (F, A)*

is called a soft set over $\mathcal{X}$, where F is a mapping given by $F : P \to P(\mathcal{X})$. In other words, a soft set over $\mathcal{X}$ is a parameterized family of subsets of the universe $\mathcal{X}$. For $e \in P$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 1.1.41. *[61] For two soft sets (F, A) and (G, B) over a common universe $\mathcal{X}$, we say that (F, P) is a soft subset of (G, Q), if*

(i) $P \subseteq Q$

(ii) for all $e \in P$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, P) \subseteq (G, Q)$.

(F, P) is said to be a soft super set of (G, Q), if (G, Q) is a soft subset of (F, P). We denote it by $(F, P) \supset (G, Q)$.

Definition 1.1.42. *[61] Two sets (F, P) and (G, Q) over a common universe $\mathcal{X}$ are said to be soft equal, if (F, P) is a soft subset of (G, Q) and (G, Q) is a soft subset of (F, P).*

Definition 1.1.43. *[61] The union of two soft sets of (F, P) and (G, Q) over the common universe $\mathcal{X}$ is the soft set (H, R) where $R = P \cup Q$ and for all $e \in R$,*

$$H(e) = \begin{cases} F(e), \text{ if } e \in P - Q \\ G(e), \text{ if } e \in Q - P \\ F(e) \cup G(e), \text{ if } e \in P \cap Q \end{cases}$$

We write $(F, P) \cup (G, Q) = (H, R)$.

Definition 1.1.44. *[61] The intersection (H, R) of two soft sets (F, P) and (G, Q) over a common universe $\mathcal{X}$, denoted $(F, P) \cap (G, Q)$, is defined as $R = P \cap Q$ and $H(e) = F(e) \cap G(e)$, for all $e \in R$.*

Definition 1.1.45. *[61] The difference (H, E) of two sets (F, E) and (G, E) over $\mathcal{X}$, denoted by $(F, E)/(G, E)$, is defined as $H(e) = F(e)/G(e)$, for all $e \in E$.*

Definition 1.1.46. *[61] Let (F, E) be a soft set over $\mathcal{X}$ and $\mathcal{Y}$. Then the soft subset of (F, E) over $\mathcal{Y}$, denoted by $(\mathcal{Y}_F, E)$, is defined as follows: $F_\gamma(\alpha) = \gamma \cap F(\alpha)$, for all $\alpha \in E$. In other words, $(\gamma_F, E) = \gamma \cap (F, E)$.*

Definition 1.1.47. *[61] The relative complement of a soft set* (F, A) *is denoted by* $(F, P)^{'}$ *and is defined by* $(F, A)^{'} = (F^{'}, A^{'})$*, where* $F^{'} : P \to P(U)$ *is a mapping given by* $F^{'}(\alpha) = U \backslash F(\alpha)$*, for all* $\alpha \in P$*.*

Definition 1.1.48. *[84] Let* τ *be the collection of soft sets over* $\mathcal{X}$*, then* τ *is said to be a soft topology on* $\mathcal{X}$*, if*

(i) $\emptyset, \overline{\mathcal{X}}$ *belong to* τ*.*

(ii) the union of any number of soft sets in τ *belongs to* τ*.*

(iii) the intersection of any two soft sets in τ *belongs to* τ*.*

The triplet $(\mathcal{X}, \tau, E)$ *is called a soft topological space over* $\mathcal{X}$*.*

Definition 1.1.49. *[27] Let* $(\mathcal{X}, \tau, E)$ *be a soft space over* $\mathcal{X}$*. A soft set* (F, E) *over* $\mathcal{X}$ *is said to be a soft closed set in* $\mathcal{X}$*, if its relative complement* $(F, E)^{'}$ *belongs to* τ*.*

Definition 1.1.50. *[27] Let* $(\mathcal{X}, \tau, E)$ *be a soft topological space over* $\mathcal{X}$*,* (G, E) *be a soft set over* $\mathcal{X}$ *and* $x \in \mathcal{X}$*. Then* (G, E) *is said to be a soft neighborhood of* x*, if there exists a soft open set* (F, E) *such that* $x \in (F, E) \subset (G, E)$*.*

Definition 1.1.51. *[31] Let* $(\mathcal{X}, \tau, E)$ *be a soft topological space over* $\mathcal{X}$ *and* (F, E) *be a soft set over* $\mathcal{X}$*. Then the soft closure of* (F, E) *denoted by* $\overline{(F, E)}$ *is the intersection of all soft closed super sets of* (F, E)*. Clearly* $\overline{(F, E)}$ *is the smallest soft closed set over* $\mathcal{X}$ *which contains* (F, E)*.*

Definition 1.1.52. *[61] Let* $(\mathcal{X}, \tau, E)$ *be a soft topological space over* $\mathcal{X}$*. Then soft boundary of soft set* (F, E) *over* $\mathcal{X}$ *is denoted by* $\underline{(F, E)}$ *and is defined as* $\underline{(F, E)} = \overline{(F, E)} \cap \overline{(F, E)^{'}}$*. Obviously,* $\underline{(F, E)}$ *is a smallest soft closed set over* $\mathcal{X}$ *containing* (F, E)*.*

Definition 1.1.53. *[60] A* α*-cut of a fuzzy set A is a crisp set A*α *that containing all elements in U that have membership values in A greater than or equal to* α*.*

Definition 1.1.54. *[41] A membership function is a curve that defines how each point in the input space is mapped to a membership value between 0 and 1.*

The following notations are carried out in the thesis:

- $(\mathcal{X}, \tau_{\mathcal{X}})$ - Topological Space.
- $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ - Neutrosophic Topological Space.
- $\mathcal{N}SO(\mathcal{X})$ - The collection of neutrosophic semi open sets.
- $\mathcal{N}SJO(\mathcal{X})$ - The collection of neutrosophic semi j-open sets.
- $\mathcal{N}SJO(\mathcal{X})$ - The collection of neutrosophic semi j-closed sets.
- $JO(\mathcal{X})$ - The collection of neutrosophic semi j-open sets.
- $(\mathcal{X}, \tau_j)$ - Topology formed by j-open sets.

1.2 Author's Contributions

With the resource-rich studies reviewed, the researcher records that the present thesis is based on these significant outcomes.

1. j-connectedness and j-disconnectedness.
2. Semi j-hyperconnected spaces.
3. Neutrosophic hyperconnected spaces.
4. Neutrosophic resolvable sets and neutrosophic resolvable functions in neutrosophic hyperconnected spaces.
5. An expert system design to diagnose a coronary artery disease using soft sets.