# Chapter 3

# Semi j-hyperconnected Spaces

## 3.1 Introduction

This chaper begins with the new class of sets called semi j-open set and semi j-closed set and $\frac{1}{2}$ semi j-separated sets in topological spaces exercised with theorems and suitable examples. We established the relationship between these sets and some existing sets like semi open, pre open, alpha open, j-open etc. Semi j-continuous function, semi j-open function and semi j-closed function are defined and analysed with some theorems and examples. Consequently, we proposed the novel spaces namely semi j-connected spaces, $\frac{1}{2}$ semi j-connected and semi j-hyperconnected spaces with the help of semi j-open sets. We also attempted to explore the characteristics of these spaces.

## 3.2 Semi j-open sets

**Definition 3.2.1.** *A subset R of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$ *is called semi j-open if there exist a j-open set J in* $(\mathcal{X}, \tau_{\mathcal{X}})$ *such that* $J \subset R \subset cl(J)$ *or equivalently* $R \subseteq cl(int(pcl(R)))$*. The family of all semi j-open sets of* $(\mathcal{X}, \tau_{\mathcal{X}})$ *is denoted by* $SJO(\mathcal{X})$*.*

**Example 3.2.2.** *Let* $\mathcal{X} = \{q, r, s, t\}$ *with* $\tau_{\mathcal{X}} = \{\emptyset, \{q, r\}, \{q, r, s\}, \mathcal{X}\}$*.*
*For this topology, we obtain the semi j-open sets :* $\emptyset, \{q, r\}, \{q, r, s\}, \{q, r, t\}$ *and* $\mathcal{X}$*.*

**Theorem 3.2.3.** *Let R be a subset of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*. If R is a open set in*

$(\mathcal{X}, \tau_{\mathcal{X}})$*, then R is a semi j-open set in* $(\mathcal{X}, \tau_{\mathcal{X}})$*.*

*Proof.* Since R is open in $(\mathcal{X}, \tau_{\mathcal{X}})$, then $int(R) = R$, we have $R \subseteq pcl(R) \subseteq cl(R)$. Now $R \subseteq pcl(R) \implies int(R) \subseteq int(pcl(R)) \implies R \subseteq int(pcl(R)) \implies cl(R) \subseteq cl(int(pcl(R))) \implies R \subseteq cl(int(pcl(R)))$. Hence R is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Remark 3.2.4.** *The reverse of the previous theorem need not be true as seen in the following example.*

**Example 3.2.5.** *Let* $\mathcal{X} = \{q, r, s, t\}$ *with* $\tau_{\mathcal{X}} = \{\emptyset, \{q\}, \{r\}, \{q, r\}, \{q, s\}, \{q, r, s\}, \mathcal{X}\}$*. For this topology, we have semi j-open sets:* $\emptyset, \{q\}, \{r\}, \{q, r\}, \{q, s\}, \{q, t\}, \{r, t\}$, $\{q, r, s\}, \{q, s, t\}, \{q, r, t\}$ *and* $\mathcal{X}$ *. Clearly* $\{q, t\}, \{r, t\}, \{q, s, t\}, \{q, r, t\}$ *are open in* $(\mathcal{X}, \tau_{\mathcal{X}})$*.*

**Theorem 3.2.6.** *In a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, R is semi j-open if and only if R is semi open.*

*Proof.* Let R be a semi open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R \subseteq cl(int(R))$. Since $R \subseteq pcl(R) \subseteq cl(R)$. Therefore, $R \subseteq cl(int(pcl(R)))$. Thus R is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$.
Conversely, suppose R is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$.
Then $R \subseteq cl(int(pcl(R)))$. By lemma 1.1.17, $R \subseteq cl(int[R \cup (cl(int(R)))]) \subseteq cl(int(R)) \cup cl(int(cl(int(R)))) \subseteq cl(int(R)) \cup cl(int(R)) = cl(int(R))$ which implies $R \subseteq cl(int(R))$. Hence R is semi open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Remark 3.2.7.** *In general, semi j-open sets and pre open sets are independent i.,e every preopen sets need not be semi j-open and every semi j-open sets need not be preopen in* $(\mathcal{X}, \tau_{\mathcal{X}})$*.*

**Example 3.2.8.** *Consider* $\mathcal{X}_1 = \{u, v, w\}$ *with the topology* $\tau_{\mathcal{X}_1} = \{\emptyset, \{u\}, \{v\}, \{u, v\}, \mathcal{X}\}$*. For this* $\tau_{\mathcal{X}_1}$*, we have the collection of preopen sets are* $\emptyset, \{u\}, \{v\}, \{u, v\}$ *and* $\mathcal{X}$*. The collection of semi j-open sets are* $\emptyset, \{u\}, \{v\}, \{u, v\}, \{u, w\}, \{v, w\}$ *and* $\mathcal{X}$*. Hence, every semi j-open sets need not be preopen.*

*Also, we take* $\mathcal{X}_2 = \{u, v, w\}$ *with* $\tau_{\mathcal{X}_2} = \{\emptyset, \{u, v\}, \mathcal{X}\}$*. For this* $\tau_{\mathcal{X}_2}$*,* $\emptyset, \mathcal{X}, \{u\}, \{v\}$, $\{u, v\}, \{u, w\}, \{v, w\}$ *are the collection of preopen sets and* $\emptyset, \mathcal{X}, \{u, v\}$ *are the semi j-open sets. Therefore, every preopen sets need not be semi j-open.*

**Theorem 3.2.9.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, every j-open sets are semi j-open.*

*Proof.* Let R be a j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R \subseteq int(pcl(R))$. Therefore, $cl(R) \subseteq cl(int(pcl(R)))$. This implies $R \subseteq cl(int(pcl(R)))$. Thus R is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Hence every j-open sets are semi j-open. □

**Remark 3.2.10.** *The reverse of the above theorem need not be true as seen in the following example.*

**Example 3.2.11.** *Let $\mathcal{X} = \{q, r, s, t\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}, \mathcal{X}\}$. For $\tau_{\mathcal{X}}$, we obtain the j-open sets are $\emptyset, \{q\}, \{r\}, \{q, r\}, \{q, r, s\}, \{q, r, t\}$ and $\mathcal{X}$. Also, we get $\emptyset, \{q\}, \{r\}, \{q, r\}, \{q, s\}, \{q, t\}, \{r, s\}, \{r, t\}, \{q, r, s\}, \{q, r, t\}$, $\{q, s, t\}, \{r, s, t\}$ and $\mathcal{X}$ are semi j-open sets. Clearly $\{q, s\}, \{q, t\}, \{r, s\}, \{r, t\}$, $\{q, s, t\}, \{r, s, t\}$ are semi j-open sets but not j-open sets.*

**Theorem 3.2.12.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, every semi j-open sets are semi pre-open.*

*Proof.* Let R be a semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R \subseteq cl(int(pcl(R)))$, we have $pcl(R) \subseteq cl(R)$. This implies $R \subseteq cl(int(pcl(R))) \subseteq cl(int(cl(R)))$. Therefore, R is semi pre open in $(\mathcal{X}, \tau_{\mathcal{X}})$. Hence every semi j-open sets are semi preopen in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Remark 3.2.13.** *The following example shows that every semi preopen sets need not be semi j-open sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Example 3.2.14.** *Let $\mathcal{X} = \{q, r, s, t\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{q, t\}, \{r, s\}, \mathcal{X}\}$. For this $\tau_{\mathcal{X}}$, $\emptyset, \{q, t\}, \{r, s\}$ and $\mathcal{X}$ are the semi j-open sets. $\emptyset, \{q\}, \{r\}, \{s\}, \{t\}, \{q, r\}, \{q, s\}, \{q, t\}$, $\{r, s\}, \{r, t\}, \{s, t\}, \{q, r, s\}, \{q, r, t\}, \{q, s, t\}, \{r, s, t\}$ are semi pre open sets.*

**Theorem 3.2.15.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, arbitrary union of semi j-open set is also semi j-open.*

*Proof.* Let $\{R_i\}_{i \in \Delta}$ be a family of semi j-open sets in $(\mathcal{X}, \tau_{\mathcal{X}})$. Since $R_i$ is semi j-open. Then $R_i \subseteq cl(int(pcl(R_i)))$. Now, $\cup_{i \in \Delta} R_i \subseteq \cup_{i \in \Delta} cl(int(pcl(R_i))) \subseteq cl(\cup_{i \in \Delta} int$

$(pcl(R_i))) \subseteq cl(int(\cup_{i\in\Delta} pcl(R_i))) \subseteq cl(int(pcl(\cup_{i\in\Delta} R_i)))$. Hence $\cup_{i\in\Delta} R_i$ is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Remark 3.2.16.** *In general, the intersection of any two semi j-open sets are not semi j-open. The following example verifies this statement.*

**Example 3.2.17.** *Let* $\mathcal{X} = \{r, s, t\}$ *with* $\tau_{\mathcal{X}} = \{\emptyset, \{r\}, \{s\}, \{r, s\}, \mathcal{X}\}$. *Here* $\emptyset, \{r\}, \{s\}, \{r, s\}, \{r, t\}, \{s, t\}$ *and* $\mathcal{X}$ *are the collection of semi j-open sets. Clearly,* $\{r, t\} \cap \{s, t\} = \{t\}$, *which is not semi j-open in* $(\mathcal{X}, \tau_{\mathcal{X}})$.

**Theorem 3.2.18.** *In a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$, *if Q is open and R is semi j-open, then* $Q \cap R$ *is semi j-open.*

*Proof.* Since R is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R \subseteq cl(int(pcl(R)))$. Now, we take $Q \cap R \subseteq Q \cap cl(int(pcl(R))) \subseteq cl(Q \cap int(pcl(R))) \subseteq cl(int(Q \cap pcl(R))) \subseteq cl(int(pcl(Q \cap R)))$. Hence $Q \cap R$ is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Theorem 3.2.19.** *Let* $(\mathcal{X}, \tau_{\mathcal{X}})$ *be a topological space. If S is any subset of* $\mathcal{X}$ *and R is j-open set in* $\mathcal{X}$ *such that* $R \subseteq S \subseteq cl(R)$, *then S is semi j-open in* $(\mathcal{X}, \tau_{\mathcal{X}})$.

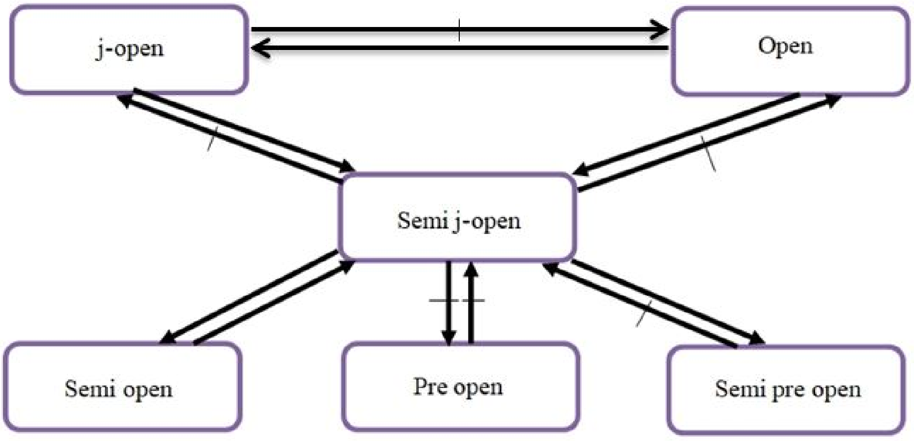
*Proof.* Since R is a j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. then $R \subseteq int(pcl(R))$. Now, we take $S \subseteq cl(R)$. Hence $S \subseteq cl(int(pcl(R))) \subseteq cl(int(pcl(S)))$. Hence S is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

The following diagram indicates the above theorems and examples.

## 3.3 Semi j-closed sets

**Definition 3.3.1.** *A subset S of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$ *is semi j-closed if* $\mathcal{X} - S$ *is semi j-open or equivalently* $int(cl(pint(S))) \subseteq S$. *The family of all semi j-closed sets of* $(\mathcal{X}, \tau_{\mathcal{X}})$ *is denoted by* $SJC(\mathcal{X})$.

**Example 3.3.2.** *Let* $\mathcal{X} = \{3, 4, 5, 6\}$ *with* $\tau_{\mathcal{X}} = \{\emptyset, \{3\}, \{3, 4, 5\}, \{3, 6\}, \mathcal{X}\}$. *In this space* $\tau_{\mathcal{X}}$, *the semi j-closed sets are* $\emptyset, \{4, 5, 6\}, \{5, 6\}, \{4, 6\}, \{4, 5\}, \{6\}, \{5\}, \{4\}$ *and* $\mathcal{X}$.




**Theorem 3.3.3.** *Let S be any subset of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$. Then S is a semi j-closed set if and only if $\mathcal{X} - S$ is semi j-open.*

*Proof.* Assume S is a semi j-closed subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Then, we have $int(cl(pint(S))) \subseteq S$. Now, we taking the complements on both sides, we have $(\mathcal{X} - S) \subseteq \mathcal{X} - [int(cl(pint(S)))] = cl(int(pcl(\mathcal{X} - S)))$. Hence $\mathcal{X} - S$ is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$.
Conversely, Suppose $\mathcal{X} - S$ is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore, $\mathcal{X} - S \subseteq cl(int(pcl(\mathcal{X} - S)))$.Taking the complements on both sides, we have $\mathcal{X} - (cl(int(pcl(\mathcal{X} - S)))) \subseteq (\mathcal{X} - S) \implies int(cl(pint(\mathcal{X} - (\mathcal{X} - S)))) \subseteq S \implies int(cl(pint(S))) \subseteq S$. Hence S is semi j-closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$.

□

**Theorem 3.3.4.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, every closed sets are semi j-closed.*

*Proof.* Let S be a closed subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $cl(S) = S$. We have $intS \subseteq pint(S) \subseteq S \subseteq pcl(S) \subseteq cl(S)$. Now we take the relation $pint(S) \subseteq S \implies cl(pint(S)) \subseteq cl(S) = S \implies int(cl(pint(S))) \subseteq int(S) \subseteq S$. Hence every closed sets are semi j-closed in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Remark 3.3.5.** *The reverse of the above theorem need not be true as verified by the following example.*

**Example 3.3.6.** *From example 3.3.2, clearly* $\{4\}, \{5\}, \{4, 6\}, \{5, 6\}$ *are semi j-closed sets but not closed in* $(\mathcal{X}, \tau_{\mathcal{X}})$

**Theorem 3.3.7.** *In a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, every j-closed sets are semi j-closed.*

*Proof.* Let S be a j-closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $cl(pint(S)) \subseteq S$. This implies $int(cl(pint(S))) \subseteq int(S) \subseteq S$. Thus S is semi j-closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Hence every j-closed sets are semi j-closed. □

**Remark 3.3.8.** *The reverse of the previous theorem may not be true as seen in the following example.*

**Example 3.3.9.** *Consider* $\mathcal{X} = \{3, 4, 5, 6\}$ *with the topology* $\tau_{\mathcal{X}} = \{\emptyset, \{3\}, \{4\}, \{3, 4\}, \mathcal{X}\}$*. For* $\tau_{\mathcal{X}}$*,* $\emptyset, \{4, 5, 6\}, \{3, 5, 6\}, \{5, 6\}, \{5\}, \{6\}, \mathcal{X}$ *are j-closed sets and* $\emptyset, \{4, 5, 6\}, \{3, 5, 6\}, \{5, 6\}, \{4, 6\}, \{4, 5\}, \{3, 6\}, \{3, 5\}, \{6\}, \{5\}, \{4\}, \{3\}, \mathcal{X}$ *are semi j-closed sets. Clearly,* $\{3\}, \{4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}$ *are semi j-closed sets but not j-closed in* $(\mathcal{X}, \tau_{\mathcal{X}})$.

**Theorem 3.3.10.** *If* $\{S_i\}_{i\in\Delta}$ *is the family of semi j-closed sets in a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, then the arbitrary intersection of semi j-closed sets is also semi j-closed.*

*Proof.* Let $\{S_i\}_{i\in\Delta}$ be the family of semi j-closed sets in $(\mathcal{X}, \tau_{\mathcal{X}})$, We have $R_i = \mathcal{X} - S_i$. Therefore, $\{R_i\}_{i\in\Delta}$ is the family of semi j-open sets in $(\mathcal{X}, \tau_{\mathcal{X}})$. Using (3.2.15), $\cup_{i\in\Delta} R_i$ is semi j-open. Therefore, $\mathcal{X} - \cup_{i\in\Delta} R_i$ is semi j-closed. This implies $\cap_{i\in\Delta}[\mathcal{X} - \{R_i\}]$ is semi j-closed. Hence $\cap_{i\in\Delta} S_i$ is semi j-closed in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

**Definition 3.3.11.** *The union of all semi j-open sets in a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$ *contained in R is called as semi j-interior of R and is denoted by* $int_{sj}(R)$*. Equivalently,* $int_{sj}(R) = \cup\{S : S \subseteq R$*, S is a semi j-open set*$\}$.

**Definition 3.3.12.** *The intersection of all semi j-closed sets in a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$ *containing R is called as semi j-closure of R and is denoted by* $cl_{sj}(R)$*. Equivalently,* $cl_{sj}(R) = \cap\{S : R \subseteq S$*, S is a semi j-closed set*$\}$.

**Proposition 3.3.13.** *Let R be any subset of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, then the following statements hold:*

*(i)* $int_{sj}(R) = R$ *iff R is a semi j-open set.*

*(ii)* $cl_{sj}(R) = R$ *iff R is a semi j-closed set.*

*(iii)* $int_{sj}(R)$ *is the biggest semi j-open set contained in R.*

*(iv)* $cl_{sj}(R)$ *is the smallest semi j-closed set containing R.*

*Proof.* Proof is obvious. □

**Proposition 3.3.14.** *Let R be any subset of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, then the following statements are true.*

*(i)* $int_{sj}(\mathcal{X} - R) = \mathcal{X} - cl_{sj}(R)$.

*(ii)* $cl_{sj}(\mathcal{X} - R) = \mathcal{X} - int_{sj}(R)$.

*Proof.* (i) We have, $cl_{sj}(R) = \cap\{S : R \subseteq S$, S is a semi j-closed set$\}$ $\mathcal{X} - cl_{sj}(R) = \mathcal{X} - \cap\{S : R \subseteq S$, S is a semi j-closed set$\} = \cup\{\mathcal{X} - S : R \subseteq S$, S is a semi closed set. $= \cup\{\mathcal{X} - S : \mathcal{X} - S \subseteq \mathcal{X} - R, \mathcal{X} - S$ is a semi j-open$\} = int_{sj}(\mathcal{X} - R)$
(ii) We have, $int_{sj}(R) = \cup\{S : S \subseteq R$, S is a semi j-open set$\}$ $\mathcal{X} - int_{sj}(R) = \mathcal{X} - \cup\{S : S \subseteq R$, S is a semi j-open set$\} = \cap\{\mathcal{X} - S : S \subseteq R$, S is a semi j-open set$\}$ $= \cap\{\mathcal{X} - S : \mathcal{X} - R \subseteq \mathcal{X} - S, \mathcal{X} - S$ is a semi j-closed set$\} = cl_{sj}(\mathcal{X} - R)$. □

**Theorem 3.3.15.** *Let R and S be any two subsets of a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*, then the following conditions hold:*

*(i)* $int_{sj}(\emptyset) = \emptyset$, $int_{sj}(\mathcal{X}) = \mathcal{X}$.

*(ii)* $int_{sj}(R) \subseteq R$

*(iii)* $R \subseteq S$ *implies* $int_{sj}(R) \subseteq int_{sj}(S)$

*(iv)* $int_{sj}(int_{sj}(R)) = int_{sj}(R)$

*(v)* $int_{sj}(R \cap S) \subseteq int_{sj}(R) \cap int_{sj}(S)$

*(vi)* $int_{sj}(R \cup S) \supseteq int_{sj}(R) \cup int_{sj}(S)$

*Proof.* It is evident. □

**Remark 3.3.16.** *In general, the equality of (v) and (vi) is not true. It is verified by the following example.*

**Example 3.3.17.** *For (iv), let* $\mathcal{X} = \{3, 4, 5, 6\}$ *with* $\tau_{\mathcal{X}} = \{\emptyset, \{3\}, \{4\}, \{3, 4\}, \{3, 4, 5\}, \mathcal{X}\}$*. For* $\tau_{\mathcal{X}}$*, we have the collection* $\emptyset, \{3\}, \{4\}, \{3, 4\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}, \{3, 4, 5\}, \{3, 4, 6\}, \{3, 5, 6\}, \{4, 5, 6\}$ *and* $\mathcal{X}$ *are semi j-open sets. Let* $R = \{3, 6\}$ *and* $S = \{4, 5, 6\}$*. Then* $R \cap S = \{6\}$*,* $int_{sj}(R) = \{3, 6\}, int_{sj}(S) = \{4, 5, 6\}$*. Now* $int_{sj}(R \cap S) = \emptyset$ *and* $int_{sj}(R) \cap int_{sj}(S) = \{3, 6\} \cap \{4, 5, 6\} = \{6\}$*. Clearly* $\{6\} \nsubseteq \emptyset$*. Therefore* $int_{sj}(R) \cap int_{sj}(S) \nsubseteq int_{sj}(R \cap S)$*.*
*For (v), Taking* $R = \{3, 4\}$ *and* $S = \{5\}$*. Then* $R \cup S = \{3, 4, 5\}$*,* $int_{sj}(R) = \{3, 4\}, int_{sj}(S) = \emptyset$*. Therefore,* $int_{sj}(R \cup S) = \{3, 4, 5\}$ *and* $int_{sj}(R) \cup int_{sj}(S) = \{3, 4\}$*. Clearly,* $int_{sj}(R \cup S) \nsubseteq int_{sj}(R) \cup int_{sj}(S)$*.*

**Theorem 3.3.18.** *Let R and S be any two subsets in a topological space* $(\mathcal{X}, \tau_{\mathcal{X}})$*. Then the following properties hold:*

*(i)* $cl_{sj}(\emptyset) = \emptyset$ *and* $cl_{sj}(\mathcal{X}) = \mathcal{X}$

*(ii)* $R \subseteq cl_{sj}(R)$

*(iii)* $R \subseteq S$ *implies* $cl_{sj}(R) \subseteq cl_{sj}(S)$

*(iv)* $cl_{sj}(cl_{sj}(R)) = cl_{sj}(R)$

*(v)* $cl_{sj}(R \cap S) \subseteq cl_{sj}(R) \cap cl_{sj}(S)$

*(vi)* $cl_{sj}(R) \cup cl_{sj}(S) \subseteq cl_{sj}(R \cup S)$

*Proof.* (i) and (ii) are clear.
(iii) Let $S \subseteq cl_{sj}(S)$ and $R \subseteq S$. This implies $R \subseteq cl_{sj}(S)$. Since $cl_{sj}(R)$ is the smallest semi j-closed set containing R. Therefore, $cl_{sj}(R) \subseteq cl_{sj}(S)$. Hence $R \subseteq S$ implies $cl_{sj}(R) \subseteq cl_{sj}(S)$.
(iv) Since $cl_{sj}(R)$ is semi j-closed. Hence, $cl_{sj}(cl_{sj}(R)) = cl_{sj}(R)$.
(v) We know that $R \cap S \subseteq R$ and $R \cap S \subseteq S$. By (iii) $cl_{sj}(R \cap S) \subseteq cl_{sj}(R)$ and $cl_{sj}(R \cap S) \subseteq cl_{sj}(S)$. This implies $cl_{sj}(R \cap S) \subseteq cl_{sj}(R) \cap cl_{sj}(S)$.
(vi) Since, $R \subseteq R \cup S$ and $S \subseteq R \cup S$. By (iii)$cl_{sj}(R) \subseteq cl_{sj}(R \cup S)$ and $cl_{sj}(S) \subseteq cl_{sj}(R \cup S)$. Hence, $cl_{sj}(R) \cup cl_{sj}(S) \subseteq cl_{sj}(R \cup S)$. □

**Remark 3.3.19.** *In general the equality (iv) and (v) may not be true as verified by the following example.*

**Example 3.3.20.** *Using 3.3.17, Take $R = \{3, 6\}$ and $S = \{3, 4, 5\}$. Then $R \cap S = \{3\}$, $cl_{sj}(R) = \{3, 6\}$, $cl_{sj}(S) = \mathcal{X}$. Now $cl_{sj}(R \cap S) = \{3\}$ and $cl_{sj}(R) \cap cl_{sj}(S) = \{3, 6\} \cap \mathcal{X} = \{3, 6\}$. Clearly $cl_{sj}(R) \cap cl_{sj}(S) \nsubseteq cl_{sj}(R \cap S)$.*
*For (v), let $R = \{3\}$ and $S = \{4, 5\}$. Then $R \cup S = \{3, 4, 5\}$, $cl_{sj}(R) = \{3\}$ and $cl_{sj}(S) = \{4, 5\}$. Now $cl_{sj}(R \cup S) = \mathcal{X}$ and $cl_{sj}(R) \cup cl_{sj}(S) = \{3\} \cup \{4, 5\} = \{3, 4, 5\}$. Clearly, $cl_{sj}(R \cup S) \nsubseteq cl_{sj}(R) \cup cl_{sj}(S)$.*

**Definition 3.3.21.** *A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called as semi j-continuous if $f^{-1}(R)$ is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$ for every open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

**Remark 3.3.22.** *In general, every continuous function is semi j-continuous function but the reverse need not be true as verified by the following example.*

**Example 3.3.23.** *Let $\mathcal{X} = \{r, s, t\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{r\}, \{s\}, \{r, s\}, \mathcal{X}\}$ and $\mathcal{Y} = \{3, 4, 5, 6\}$ and $\tau_{\mathcal{Y}} = \{\emptyset, \{3\}, \{4, 5\}, \{3, 4, 5\}, \mathcal{Y}\}$. A mapping $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is defined by $f(r) = 3, f(s) = 5, f(t) = 4$. For $\tau_{\mathcal{X}}$, $\emptyset, \{r\}, \{s\}, \{r, s\}, \{r, t\}, \{s, t\}$ are semi j-open sets. Here we have $f^{-1}(3) = \{r\}$, $f^{-1}(4, 5) = \{s, t\}$, $f^{-1}\{3, 4, 5\} = \{r, s, t\}$. This implies $\{r\}, \{s, t\}, \{r, s, t\}$ are semi j-open sets. Therefore, f is semi j-continuous. But f is not continuous because the set $f^{-1}\{4, 5\} = \{s, t\}$ is not open set in $\tau_{\mathcal{X}}$.*

**Theorem 3.3.24.** *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a single valued function from a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ into the another topological space $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then f is semi j-continuous if and only if, for each point $r \in \mathcal{X}$ and for each open set R in $\mathcal{Y}$ with $f(r) \in R$, there exist a semi j-open set S in $\mathcal{X}$ such that $r \in S$ and $f(S) \subseteq R$.*

*Proof.* Suppose $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is semi j-continuous. Take $f(r) \in R$ and $R \subset \mathcal{Y}$, where $R$ is a open set, then $r \in f^{-1}(R) \in SJO(\mathcal{X})$. Since $f$ is semi j-continuous. Put $S = f^{-1}(R)$, therefore $r \in S$ and $f(r) \subseteq R$.
Conversely, let R be a open set in $\mathcal{Y}$ and $r \in f^{-1}(R)$. Then $f(r) \in R$, there exist a neighborhood $S_r \in SJO(\mathcal{X})$ such that $r \in S_r$ and $f(S_r) \subset R$. Therefore, $r \in S_r \subset f^{-1}(R)$ and $f^{-1}(R) = \cup S_r$. Thus $f^{-1}(R) = SJO(\mathcal{X})$. Hence f is semi j-continuous function. □

**Theorem 3.3.25.** *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a single valued function, then f is semi j-continuous iff $f^{-1}(S)$ is semi j-closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$ for every closed set S in $(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

*Proof.* Suppose $f$ is a semi j-continuous function and let S be a closed subset of $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then $\mathcal{Y}-S$ is open in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ and $f^{-1}(\mathcal{Y}-S) = \mathcal{X}-f^{-1}(S) \in SJO(\mathcal{X}$. Hence $f^{-1}(S)$ is semi j-closed in $(\mathcal{X}, \tau_{\mathcal{X}})$.
Conversely, let R be an arbitrary open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$, then $(\mathcal{Y}-R)$ is closed set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Therefore $f^{-1}(\mathcal{Y} - R) = \mathcal{X} - f^{-1}(R)$. Thus $f^{-1}(R)$ is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $f$ is semi j-continuous. □

**Theorem 3.3.26.** *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a single valued function. Then the following properties are equivalent.*

*(i) $f^{-1}(S)$ is semi j-closed for each closed set S in $(\mathcal{Y}, \tau_{\mathcal{Y}})$*

*(ii) $f[cl_{sj}(S)] \subseteq cl[f(S)]$ for each subset S of $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* If (i) is true. Let S be a subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Since $S \subset f^{-1}[f(S)]$, we have $S \subset f^{-1}[cl(f(S))]$, $cl[f(S)]$ is a closed set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Hence $f^{-1}[cl(f(S))]$ is semi j-closed set containing S.
Conversely, let S be an arbitrary closed subset of $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then $f[cl_{sj}(f^{-1}(S))] \subset cl[f(f^{-1}(S)] \subset cl_{sj}(S) = S$. Therefore, $cl_{sj}[f^{-1}(S)] \subseteq f^{-1}(S)$. This implies $f^{-1}(S)$ is semi j-closed for each closed set S in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. □

**Theorem 3.3.27.** *For the function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ the following statements are equivalent:*

*(i) $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is semi j-continuous.*

*(ii) $f^{-1}(int(R)) \subset int_{sj}(f^{-1}(R))$ for every subset R in $(\mathcal{Y}, \tau_{\mathcal{Y}})$*

*Proof.* Let R be any subset of $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then $intR$ is an open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ and $f^{-1}(intR)$ is a semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore, $f$ is semi j-continuous.
Conversely, suppose $f$ is semi j-continuous. Then $f^{-1}(intR)$ is semi j-open and $f^{-1}(intR) \subset f^{-1}(R)$. Hence $f^{-1}(intR) \subseteq int_{sj}[f^{-1}(R)]$. □

**Theorem 3.3.28.** *Let $R \subset \mathcal{X}_0 \subset \mathcal{X}$ and $\mathcal{X}_0 \in SJO(\mathcal{X})$. Then $R \in SJO(\mathcal{X}_0)$.*

*Proof.* Suppose $R \in SJO(\mathcal{X})$, then there is a j-open set $J \subset \mathcal{X}$ such that $J \subset R \subset cl(J)$. Let $cl_{\mathcal{X}}$ be a closure operator in $\mathcal{X}$ and $cl_{\mathcal{X}_0}$ be a closure operator in $\mathcal{X}_0$. Therefore, $J \subset \mathcal{X}_0$ as $\mathcal{X}_0 \subset \mathcal{X}$. Then $J = J \cap \mathcal{X}_0 \subset R \cap \mathcal{X}_0 \subset \mathcal{X}_0 \cap cl_{\mathcal{X}}(J)$ or $J \subset R \subset cl_{\mathcal{X}_0}(J)$. Thus J is j-open in $\mathcal{X}_0$. Hence $R \in SJO(\mathcal{X}_0)$.
Conversely, $R \in SJO(\mathcal{X}_0)$, then there exist a j-open set J such that $J = J \cap \mathcal{X}_0 \subset R \cap \mathcal{X}_0 \subset R \cap \mathcal{X} \subset \mathcal{X} \cap cl_{\mathcal{X}}(J)$ or $J \subset R \subset cl_{\mathcal{X}}(J)$. Hence $R \in SJO(\mathcal{X})$. □

**Theorem 3.3.29.** *If $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is a semi j-continuous function and R is an open set in $(\mathcal{X}, \tau_{\mathcal{X}})$, then $f/R : R \to \mathcal{Y}$ is semi j-continuous function.*

*Proof.* Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is semi j-continuous. Then $f^{-1}(w)$ is a semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$ for each open set $w$ in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Using theorem 3.2.18, $R \cap f^{-1}(w)$ is semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Also, $(f/R)^{-1}(w) = f^{-1}(w) \cap R$ is a semi j-open set in R. Hence $f/R$ is semi j-continuous. □

**Definition 3.3.30.** *A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called semi j-open if $f(R)$ is semi j-open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ for every open set R in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Remark 3.3.31.** *In general, every open function is semi j-open function. But the reverse need not be true as seen in the following example.*

**Example 3.3.32.** *Consider $\mathcal{X} = \{q, r, s\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{q\}, \{q, r\}, \mathcal{X}\}$ and*
*$\mathcal{Y} = \{4, 5, 6\}$ with $\tau_{\mathcal{Y}} = \{\emptyset, \{4\}, \{4, 6\}, \mathcal{Y}\}$.*
*A mapping $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is defined as $f(q) = 4, f(r) = 5, f(s) = 6$.*
*For $\tau_{\mathcal{Y}}$, $\emptyset, \{4\}, \{4, 5\}, \{4, 6\}$ and $\mathcal{X}$ are the collection of semi j-open sets. Clearly f is semi j-open function but not open function. Since $f(q, r) = \{4, 5\}$ is not open in $\tau_{\mathcal{Y}}$.*

**Theorem 3.3.33.** *Let $(\mathcal{X}, \tau_{\mathcal{X}})$, $(\mathcal{Y}\tau_{\mathcal{Y}})$ and $(\mathcal{Z}, \tau_{\mathcal{Z}})$ be three topological spaces and $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$, $g : (\mathcal{Y}, \tau_{\mathcal{Y}}) \to (\mathcal{Z}, \tau_{\mathcal{Z}})$ be the function and $g \circ f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Z}, \tau_{\mathcal{Z}})$ is semi j-open function, then the following statements are true:*

*(i) If f is continuous and onto then g is semi j-open function.*

*(ii) If g is j-open, j-irresolute and one-to-one then f is semi j-open function.*

*Proof.* (i) Let R be an open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Since $g \circ f$ is semi j-open function and $f$ is onto, then $g(R) = (g \circ f)\{f^{-1}(R)\}$ is semi j-open set in $(\mathcal{Z}, \tau_{\mathcal{Z}})$. This implies g is semi j-open function.

(ii) Since g is one-to-one and for every subset S of $(\mathcal{X}, \tau_{\mathcal{X}})$, we have $f(S) = g^{-1}\{g(f(S)\}$. Let $w$ be an open set in $(\mathcal{X}, \tau_{\mathcal{X}})$, then $(g \circ f)(w)$ is semi j-open set in $(\mathcal{Z}, \tau_{\mathcal{Z}})$. $f(w) = g^{-1}(g \circ f)(w) \in SJO(\mathcal{Y})$. This implies $f(w)$ is semi j-open set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Hence $f$ is semi j-open function.

□

**Definition 3.3.34.** *A function $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ is called semi j-closed if $f(S)$ is semi j-closed set in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ for every closed set S in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Remark 3.3.35.** *In general, every closed function is semi j-closed function. But the converse may not be true as shown by the following example.*

**Example 3.3.36.** *Let $\mathcal{X} = \{u, v, w\}$ with $\tau_{\mathcal{X}_1} = \{\emptyset, \{u\}, \{u, v\}, \{u, w\}, \mathcal{X}\}$ and $\tau_{\mathcal{X}_2} = \{\emptyset, \{u\}, \{u, v\}, \mathcal{X}\}$. Define an identity function $f : (\mathcal{X}, \tau_{\mathcal{X}_1}) \to (\mathcal{X}, \tau_{\mathcal{X}_2})$ as $f(u) = u, f(v) = v$ and $f(w) = w$. Then $\tau^c_{\mathcal{X}_1} = \{\emptyset, \{v, w\}, \{v\}, \{w\}, \mathcal{X}\}$ and $\tau^c_{\mathcal{X}_2} = \{\emptyset, \{v, w\}, \{w\}, \mathcal{X}\}$. Clearly $f(v) = \{v\}$ is semi j-closed but not closed in $\tau^c_{\mathcal{X}_2}$.*

**Theorem 3.3.37.** *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a function from a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ into another topological space $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then f is semi j-closed function if and only if $cl_{sj}(f(R)) \subset f(cl_{sj}(R))$ for each subset R of $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* Let R be any subset of $(\mathcal{X}, \tau_{\mathcal{X}})$ and $f$ be semi j-closed function. Then $f(cl_{sj}(R)) \in SJC(\mathcal{Y})$. We know that $f(R) \subset f(cl_{sj}(R))$ which implies that $cl_{sj}(f(R)) \subset f(cl_{sj}(R))$. Conversely, $S \in SJC(\mathcal{X})$. Then $f(S) = f(cl_{sj}(S)) \supset cl_{sj}[f(S)]$. Therefore, $cl_{sj}[f(S)] = f(S)$. Hence $f$ is semi j-closed function. □

## 3.4 $\frac{1}{2}$ semi j-separated sets

**Definition 3.4.1.** *Let $H_1$ and $H_2$ be any two nonempty subsets of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $H_1$ and $H_2$ are called half semi j-separated if $H_1 \cap cl_{sj}(H_2) = \emptyset$ or $cl_{sj}(H_1 \cap H_2) = \emptyset$.*

**Example 3.4.2.** *Let $\mathcal{X} = \{e, f, g\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{e\}, \{f\}, \{e, f\}, \{f, g\}, \mathcal{X}\}$. For $\tau_{\mathcal{X}}$, semi j-open sets are $\emptyset, \{e\}, \{f\}, \{e, f\}, \{f, g\}, \mathcal{X}$. Here, we obtain $\{f\}$ and $\{g\}$ are $\frac{1}{2}$semi j-separated sets. Since $\{f\} \cap cl_{sj}\{g\} = \emptyset$ and $cl_{sj}\{f\} \cap \{g\} = \{f, g\} \cap \{g\} \neq \emptyset$.*

**Theorem 3.4.3.** *Let $\emptyset \neq H_1$ and $\emptyset \neq H_2$ be two subsets in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then the following properties hold:*

*(i) If $H_1$ and $H_2$ are half semi j-separated sets and $G_1 \subseteq H_1$ and $G_2 \subseteq H_2$, then $G_1$ and $G_2$ are also half semi j-separated sets.*

*(ii) If $H_1 \cap H_2 = \emptyset$ and either $H_1$ is semi j-open or semi j-closed or $H_2$ is semi j-open or semi j-closed, then $H_1$ and $H_2$ are half semi j-separated sets.*

*(iii) If either one of $H_1$ and $H_2$ is semi j-open or semi j-closed and if $P = H_1 \cap (\mathcal{X} - H_2)$ and $Q = H_2 \cap (\mathcal{X} - H_1)$, then $P$ and $Q$ are half semi j-separated.*

*Proof.* (i) Let $H_1$ and $H_2$ are half semi j-separated sets. Then we have $H_1 \cap cl_{sj}(H_2) = \emptyset$ or $cl_{sj}(H_1) \cap H_2 = \emptyset$. Suppose $H_1 \cap cl_{sj}(H_2) = \emptyset$. Since $G_1 \subseteq H_1$ and $G_2 \subseteq H_2$. This implies $G_1 \cap cl_{sj}(G_2) \subseteq H_1 \cap cl_{sj}(H_2) = \emptyset$. Therefore, $G_1 \cap cl_{sj}(G_2) = \emptyset$. Hence $G_1$ and $G_2$ are half semi j-separated sets.

(ii) Assume $H_1$ is semi j-open and $H_1 \cap H_2 = \emptyset$. This implies $H_1 \cap cl_{sj}(H_2) = H_1 = \emptyset$. In case $H_1$ is semi j-closed. This implies $cl_{sj}(H_1) \cap H_2 = H_1 \cap H_2 = \emptyset$. Therefore, $H_1$ and $H_2$ are half semi j-separated sets. If $H_2$ is semi j-open or semi j-closed, similarly we obtain $H_1$ and $H_2$ are half semi j-separated sets.

(iii) Case(i)

Let $H_1$ be semi j-closed and $P = H_1 \cap (\mathcal{X} - H_2)$. Now $cl_{sj}(P) \cap Q \subset cl_{sj}(H_1) \cap H_2 \cap (\mathcal{X} - H_1) = H_1 \cap H_2 \cap ((\mathcal{X} - H_1) = \emptyset$. Thus $cl_{sj}(P) \cap Q = \emptyset$. Similarly we obtain $P \cap cl_{sj}(Q) = \emptyset$. Therefore $P$ and $Q$ are half semi j-separated sets.

Case(ii)

Let $H_1$ be semi j-open and $Q = H_2 \cap (\mathcal{X} - H_1)$. Now $P \cap cl_{sj}(H_2) \subset H_1 \cap (\mathcal{X} - H_2) \cap H_2 = \emptyset$. This implies $P \cap cl_{sj}(Q) = \emptyset$. Hence $P$ and $Q$ are half semi j-separated sets.

□

**Theorem 3.4.4.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, the subsets $H_1$ and $H_2$ are half semi j-separated if and only if there exist $M \in SJO(\mathcal{X})$ such that $H_1 \subset M$ and $H_2 \cap M = \emptyset$ or there exist $N \in SJO(\mathcal{X})$ such that $H_2 \subset N$ and $H_1 \cap N = \emptyset$.*

*Proof.* Let $H_1$ and $H_2$ be half semi j-separated sets. Therefore, we have $H_1 \cap cl_{sj}(H_2) = \emptyset$ or $cl_{sj}(H_1) \cap H_2 = \emptyset$. Assume $H_1 \cap cl_{sj}(H_2) = \emptyset$ and take $M = \mathcal{X} - cl_{sj}(H_2)$. This implies $M \in SJO(\mathcal{X})$, $H_1 \subset M$ and $H_2 \cap M = \emptyset$. Suppose $cl_{sj}(H_1) \cap H_2 = \emptyset$ and take $N = \mathcal{X} - cl_{sj}(H_1)$. This implies $N \in SJO(\mathcal{X})$, $H_2 \subset N$ and $H_1 \cap N = \emptyset$. Conversely, assume there exists $M \in SJO(\mathcal{X})$ such that $H_1 \subset M$ and $H_2 \cap M = \emptyset$. This implies $cl_{sj}(H_2) \cap M = \emptyset$ and hence $H_1 \cap cl_{sj}(H_2) \subset M \cap cl_{sj}(H_2) = \emptyset$. Thus $H_1$ and $H_2$ are half semi j-separated sets. The proof is similar to the another case. □

## 3.5 $\frac{1}{2}$ semi j-connected spaces

**Definition 3.5.1.** *A subset $C$ of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is called as $\frac{1}{2}$semi j-connected if $C \neq H_1 \cup H_2$ such that $H_1$ and $H_2$ are non empty $\frac{1}{2}$semi j-separated sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Theorem 3.5.2.** *A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is $\frac{1}{2}$ semi j-connected iff $\mathcal{X} \neq H_1 \cup H_2$ and $H_1 \cap H_2 = \emptyset$ such that $H_1$ and $H_2$ are non empty semi j-open and semi j-closed sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* Let $(\mathcal{X}, \tau_{\mathcal{X}})$ be a $\frac{1}{2}$ semi j-connected space. Assume $\mathcal{X} = H_1 \cup H_2$ and $H_1 \cap H_2 = \emptyset$. Also $\emptyset = R$ and $\emptyset = S$ be a semi j-open set and semi j-closed sets in $(\mathcal{X}, \tau_{\mathcal{X}})$ respectively. Therefore, $H_1 \cap cl_{sj}(H_2) = \emptyset$. This implies $H_1$ and $H_2$ are $\frac{1}{2}$semi j-separated. Thus $(\mathcal{X}, \tau_{\mathcal{X}})$ is not a $\frac{1}{2}$semi j-connected, which is a contradiction. Hence $\mathcal{X} \neq H_1 \cup H_2$.

Conversely, assume $(\mathcal{X}, \tau_{\mathcal{X}})$ is not a $\frac{1}{2}$semi j-connected. Then we obtain $\mathcal{X} = H_1 \cup H_2$, where $H_1$ and $H_2$ are nonempty $\frac{1}{2}$semi j-separated sets. Therefore, $H_1 \cap cl_{sj}(H_2) = \emptyset$ and $cl_{sj}(H_1) \cap H_2 = \emptyset$. Taking $H_1 \cap cl_{sj}(H_2) = \emptyset$, $H_1 = \mathcal{X} - cl_{sj}(H_2)$ and $H_2 = \mathcal{X} - H_1$. This implies $H_1 \cup H_2 = \mathcal{X}$ and $H_1 \cap H_2 = \emptyset$ where $H_1$ and $H_2$ are semi j-open and semi j-closed sets in $(\mathcal{X}, \tau_{\mathcal{X}})$ respectively, which is a contradiction to $\mathcal{X} \neq H_1 \cup H_2$. Similarly we have for $cl_{sj}(H_1) \cap H_2 = \emptyset$. ☐

**Theorem 3.5.3.** *If a subset $C$ of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is $\frac{1}{2}$semi j-connected, then $cl_{sj}(C)$ is also $\frac{1}{2}$ semi j-connected.*

*Proof.* Let $C$ is $\frac{1}{2}$semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$. Assume $cl_{sj}(C)$ is not $\frac{1}{2}$semi j-connected, then we have two $\frac{1}{2}$semi j-separated sets $H_1$ and $H_2$ in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $cl_{sj}(C) = H_1 \cup H_2$. Taking $C = (H_1 \cap C) \cap (H_2 \cap C)$ and $cl_{sj}(H_1) \cap H_2 = \emptyset$. This implies $cl_{sj}(H_1 \cap C) \cap (H_2 \cap C) = \emptyset$. Thus $C$ is not $\frac{1}{2}$semi j-connected, which is a contradiction. Hence $cl_{sj}(C)$ is $\frac{1}{2}$semi j-connected. ☐

**Theorem 3.5.4.** *If $C$ is a $\frac{1}{2}$semi j-connected subset of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ and $H_1$, $H_2$ are the $\frac{1}{2}$semi j-separated subsets of $(\mathcal{X}, \tau_{\mathcal{X}})$ with $C \subset H_1 \cup H_2$ then either $C \subset H_1$ or $C \subset H_2$.*

*Proof.* Let $C$ be a $\frac{1}{2}$semi j-connected subset in $(\mathcal{X}, \tau_{\mathcal{X}})$. Taking $C \subset H_1 \cup H_2$, where $H_1$ and $H_2$ are $\frac{1}{2}$semi j-separated. Therefore, we have $cl_{sj}(H_1) \cap H_2 = \emptyset$. Put $cl_{sj}(H_1) \cap H_2 = \emptyset$ and $C = (C \cap H_1) \cup (C \cap H_2)$. This implies $(C \cap H_2) \cap cl_{sj}(C \cap H_1) \subset H_2 \cap cl_{sj}(H_1) = \emptyset$. Suppose $(C \cap H_1) \neq \emptyset$. and $(C \cap H_2) = \emptyset$. Therefore $C$ is not $\frac{1}{2}$semi j-connected, which contradicts our assumption. Thus either $C \cap H_1 = \emptyset$ or $C \cap H_2 = \emptyset$, which implies $C \subset H_1$ or $C \subset H_2$. Similarly we obtain the result for $H_1 \cap cl_{sj}(H_2) = \emptyset$. ☐

**Theorem 3.5.5.** *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a semi j-continuous function. If $\mathcal{X}$ is $\frac{1}{2}$semi j-connected, then continuous image $f(\mathcal{X})$ is also $\frac{1}{2}$semi j-connected.*

*Proof.* Let $f$ be a semi j-continuous function and $\mathcal{X}$ be $\frac{1}{2}$semi j-connected. Assume $f(\mathcal{X})$ is not $\frac{1}{2}$semi j-connected subset of $(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then, we obtain two $\frac{1}{2}$semi j-separated sets $H_1$ and $H_2$ in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ such that $f(\mathcal{X}) = H_1 \cup H_2$. Therefore, we have $cl_{sj}(H_1) \cap H_2 = \emptyset$ or $H_1 \cap cl_{sj}(H_2)n = \emptyset$. Since $f$ is semi j-continuous, therefore $cl_{sj}(f^{-1}(H_1)) \cap f^{-1}(H_2) \subset f^{-1}(cl_{sj}(H_1)) \cap f^{-1}(H_2) = f^{-1}(cl_{sj}(H_1) \cap H_2) = \emptyset$. Also, $f^{-1}(H_1) \cap cl_{sj}(f^{-1}(H_2)) \subset f^{-1}(H_1) \cap f^{-1}(cl_{sj}(H_2)) = f^{-1}(H_1 \cap cl_{sj}(H_2)) = \emptyset$. Since $H_1 \neq H_2$, then there exist a point $h \in \mathcal{X}$ such that $f(h) \in H_1$ and $f^{-1}(H_1) \neq \emptyset$. Equivalently, $f^{-1}(H_2) \neq \emptyset$. Therefore, $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are $\frac{1}{2}$semi j-separated such that $\mathcal{X} = f^{-1}(H_1) \cup f^{-1}(H_2)$. This implies $\mathcal{X}$ is not $\frac{1}{2}$semi j-connected, which is a contradiction. Hence $f(\mathcal{X})$ is $\frac{1}{2}$semi j-connected in $(\mathcal{Y}, \tau_{\mathcal{Y}})$ □

**Theorem 3.5.6.** *If $C_1$ and $C_2$ are $\frac{1}{2}$semi j-connected subsets of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ and $C_1$, $C_2$ are not $\frac{1}{2}$semi j-separated, then $C_1 \cup C_2$ is $\frac{1}{2}$semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* Let $C_1$ and $C_2$ be $\frac{1}{2}$semi j-connected sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.Assume $C_1 \cup C_2$ is not $\frac{1}{2}$semi j-connected. Then, there exist two $\frac{1}{2}$semi j-separated sets $H_1$ and $H_2$ such that $C_1 \cup C_2 = H_1 \cup H_2$ and $cl_{sj}(H_1) \cap H_2 = \emptyset$ or $H_1 \cap cl_{sj}(H_2) = \emptyset$. Taking $H_1 \cap cl_{sj}(H_2) = \emptyset$. Since $C_1$ and $C_2$ are $\frac{1}{2}$semi j-connected sets, we have $C_1 \subset H_1$ and $C_2 \subset H_2$ or $C_2 \subset H_1$ and $C_1 \subset H_2$. In the first case, $C_1 \subset H_1$ and $C_2 \subset H_2$, we have $C_1 \cap cl_{sj}(C_2) \subset H_1 \cap cl_{sj}(H_2) = \emptyset$. This implies $C_1$ and $C_2$ are $\frac{1}{2}$semi j-separated, which is a contradiction. Hence $C_1 \cup C_2$ is $\frac{1}{2}$semi j-connected.
In the second case, if $C_2 \subset H_1$ and $C_1 \subset H_2$, then $cl_{sj}(C_1) \cap C_2 \subset cl_{sj}(H_2) \cap H_1 = \emptyset$. This implies $C_1$ and $C_2$ are $\frac{1}{2}$semi j-separated sets which contradicts assumption. Hence $C_1 \cup C_2$ is $\frac{1}{2}$semi j-sonnected. Similarly, we prove for $cl_{sj}(H_1) \cap H_2 = \emptyset$. □

**Theorem 3.5.7.** *Let $\{C_\alpha : \alpha \in \Delta\}$ be a non empty family of half semi j-connected subsets in a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ and $\bigcup\limits_{\alpha\in\Delta} C_\alpha \neq \emptyset$, then $\bigcup\limits_{\alpha\in\Delta} C_\alpha$ is also half semi j-connected.*

*Proof.* Assume $\bigcup\limits_{\alpha\in\Delta} C_\alpha$ is not half semi j-connected. Then we obtain $\bigcup\limits_{\alpha\in\Delta} C_\alpha = H_1 \cup H_2$, where $H_1$ and $H_2$ are non-empty half semi j-separated sets in $(\mathcal{X}, \tau_{\mathcal{X}})$. Since $\bigcap_{\alpha\in\Delta} C_\alpha \neq \emptyset$, there exists a point $h \in \bigcap_{\alpha\in\Delta} C_\alpha$. This implies $h \in \bigcup\limits_{\alpha\in\Delta} C_\alpha$, then we have either $h \in H_1$ or $h \in H_2$. Suppose $h \in H_1$. Since $h \in C_\alpha$ for each $\alpha \in \Delta$, then $C_\alpha$ and $H_1$ intersect for each $\alpha \in \Delta$. By theorem 3.5.4, $C_\alpha \subset H_1$ or $C_\alpha \subset H_2$. Since $H_1 \cap H_2 = \emptyset$, $C_\alpha \in H_1$ for all $\alpha \in \Delta$. Thus $\bigcup_{\alpha\in\Delta} C_\alpha \subset H_1$. This implies $H_2 = \emptyset$, which is a contradiction. Suppose $h \in H_2$, similarly we obtain $H_1 = \emptyset$, which is a contradiction. Thus $\bigcup_{\alpha\in\Delta} C_\alpha$ is half semi j-connected. □

**Theorem 3.5.8.** *Let $\{D_\alpha : \alpha \in \Delta\}$ be a family of half semi j-connected sets and $D$ be a half semi j-connected set. If $D \cap D_\alpha \neq \emptyset$ for every $\alpha \in \Delta$ then $D \cup (\bigcup_{\alpha\in\Delta} D_\alpha)$ is half semi j-connected.*

*Proof.* Let $D \cap D_\alpha \neq \emptyset$ for each $\alpha \in \Delta$. Using theorem 3.5.7, $\cup D_\alpha$ is half semi j-connected for each $\alpha \in \Delta$. Now, $D \cup (\cup D_\alpha) = \cup(D \cup D_\alpha)$ and $\cap(D \cup D_\alpha) \supset D \neq \emptyset$. Hence $D \cup (\cup D_\alpha)$ is half semi j-connected. □

## 3.6 Semi j-Connected Spaces

**Definition 3.6.1.** *Let R and S be the semi j-open subsets of a topological space $(\mathcal{X}, \tau_\mathcal{X})$. Then R and S are said to be semi j-separated if $R \cap cl_{sj}(S) = \emptyset$ and $cl_{sj}(R) \cap S = \emptyset$.*

**Theorem 3.6.2.** *If R and S are semi j-separated sets in $(\mathcal{X}, \tau_\mathcal{X})$, then R and S are disjoint.*

*Proof.* Let R and S be semi j-separated sets in $(\mathcal{X}, \tau_\mathcal{X})$. Then $cl_{sj}(R) \cap S = \emptyset$ and $R \cap cl_{sj}(S) = \emptyset$, we have $R \subseteq cl_{sj}(R) \subseteq cl(R)$. Therefore $R \cap S \subseteq cl_{sj}(R) \cap S = \emptyset$. Thus $R \cap S = \emptyset$. Hence R and S are disjoint in $(\mathcal{X}, \tau_\mathcal{X})$. □

**Remark 3.6.3.** *The reverse of the previous theorem may not be true as verified by the following example.*

**Example 3.6.4.** *Let $\mathcal{X} = \{m, n, o, p\}$ with $\tau_\mathcal{X} = \{\emptyset, \{m\}, \{n\}, \{m, n\}, \mathcal{X}\}$. Here the subsets $\{m, n\}$ and $\{o, p\}$ are disjoint sets but not j-separated. Since $cl_{sj}\{m, n\} \cap \{o, p\} = \mathcal{X} \cap \{o, p\} = \{o, p\} \neq \emptyset$.*

**Definition 3.6.5.** *Let $(\mathcal{X}, \tau_\mathcal{X})$ be the topological space. Then $(\mathcal{X}, \tau_\mathcal{X})$ is said to be semi j-connected if there is no such pair R and S of non-empty disjoint semi j-open subsets of*

*$(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{X} = R \cup S$, otherwise $(\mathcal{X}, \tau_{\mathcal{X}})$ is called as semi j-disconnected. In this case, R and S is called as semi j-disconnection of $\mathcal{X}$.*

**Theorem 3.6.6.** *For any topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, the statements below are equivalent:*

*(i) $\mathcal{X}$ is semi j-connected.*

*(ii) The only semi j-open and semi j-closed sets are $\emptyset$ and $\mathcal{X}$.*

*(iii) $\mathcal{X} \neq R \cup S$, for each non-empty disjoint semi j-open sets R and S.*

*(iv) $\mathcal{X} \neq P \cup Q$, for each non-empty disjoint semi j-closed sets P and Q.*

*(v) $\mathcal{X} \neq R \cup S$, where R and S are the disjoint non-empty semi j-separated sets.*

*Proof.* $(i) \implies (ii)$
Let $\emptyset \neq R$ be a proper subset of $(\mathcal{X}, \tau_{\mathcal{X}})$, which is both semi j-open and semi j-closed in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then there exists a two sets M and N such that $M = R$ and $N = \mathcal{X} - R$, which forms a semi j-separation of $\mathcal{X}$. Therefore $\mathcal{X}$ and $\emptyset$ are the only semi j-open and semi j-closed sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.
$(ii) \implies (iii)$
Assume (iii) is not true. Then $\mathcal{X} = R \cup S$, R and S are disjoint non-empty semi j-open sets. Since $\mathcal{X} - R = S$ and $\mathcal{X} - S = R$. This implies R is both semi j-open and semi j-closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$ which contradicts (ii). Therefore, $\mathcal{X} \neq R \cup S$ for each nonempty disjoint semi j-open sets R and S in $(\mathcal{X}, \tau_{\mathcal{X}})$.
$(iii) \implies (ii)$
Similar to $(ii) \implies (iii)$.
$(iv) \implies (v)$
Suppose (v) is not true, then $\mathcal{X} = R \cup S$, where R and S are nonempty semi j-separated sets. Since $R \cap cl_{sj}(S) = \emptyset$, we have $cl_{sj}(S) \subseteq cl(S)$. This implies S is semi j-closed. Similarly, R is also semi j-closed which contradicts (iv). Therefore, $\mathcal{X} \neq R \cup S$, where R and S are nonempty semi j-separated set in $(\mathcal{X}, \tau_{\mathcal{X}})$.
$(v) \implies (i)$
Assume the contrary, $\mathcal{X}$ is not semi j-connected. Then $\emptyset \neq R$ is a proper semi j-open

and semi j-closed subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Let $S = \mathcal{X} - R$. Therefore, $R$ forms a semi j-separation of $\mathcal{X}$ which contradicts $(v)$. Thus $\mathcal{X}$ is semi j-connected. □

**Theorem 3.6.7.** *If R is a semi j-connected set, D and E are the semi j-separated sets in a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, such that $R \subseteq D \cup E$. Then either $R \subseteq D$ or $R \subseteq E$.*

*Proof.* Let $R = (R \cap D) \cup (R \cap E)$. This implies $(R \cap D) \cap cl_{sj}(R \cap E) \subseteq (R \cap D) \cap cl_{sj}(R) \cap cl_{sj}(E) = \emptyset$. Similarly, $cl_{sj}(R \cap D) \cap (R \cap E) \subseteq cl_{sj}(R) \cap cl_{sj}(D) \cap R \cap E = \emptyset$, since D and E are separated sets. If $R \cap D = \emptyset$ and $R \cap E = \emptyset$, then R is not semi j-connected sets which contradicts our assumption. Thus either $R \cap D = \emptyset$ or $R \cap E = \emptyset$, which implies $R \subset D$ or $R \subset E$. □

**Theorem 3.6.8.** *If R is semi j-connected subset of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, then $cl_{sj}(R)$ is also semi j-connected set in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* Suppose $cl_{sj}(R)$ is not semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then, there exists the semi j-separated sets D and E such that $cl_{sj}(R) = D \cup E$, we have $R \subseteq cl_{sj}(R)$. Taking $R = (R \cap D) \cup (R \cap E)$. Using 3.6.7, we have $R \subseteq D$ or $R \subseteq E$.

(i) If $R \subseteq D$, then $cl_{sj}(R) \subseteq cl_{sj}(D)$, we have $cl_{sj}(D) \cap E = \emptyset$. This implies $cl_{sj}(R) \cap E = \emptyset$, since $E \subseteq cl_{sj}(R)$. Therefore $E = \emptyset$, which is a contradiction.

(ii) If $R \subseteq E$, similarly we have $D = \emptyset$, which is a contradiction. Hence $cl_{sj}(R)$ is semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$.

□

**Theorem 3.6.9.** *If R is semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $R \subseteq S \subseteq cl_{sj}(R)$, then S is also semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* Assume S is not semi j-connected, then there exist two sets D and E such that $cl_{sj}(D) \cap E = \emptyset$, $D \cap cl_{sj}(E) = \emptyset$ and $S = D \cup E$. Since, $R \subseteq S$, then $R \subseteq D$ or $R \subseteq E$. Suppose $R \subseteq D$, then $cl_{sj}(R) \subseteq cl_{sj}(D)$ and $E \cap cl_{sj}(R) = \emptyset$. By hypothesis, $E \subseteq S \subseteq cl_{sj}(R)$. This implies $E = E \cap cl_{sj}(R) = \emptyset$. But $E \neq \emptyset$. Similarly, we have $D = \emptyset$, $E = E \cap cl_{sj}(R) = \emptyset$. But $E \neq \emptyset$. Similarly, we have $D = \emptyset$, which is a contradiction. Hence S is semi j-connected set in $(\mathcal{X}, \tau_{\mathcal{X}})$.

□

## 3.7 Semi j hyperconnected spaces

**Definition 3.7.1.** *A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j hyperconnected if the intersection of any two non empty semi j open sets is non empty.Equivalently, $cl_{sj}(R) = \mathcal{X}$ for every semi j-open sets in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Example 3.7.2.** *Let $\mathcal{X} = \{1, 2, 3, 4\}$,$\tau = \{\emptyset, \{2\}, \{2, 3, 4\}, \mathcal{X}\}$ be a topology on $\mathcal{X}$. $SJO(\mathcal{X}) = \{\emptyset, \{2\}, \{1, 2\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \mathcal{X}\}$ is semi j hyperconnected.*

**Definition 3.7.3.** *A subset R of $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be*

*(i) semi j-regular open if $A = int_{sj}(cl_{sj}(R))$.*

*(ii) semi j-regular closed if $R = cl_{sj}(int_{sj}(S))$.*

**Definition 3.7.4.** *A subset $R$ of topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be semi j-boundary of $R$ [simply $bd_{sj}(R)$] if $bd_{sj}(R) = cl_{sj}(R) \cap cl_{sj}(\mathcal{X} - R)$.*

**Definition 3.7.5.** *A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is called semi j-extremally disconnected iff semi j-closure of each semi j-open set is semi j-open in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

**Theorem 3.7.6.** *Every semi j-hyperconnected space $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-extremally disconnected space.*

*Proof.* Assume $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-hyperconnected space. Then for every open set R in $(\mathcal{X}, \tau_{\mathcal{X}})$, we have $cl_{sj}(R) = \mathcal{X}$. This implies that $cl_{sj}(R)$ is semi j-open. Hence $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-extremally disconnected spaces. □

**Remark 3.7.7.** *The reverse of the above theorem need not be true as seen in the following example.*

**Example 3.7.8.** *Let $\mathcal{X} = \{r,s,t\}$ with $\tau_{\mathcal{X}} = \{\emptyset, \{r\}, \{s\}, \{r,s\}, \mathcal{X}\}$. For this $\tau_{\mathcal{X}}$, we have the semi j-open sets are $\emptyset, \{r\}, \{s\}, \{r,s\}, \{r,t\}$ and {s,t}. Clearly $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-extremally disconnected space but not semi j-hyperconnected space, since $cl_{sj}\{r\} = \{r\} \neq \mathcal{X}$.*

**Definition 3.7.9.** *A subset $R$ of a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be*

*(i) semi j-dense if $cl_{sj}(R) = \mathcal{X}$.*

*(ii) semi j-nowhere dense if $int_{sj}(cl_{sj}(R)) = \emptyset$.*

**Theorem 3.7.10.** *For any topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, the following statements are equivalent.*

*(i) $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-hyperconnected space.*

*(ii) Each subset R of $(\mathcal{X}, \tau_{\mathcal{X}})$ is either semi j-dense or semi j-nowhere dense in $(\mathcal{X}, \tau_{\mathcal{X}})$.*

*Proof.* $(i) \implies (ii)$
Let R be any subset of a semi j-hyperconnected space $(\mathcal{X}, \tau_{\mathcal{X}})$. Suppose R is not semi j-nowhere dense set. Then $int_{sj}(cl_{sj}(R)) \neq \emptyset$. This implies $cl_{sj}(int_{sj}(cl_{sj}(R))) = \mathcal{X} \subseteq cl_{sj}(R)$. Therefore, $cl_{sj}(R) = \mathcal{X}$. Hence R is semi j-dense set.
$(ii) \implies (i)$
Let $\emptyset \neq R$ be any semi j-open set in $(\mathcal{X}, \tau_{\mathcal{X}})$, we have $R \subseteq int_{sj}(cl_{sj}(R))$. This implies that R is not semi j-nowhere dense set. By hypothesis R is semi j-dense. Hence $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-hyperconnected space. □

**Theorem 3.7.11.** *In a topological space $(\mathcal{X}, \tau_{\mathcal{X}})$, each of the following statements are equivalent.*

*(i) $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j hyperconnected.*

*(ii) cl(R)=$\mathcal{X}$ for every non empty set $R \in SJO(\mathcal{X})$.*

*(iii) scl(R)=$\mathcal{X}$ for every non empty set $R \in SJO(\mathcal{X})$.*

*Proof.* $(i) \implies (ii)$
Let $\emptyset \neq R$ be any semi j open set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R \subseteq cl(int(pcl(R)))$. This implies

$int(pcl(R)) \neq \emptyset$. Hence $cl(int(pcl(R))) = \mathcal{X} = cl(R)$. Since $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j hyperconnected.

$(ii) \implies (iii)$

Let R be any non empty semi j open set in $\mathcal{X}$. Then $scl(R) = R \cup int(cl(R)) = R \cup int(\mathcal{X}) = \mathcal{X}$. Since $cl(R) = \mathcal{X}$ for every non empty semi j open set in $(\mathcal{X}, \tau_{\mathcal{X}})$.

$(iii) \implies (i)$

For every non empty semi j open set $R$ in $(\mathcal{X}, \tau_{\mathcal{X}})$ and $scl(R) = \mathcal{X}$. Clearly $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j hyperconnected. □

**Theorem 3.7.12.** *Let $(\mathcal{X}, \tau_{\mathcal{X}})$ be a topological space, then the following statements are equivalent.*

*(i)* $(\mathcal{X}, \tau_{\mathcal{X}})$ *is semi j hyperconnected.*

*(ii)* $(\mathcal{X}, \tau_{\mathcal{X}})$ *does not have proper semi j regular open or proper semi j regular closed subset in* $\mathcal{X}$.

*(iii)* $(\mathcal{X}, \tau_{\mathcal{X}})$ *has no proper disjoint semi j open subsets E and F such that* $\mathcal{X} = cl_{sj}(E) \cup F = E \cup cl_{sj}(F)$.

*(iv)* $(\mathcal{X}, \tau_{\mathcal{X}})$ *does not have proper semi j closed subsets M and N such that* $X = M \cup N$ *and* $int_{sj}(M) \cap N = M \cap int_{sj}(N) = \emptyset$.

*Proof.* $(i) \implies (ii)$

Let $\emptyset \neq R$ be any semi j regular open subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Then $R = int_{sj}(cl_{sj}(R))$. Since $\mathcal{X}$ is semi j hyperconnected. Therefore $cl_{sj}(R) = \mathcal{X}$. This implies $R = \mathcal{X}$. Hence $R$ cannot be a proper semi j regular open subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. Clearly $(\mathcal{X}, \tau_{\mathcal{X}})$ cannot have a proper semi j regular closed subset.

$(ii) \implies (iii)$

Assume that there exist two disjoint proper semi j open subsets $\emptyset \neq E$ and $\emptyset \neq F$ such that $\mathcal{X} = cl_{sj}(E) \cup F = E \cup cl_{sj}(F)$. Then $cl_{sj}(E) = cl_{sj}(int_{sj}(E)) \neq \emptyset$ is the semi j regular closed set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Since $E \cap F = \emptyset$ and $cl_{sj}(E) \cap F = \emptyset$. This implies $cl_{sj}(E) \neq \mathcal{X}$. Therefore $\mathcal{X}$ has a proper semi j regular closed subset E which is a contradiction to (ii).

$(iii) \implies (iv)$

Suppose there exist two proper non empty semi j closed subsets M and N in $\mathcal{X}$ such that $\mathcal{X} = M \cup N$, $int_{sj}(M) \cap N = M \cap int_{sj}(N) = \emptyset$ then $E = \mathcal{X} - M$ and $F = \mathcal{X} - N$ are two disjoint non empty semi j open sets such that $\mathcal{X} = cl_{sj}(E) \cup F = E \cup cl_{sj}(E)$ which is prohibitive to (iii).

$(iv) \implies (i)$

Assume that there exist a proper semi j open subset $\emptyset \neq R$ of $\mathcal{X}$ such that $cl_{sj}(R) \neq \mathcal{X}$. Then $int_{sj}(cl_{sj}(R)) \neq \mathcal{X}$. Put $cl_{sj}(R) = M$ and $N = \mathcal{X} - int_{sj}(cl_{sj}(R))$. Thus $\mathcal{X}$ has two proper semi j closed subsets M and N such that $\mathcal{X} = M \cup N$, $int_{sj}(M) \cap N = M \cap int_{sj}(N) \neq \emptyset$. This contradicts $(iv)$. □

**Theorem 3.7.13.** *A topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ is semi j-hyperconnected if and only if the intersection of any two semi j open set is also semi j open and it is semi j connected.*

*Proof.* In a semi j-hyperconnected space, we have $cl(R \cap S) = cl(R) \cap cl(S)$, where $R$ and $S$ are semi j open sets. It follows that $R \cap S \subseteq cl(int(pcl(R))) \cap cl(int(pcl(S))) = cl[int(pcl(R)) \cap int(pcl(S))] = cl(int[pcl(R) \cap pcl(S)]) = cl(int(pcl(R \cap S)))$. Hence $R \cap S$ is semi j open in $(\mathcal{X}, \tau_{\mathcal{X}})$.

Suppose $\mathcal{X}$ is not semi j-hyperconnected. Then there exist a proper semi j regular closed subset R in $(\mathcal{X}, \tau_{\mathcal{X}})$ and take $S = cl(\mathcal{X} - R)$. This implies R and S are non empty semi j open subset of $(\mathcal{X}, \tau_{\mathcal{X}})$. If $R \cap S = \emptyset$, then $R \cup S = \mathcal{X}$ implies R is a proper semi j open, semi j closed in $\mathcal{X}$. This is contradiction to $\mathcal{X}$ is semi j connected. Therefore $R \cap S \neq \emptyset$. Hence $R \cap S = R \cap cl_{sj}(\mathcal{X} - R) = R - int_{sj}(R)$=semi j boundary of R. Therefore $R \cap S$ is not semi j open. □

**Definition 3.7.14.** *A subset $S$ of $\mathcal{X}$ is said to be subspace of semi j-hyperconnected space if $S \subseteq \mathcal{X}$ and $S$ is semi j-hyperconnected on $S$.*

**Theorem 3.7.15.** *If $R$ and $S$ are semi j hyperconnected subsets of $(\mathcal{X}, \tau_{\mathcal{X}})$ and $int_{sj}(R) \cap S \neq \emptyset$ or $R \cap int_{sj}(S) \neq \emptyset$ then $R \cup S$ is a semi j hyperconnected subset of $\mathcal{X}$.*

*Proof.* Assume $T = R \cup S$ is not semi j hyperconnected set in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then there exist semi j open sets U and V in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $T \cap U \neq \emptyset$, $T \cap V \neq \emptyset$ and $T \cap U \cap V = \emptyset$.

Since $R$ and $S$ are semi j hyper connected subsets of $\mathcal{X}$. This implies $R \cap U \cap V = \emptyset$ and $S \cap U \cap V = \emptyset$. Without loss of generality, assume $S \cap U = \emptyset$. Then $R \cap U \neq \emptyset$, $R \cap V = \emptyset$ and $S \cap V = \emptyset$. If $R \cap int(S) \neq \emptyset$, then $R \cap int_{sj}(S)$ and $R \cap U \neq \emptyset$ are disjoint semi j open sets in the subspace $R$ of $\mathcal{X}$ which contradicts the hypothesis that $R$ is semi j hyperconnected. Similarly if $int_{sj}(R) \cap S \neq \emptyset$, then $S$ is not semi j hyperconnected. □

## 3.8 Conclusion

This chapter deals with the major findings and observations based on the semi j-open sets, semi j-closed sets, semi j-continuous function, semi j-connectedness and semi j-hyperconnectedness in topological spaces. We analyzed the characteristics of such sets and spaces by theorems and examples. Thus the study has proved that these concepts will be used in various areas of topological spaces.