Chapter 4

Neutrosophic Hyperconnected Spaces

4.1 Introduction

Neutrosophic set is a generalization of fuzzy sets and intuitionistic fuzzy sets. It is characterized by a true, indeterminate and falsity membership function respectively. This chapter classifies a new class of sets namely neutrosophic semi j-open sets, neutrosophic semi j- closed sets and neutrosophic semi j-separated sets in neutrosophic topological space. Using these sets, we present the new spaces as neutrosophic semi j-connected, neutrosophic hyperconnected and neutrosophic semi j-hyperconnected in neutrosophic topological space. We analyze the essential characteristics of such sets and spaces by the theorems and suitable examples. Also we examine the properties of neutrosophic hyperconnected spaces with some existing continuous functions.

4.2 Neutrosophic semi j-open sets

Definition 4.2.1. *A neutrosophic subset P of a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is said to be neutrosophic semi j-open set in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *if and only if* $P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$.

Example 4.2.2. *Let* $\mathcal{X} = \{s, t, r\}$ *and the neutrosophic subsets* P*,* Q*,* R *and* S *in* $\mathcal{X}$ *as follows,*

$P = \{< s, 0.4, 0.3, 0.8 >, < t, 0.5, 0.2, 0.6 >, < r, 0.4, 0.2, 0.6 >; s, t, r \in \mathcal{X}\}$,

$Q = \{< s, 0.3, 0.4, 0.5 >, < t, 0.6, 0.4, 0.6 >, < r, 0.3, 0.4, 0.6 >; s, t, r \in \mathcal{X}\}$,
$R = \{< s, 0.4, 0.4, 0.5 >, < t, 0.6, 0.4, 0.6 >, < r, 0.4, 0.4, 0.6 >; s, t, r \in \mathcal{X}\}$,
$S = \{< s, 0.3, 0.3, 0.8 >, < t, 0.5, 0.2, 0.6 >, < r, 0.3, 0.2, 0.6 >; s, t, r \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, P, Q, R, S, 1_N\}$ *is a neutrosophic topological space on* $\mathcal{X}$.
Let $E = \{< s, 0.4, 0.4, 0.5 >, < t, 0.5, 0.4, 0.7 >, < r, 0.4, 0.4, 0.7 >; s, t, r \in \mathcal{X}\}$ *be a neutrosophic subset in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*, then* $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[E]]] = \{< s, 0.5, 0.6, 0.5 >, < t, 0.6, 0.6, 0.6 >, < r, 0.6, 0.6, 0.4 >; s, t, r \in \mathcal{X}\}$. *Therefore* $E \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[E]]]$.
Hence E *is a neutrosophic semi j-open set.*

Theorem 4.2.3. *Let* $\{P_\alpha : \alpha \in \Delta\}$ *be a collection of neutrosophic semi j-open sets in neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*. Then* $\bigcup_{\alpha \in \Delta} P_\alpha$ *is also neutrosophic semi j-open in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Proof. Since P_α is neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, we have $P_\alpha \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P_\alpha]]]$. Therefore, $\bigcup_{\alpha \in \Delta} P_\alpha \leq \bigcup_{\alpha \in \Delta} \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P_\alpha]]] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[\bigcup_{\alpha \in \Delta} P_\alpha]]]$.
Hence $\bigcup_{\alpha \in \Delta} P_\alpha$ is also neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Remark 4.2.4. *The intersection of any two neutrosophic semi j-open sets of a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *need not be a neutrosophic semi j-open set as verified by the following example.*

Example 4.2.5. *Let* $\mathcal{X} = \{s, t\}$ *and the neutrosophic subsets P, Q, R and S in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *as follows,*
$P = \{< s, 0.2, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$R = \{< s, 0.2, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$S = \{< s, 0.1, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, P, Q, R, S, 1_N\}$ *is a neutrosophic topological space on* $\mathcal{X}$.
Let $E = \{< s, 0.9, 0.1, 0.6 >, < t, 0.3, 0.2, 0.6 >; s, t \in \mathcal{X}\}$ *and*
$F = \{< s, 0.6, 0.7, 0.3 >, < t, 0.5, 0.7, 0.4 >; s, t \in \mathcal{X}\}$ *be the neutrosophic subsets in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*. Then* $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[E]]] = 1_N$ *and* $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[F]]] = 1_N$*. This implies* $E \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[E]]]$ *and* $F \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[F]]]$.
Here $E \wedge F = \{< s, 0.6, 0.1, 0.6 >, < t, 0.3, 0.2, 0.6 >; s, t \in \mathcal{X}\}$.

Thus $E \wedge F \nleq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl(E \wedge F)]]$. Therefore E and F are neutrosophic semi j-open sets but $E \wedge F$ is not a neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Theorem 4.2.6. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, let P be a neutrosophic semi j-open set and $P \leq Q \leq \mathcal{N}cl[P]$. Then Q is also a neutrosophic semi j-open set in $\mathcal{X}$.*

Proof. Since P is neutrosophic semi j-open in $\mathcal{X}$, we have
$P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$. Therefore, $\mathcal{N}cl[P] \leq \mathcal{N}cl[\mathcal{N}cl[\mathcal{N}int[[\mathcal{N}pcl[P]]]]$.
This implies, $\mathcal{N}cl[P] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$. By hypothesis, $P \leq Q \leq \mathcal{N}cl[P]$, then $Q \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$. We have $P \leq Q$, therefore $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[Q]]]$, which implies $Q \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[Q]]]$. Hence Q is a neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Theorem 4.2.7. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, every neutrosophic j-open set is neutrosophic semi j-open.*

Proof. Let P be a neutrosophic j-open set in $\mathcal{X}$. Then $P \leq \mathcal{N}int[\mathcal{N}pcl[P]]$. This implies $\mathcal{N}cl[P] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$. We know that $P \leq \mathcal{N}cl[P]$. Therefore $P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$. Hence P is a neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Remark 4.2.8. *Converse of the above theorem need not be true as shown in the following example.*

Example 4.2.9. *Let $\mathcal{X} = \{s, t\}$ and the neutrosophic subsets P and Q in $\mathcal{X}$ as follows,*
$P = \{< s, 0.2, 0.2, 0.5 >, < t, 0.4, 0.3, 0.4 >; s, t \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.1, 0.8 >, < t, 0.3, 0.1, 0.4 >; s, t \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, P, Q, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$.
Let $R = \{< s, 0.2, 0.1, 0.6 >, < t, 0.4, 0.4, 0.5 >; s, t \in \mathcal{X}\}$ be a neutrosophic subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then, we have $\mathcal{N}int[\mathcal{N}pcl[R]] = P$, this implies $R \nleq P$. and $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[R]]] = P^C$. Therefore $R \leq P^C$. Hence R is neutrosophic semi j-open but not neutrosophic j-open.

Theorem 4.2.10. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, every neutrosophic open set is neutrosophic semi j-open.*

Proof. Let P be a neutrosophic open subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $P = \mathcal{N}int[P]$. We have, $\mathcal{N}int[P] \leq P \leq \mathcal{N}pcl[P] \leq \mathcal{N}cl[P]$. This implies $P \leq \mathcal{N}pcl[P] \leq \mathcal{N}cl[P]$.
Therefore, $\implies \mathcal{N}int[P] \leq \mathcal{N}int[\mathcal{N}pcl[P]]$
$\implies \mathcal{N}cl[\mathcal{N}int[P]] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$
$\implies \mathcal{N}cl[P] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$
$\implies P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$.
Hence P is a neutrospohic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

□

Remark 4.2.11. *Reverse of the previous theorem need not be true as seen in the following example.*

Example 4.2.12. *Consider $\mathcal{X} = \{s\}$ and the neutrosophic subsets P and Q as follows*
$P = \{< s, 0.4, 0.5, 0.3 >; s \in \mathcal{X}\}$,
$Q = \{< s, 0.1, 0.5, 0.5 >; s \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, P, Q, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$.
Let $R = \{< s, 0.3, 0.6, 0.5 >; s \in \mathcal{X}\}$ be a neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. We obtain $R \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl(R)]]$. Therefore R is neutrosophic semi j-open but not neutrosophic open.

4.3 Neutrosophic semi j-closed sets

Definition 4.3.1. *A neutrosophic subset S of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be neutrosophic semi j-closed set if and only if $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq S$.*

Example 4.3.2. *Let $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets Q_1, Q_2 and Q_3 as follows*
$Q_1 = \{< s_1, 0.6, 0.5, 0.6 >, < s_2, 0.7, 0.4, 0.4 >, < s_3, 0.6, 0.4, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$Q_2 = \{< s_1, 0.7, 0.6, 0.3 >, < s_2, 0.8, 0.6, 0.4 >, < s_3, 0.6, 0.6, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$Q_3 = \{< s_1, 0.6, 0.5, 0.4 >, < s_2, 0.7, 0.5, 0.4 >, < s_3, 0.6, 0.5, 0.4 >; s_1, s_2, s_3 \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, Q_1, Q_2, Q_3, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$. Put $F = \{< s_1, 0.5, 0.4, 0.5 >, < s_2, 0.6, 0.5, 0.3 >, < s_3, 0.4, 0.4, 0.3 >; s_1, s_2, s_3 \in \mathcal{X}\}$ be a

neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. We obtain $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[F]]] \leq F$. Therefore F is a neutrosophic semi j-closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Definition 4.3.3. *A neutrosophic subset P of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be neutrosophic semi j-interior of P if the union of all neutrosophic semi j-open sets of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ contained in P. It is denoted by $\mathcal{N}int_{sj}[P]$.*

Definition 4.3.4. *A neutrosophic subset Q of $\mathcal{X}$ is said to be neutrosophic semi j-closure of Q if the intersection of all neutrosophic semi j-closed sets of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ containing Q. It is denoted by $\mathcal{N}cl_{sj}[Q]$.*

Example 4.3.5. *Consider $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets S_1, S_2, S_3 in $\mathcal{X}$ as follows,*
$S_1 = \{< s_1, 0.3, 0.4, 0.3 >, < s_2, 0.6, 0.2, 0.4 >, < s_3, 0.5, 0.2, 0.3 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$S_2 = \{< s_1, 0.2, 0.6, 0.5 >, < s_2, 0.4, 0.2, 0.3 >, < s_3, 0.2, 0.3, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$S_3 = \{< s_1, 0.3, 0.6, 0.3 >, < s_2, 0.6, 0.2, 0.3 >, < s_3, 0.5, 0.3, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, S_1, S_2, S_3, 1_N\}$ be the neutrosophic topological space on $\mathcal{X}$. For this $\tau_{\mathcal{X}}$, 0_N, 1_N, S_1, S_2, $S_1 \vee S_2$, $S_1 \vee S_3$, $S_2 \vee S_3$ are the neutrosophic semi j-open sets and 0_N, 1_N, S_1^c, S_2^c, $(S_1 \vee S_2)^c$, $(S_1 \vee S_3)^c$, $(S_2 \vee S_3)^c$ are the neutrosophic semi j-closed sets. Put $T = \{< s_1, 0.5, 0.6, 0.2 >, < s_2, 0.7, 0.3, 0.3 >, < s_3, 0.6, 0.4, 0.2; s_1, s_2, s_3 \in \mathcal{X}\}$ is a neutrosophic subset in $\mathcal{X}$. Then we have $\mathcal{N}int_{sj}(T) = S_1$ and $\mathcal{N}cl_{sj}(T) = 1_N$.

Definition 4.3.6. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic topological space and P be a neutrosophic semi j-open set of $\mathcal{X}$. Then*

(a) P is said to be neutrosophic semi j-regular open set if and only if $P = \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]$.

(b) P is said to be neutrosophic semi j-regular closed set if and only if $\mathcal{N}cl_{sj}[\mathcal{N}int_{sj}[P]] = P$.

Definition 4.3.7. *A neutrosophic subset P of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is called*

(i) neutrosophic semi j-dense if $\mathcal{N}cl_{sj}(P) = 1_N$

(ii) *neutrosophic semi j-nowhere dense if* $\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}(P)] = 0_N$.

Proposition 4.3.8. *Let P be a neutrosophic subset of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$,then the following properties hold:*

(i) $\mathcal{N}int_{sj}(P) = P$ *iff P is a neutrosophic semi j-open set.*

(ii) $\mathcal{N}int_{sj}(P)$ *is the biggest neutrosophic semi j-open set contained in P.*

(iii) $\mathcal{N}cl_{sj}(P) = P$ *iff P is a neutrosophic semi j-closed set.*

(iv) $\mathcal{N}cl_{sj}(P)$ *is the smallest neutrosophic semi j-closed set containing P.*

Proof. obvious. □

Proposition 4.3.9. *Let P be any neutrosophic subset of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then the following statements are true.*

(i) $\mathcal{N}int_{sj}(1_N - P) = 1_N - [\mathcal{N}cl_{sj}(P)]$

(ii) $\mathcal{N}cl_{sj}(1_N - P) = 1_N - [\mathcal{N}int_{sj}(P)]$

Proof. obvious. □

Theorem 4.3.10. *If S is a neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then S is neutrosophic semi j-closed if and only if $C(S)$ is neutrosophic semi j-open.*

Proof. Assume S is neutrosophic semi j-closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq S$. Thus $C[S] \leq C[\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]]] = \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[C[S]]]]$. Hence $C[S]$ is neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Conversely assume $C[S]$ is a neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $C[S] \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[C[S]]]]$. We obtain $C[\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[C[S]]]]] \leq C[C[S]]$. Therefore $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq S$. Hence S is a neutrosophic semi j-closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Theorem 4.3.11. *Let $\{S_\alpha : \alpha \in \Delta\}$ be a family of neutrosophic semi j-closed sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then arbitrary intersection of neutrosophic semi j-closed sets is also neutrosophic semi j-closed.*

Proof. Let $\{S_\alpha : \alpha \in \Delta\}$ be the family of neutrosophic semi j-closed sets in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$ and $P_\alpha = \{S_\alpha\}^c$. Then $\{P_\alpha : \alpha \in \Delta\}$ is a family of neutrosophic semi j-open sets in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$.Therefore, $\bigcup_{\alpha\in\Delta} P_\alpha$ is neutrosophic semi j-open. Then $\{\bigcup_{\alpha\in\Delta} P_\alpha\}^C$ is neutrosophic semi j-closed which implies $\bigcap_{\alpha\in\Delta} {P_\alpha}^c$ is neutrosophic semi j-closed. Hence $\bigcap_{\alpha\in\Delta} S_\alpha$ is neutrosophic semi j-closed in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. □

Theorem 4.3.12. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$, every neutrosophic j-closed set is neutrosophic semi j-closed.*

Proof. Let S be a neutrosophic j-closed set in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. Then $\mathcal{N}cl[\mathcal{N}pint[S]] \leq S$. $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq \mathcal{N}int[S]$. We know that $\mathcal{N}int[S] \leq S$. Therefore $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq S$. Hence S is neutrosophic semi j-closed. □

Remark 4.3.13. *Converse of the above theorem need not be true, as verified by the following example.*

Example 4.3.14. *Let $\mathcal{X} = \{t_1, t_2, t_3\}$ and the neutrosophic subsets Q_1, Q_2 as follows,*
$Q_1 = \{< t_1, 0.2, 0.5, 0.4 >, < t_2, 0.2, 0.4, 0.5 >, < t_3, 0.1, 0.0, 0.5 >; t_1, t_2, t_3 \in \mathcal{X}\}$,
$Q_2 = \{< t_1, 0.3, 0.4, 0.5 >, < t_2, 0.4, 0.3, 0.2 >, < t_3, 0.2, 0.3, 0.4 >; t_1, t_2, t_3 \in \mathcal{X}\}$.
Put $\tau_\mathcal{X} = \{0_N, Q_1, Q_2, Q_1 \vee Q_2, 1_N\}$ is a neutrosophic topological space on $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. Let $G = \{< t_1, 0.4, 0.5, 0.4 >, < t_2, 0.3, 0.4, 0.2 >, < t_3, 0.3, 0.4, 0.5 >; t_1, t_2, t_3 \in \mathcal{X}\}$ be a neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. Then G is a neutrosophic semi j-closed but not neutrosophic j-closed. Since $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[G]]] = Q_1 \leq G$, but $\mathcal{N}cl[\mathcal{N}pint[G]] = [Q_1 \vee Q_2]^c \nleq G$.

Theorem 4.3.15. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$, every neutrosophic closed set is neutrosophic semi j-closed.*

Proof. Let S be neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. Then $S = \mathcal{N}cl[S]$. We know that $\mathcal{N}int[S] \leq \mathcal{N}pint[S]$. $\mathcal{N}cl[\mathcal{N}int[S]] \leq \mathcal{N}cl[\mathcal{N}pint[S]] \leq \mathcal{N}cl[S]$. It follows that $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}int[S]]] \leq \mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[S]]] \leq S$. Hence S is a semi j-closed set in $\mathcal{N}(\mathcal{X}, \tau_\mathcal{X})$. □

Remark 4.3.16. *The converse of the above theorem may not be true, as shown by the following example. In example 4.3.14, we have $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint(G)]] \leq G$ and $\mathcal{N}cl(G) \neq G$. This implies G is neutrosophic semi j-closed set but not neutrosophic closed set.*

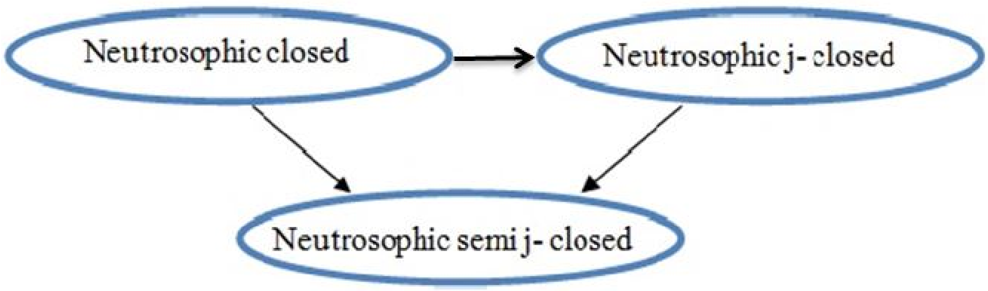
Example 4.3.17. *Let $\mathcal{X} = \{u_1, u_2, u_3\}$ and the neutrosophic subsets S_1 and S_2 defined as follows,*

$S_1 = \{< u_1, 0.4, 0.6, 0.2 >, < u_2, 0.7, 0.4, 0.3 >, < u_3, 0.4, 0.5, 0.2) >; u_1, u_2, u_3 \in \mathcal{X}\}$

$S_2 = \{< u_1, 0.4, 0.6, 0.5 >, < u_2, 0.6, 0.5, 0.1 >, < u_3, 0.6, 0.4, 0.1 >; u_1, u_2, u_3 \in \mathcal{X}\}$

Put $\tau_{\mathcal{X}} = \{0_N, S_1, S_1 \vee S_2, 1_N\}$ be the neutrosophic topological space on $\mathcal{X}$. Let $H = \{< u, 0.2, 0.4, 0.4 >, < u_2, 0.1, 0.5, 0.7 >, < u_3, 0.1, 0.5, 0.4 >; u_1, u_2, u_3 \in \mathcal{X}\}$. Here we obtain $\mathcal{N}int[\mathcal{N}cl[\mathcal{N}pint[H]]] \leq H$. Therefore H is neutrosophic semi j-closed set but not neutrosophic closed, because $\mathcal{N}cl(H) \neq H$.

From the above results, we have the following indications:



But the converse of the above indications need not be true as shown by 4.3.14 and 4.3.16.

4.4 Neutrosophic Semi j-separated sets

Definition 4.4.1. *Let P_1 and P_2 be any two subsets of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then P_1 and P_2 are called neutrosophic semi j-separated if $P_1 \wedge \mathcal{N}cl_{sj}(P_2) = \emptyset$ and $\mathcal{N}cl_{sj}(P_1) \wedge P_2 = \emptyset$.*

Theorem 4.4.2. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, any two nonempty disjoint neutrosophic semi j-closed sets are semi j-separated.*

Proof. Let Q_1 and Q_2 be two nonempty disjoint neutrosophic semi j-closed sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. $\mathcal{N}cl_{sj}(Q_1) \wedge Q_2 = Q_1 \wedge Q_2 = 0_N$ and $Q_1 \wedge \mathcal{N}cl_{sj}(Q_2) = Q_1 \wedge Q_2 = 0_N$. Hence Q_1 and Q_2 are neutrosophic semi j-separated sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 4.4.3. *Every two neutrosophic semi j-separated sets are disjoint.*

Proof. Let P_1 and P_2 be any two neutrosophic semi j-separated sets. Then, we have $P_1 \wedge \mathcal{N}cl_{sj}(P_2) = 0_N$ and $\mathcal{N}cl_{sj}(P_1) \wedge P_2 = 0_N$. Now $P_1 \wedge P_2 \leq P_1 \wedge \mathcal{N}cl_{sj}(P_2) = 0_N$. Thus $P_1 \wedge P_2 = 0_N$. Hence P_1 and P_2 are disjoint. □

Remark 4.4.4. *Every disjoint sets need not be neutrosophic semi j-separated sets as shown below.*

Example 4.4.5. *Let* $\mathcal{X} = \{r, s\}$ *with* $\tau_{\mathcal{X}} = \{0_N, Q_1, Q_2, Q_3, 1_N\}$
where $Q_1 = \{\langle r, 0.4, 0, 0.6\rangle, \langle s, 0, 0.4, 1\rangle; r, s \in \mathcal{X}\}$
$Q_2 = \{\langle r, 0, 0.5, 1\rangle, \langle s, 0.7, 0, 0.9\rangle; r, s \in \mathcal{X}\}$
$Q_3 = \{\langle r, 0.4, 0.5, 0.6\rangle, \langle s, 0.7, 0.4, 0.9\rangle; r, s \in \mathcal{X}\}$
Now, $Q_1 \wedge Q_2 = \{\langle r, 0, 0, 1\rangle, \langle s, 0, 0, 1\rangle; r, s \in \mathcal{X}\} = 0_N$
$\mathcal{N}cl_{s_j}(Q_1) = \{\langle r, 0.6, 0.5, 0.4\rangle, \langle s, 0.9, 0.6, 0.7\rangle; r, s \in \mathcal{X}\} = 0_N$
$\mathcal{N}cl_{s_j}(Q_2) = \{\langle r, 0.6, 0.5, 0.4\rangle, \langle s, 0.9, 0.6, 0.7\rangle; r, s \in \mathcal{X}\} = 0_N$
Therefore $Q_1 \wedge \mathcal{N}cl_{s_j}(Q_2) = Q_1 \neq 0_N$ *and*
$Q_2 \wedge \mathcal{N}cl_{s_j}(Q_1) = Q_2 \neq 0_N$
Hence Q_1 *and* Q_2 *are disjoint but not neutrosophic semi j-separated sets.*

Theorem 4.4.6. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, P_1 and P_2 are neutrosophic semi j-separated iff there exist neutrosophic semi j-open S_1 and S_2 in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P_1 \leq S_1$ and $P_1 \wedge S_2 = 0_N$, $P_2 \leq S_2$ and $P_2 \wedge S_1 = 0_N$.*

Proof. Let P_1 and P_2 be neutrosophic semi j-separated sets. Then, we have $P_1 \wedge \mathcal{N}cl_{sj}(P_2) = 0_N$ and $\mathcal{N}cl_{sj}(P_1) \wedge P_2 = 0_N$. Taking $S_2 = C[\mathcal{N}cl_{sj}(P_1)]$ and $S_1 =$

$C[\mathcal{N}cl_{sj}(P_2)]$. Then S_1 and S_2 are neutrosophic semi j-open sets such that $P_1 \leq S_1$ and $P_2 \leq S_2$. Also, $P_1 \wedge S_2 = 0_N$, $P_2 \wedge S_1 = 0_N$.
Conversely, let S_1 and S_2 be neutrosophic semi j-open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P_1 \leq S_1$, $P_2 \leq S_2$ and $P_1 \wedge S_2 = 0_N$, $P_2 \wedge S_1 = 0_N$. This implies $P_1 \leq C(S_2)$ and $P_2 \leq C(S_1)$. Therefore, $C(S_1)$ and $C(S_2)$ are neutrosophic semi j-closed sets. Now $\mathcal{N}cl_{sj}(P_1) \leq \mathcal{N}cl_{sj}[C(S_2)] = C[S_2] \leq C[P_2]$ and $\mathcal{N}cl_{sj}(P_2) \leq \mathcal{N}cl_{sj}[C(S_1)] = C[S_1] \leq C[P_1]$ i.,e $\mathcal{N}cl_{sj}(P_1) \leq C[P_2]$ and $\mathcal{N}cl_{sj}(P_2) \leq C[P_1]$. Therefore, $P_1 \wedge \mathcal{N}cl_{sj}(P_2) = 0_N$ and $\mathcal{N}cl_{sj}(P_1) \wedge P_2 = 0_N$. Hence P_1 and P_2 are neutrosophic semi j-separated sets. □

Theorem 4.4.7. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, the following statements hold:*

(i) If P_1 and P_2 are neutrosophic semi j-separated in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $Q_1 < P_1$, $Q_2 < P_2$, then Q_1 and Q_2 are also neutrosophic semi j-separated sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

(ii) If P_1 and P_2 are neutrosophic semi j-open and if $S = P_1 \wedge C(P_2)$ and $T = P_2 \wedge C(P_1)$, then S and T are neutrosophic semi j-separated.

Proof. (i) Let P_1 and P_2 be neutrosophic semi j-separated sets. Then $\mathcal{N}cl_{sj}(P_1) \wedge P_2 = 0_N = P_1 \wedge \mathcal{N}cl_{sj}(P_2)$. Since $Q \leq P_1$, $Q_2 \leq P_2$, we have, $\mathcal{N}cl_{sj}(Q_1) \leq \mathcal{N}cl_{sj}(P_1)$ and $\mathcal{N}cl_{sj}(Q_2) \leq \mathcal{N}cl_{sj}(P_2)$. Thus $\mathcal{N}cl_{sj}(Q_1) \wedge Q_2 \leq \mathcal{N}cl_{sj}(P_1) \wedge P_2 = 0_N$. This implies $\mathcal{N}cl_{sj}(Q_1) \wedge Q_2 = 0_N$. Similarly, $\mathcal{N}cl_{sj}(Q_2) \wedge Q_1 \leq \mathcal{N}cl_{sj}(P_2) \wedge P_1 = 0_N$. This implies $\mathcal{N}cl_{sj}(Q_2) \wedge Q_1 = 0_N$. Hence Q_1 and Q_2 are neutrosophic semi j-separated sets.
(ii) Let P_1 and P_2 be neutrosophic semi j-open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.Then $C[P_1]$ and $C[P_2]$ are neutrosophic semi j-closed sets. Since $S \leq C(P_2)$, therefore $\mathcal{N}cl_{sj}(S) \leq \mathcal{N}cl_{sj}(C[P_2])$ $= C[P_2]$. Thus $\mathcal{N}cl_{sj}(S) \wedge P_2 = 0_N$. Since $T \leq P_2$, $\mathcal{N}cl_{sj}(S) \wedge T \leq \mathcal{N}cl_{sj}(S) \wedge P_2 = 0_N$. Thus $\mathcal{N}cl_{sj}(S) \wedge T = 0_N$. Similarly $\mathcal{N}cl_{sj}(T) \wedge S = 0_N$. Hence S and T are neutrosophic semi j-separated sets. □

4.5 Neutrosophic semi j-connected spaces

Definition 4.5.1. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is called neutrosophic semi j-connected if $\mathcal{X}$ cannot be the union of two neutrosophic semi j-separated sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Definition 4.5.2. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-disconnected if there is a two neutrosophic semi j-open sets $0_N \neq P$ and $0_N \neq Q$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P \vee Q = 1_N$ and $P \wedge Q = 0_N$.*

Example 4.5.3. *Let $\mathcal{X} = \{p, q\}$ and the neutrosophic subsets M_1 and M_2 as*
$M_1 = \{< p, 0.6, 0.4, 0.5 >, < q, 0.5, 0.4, 0.4 >; p, q \in \mathcal{X}\}$
$M_2 = \{< p, 0.5, 0.3, 0.6 >, < q, 0.3, 0.2, 0.5 >; p, q \in \mathcal{X}\}$
Then $\tau_{\mathcal{X}} = \{0_N, M_1, M_2, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$. This implies $M_1 \neq 0_N$, $M_2 \neq 0_N$, $M_1 \vee M_2 = M_1 \neq 1_N$ and $M_1 \wedge M_2 = M_2 \neq 0_N$ where M_1 and M_2 are neutrosophic open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore M_1 and M_2 are neutrosophic semi j-open sets. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-connected.

Example 4.5.4. *Let $\mathcal{X} = \{q, r\}$ with $\tau_{\mathcal{X}} = \{0_N, N_1, N_2, 1_N\}$ where*
$N_1 = \{< q, 0, 1, 1 >, < r, 1, 0, 0 >; q, r \in \mathcal{X}\}$
$N_2 = \{< q, 1, 0, 0 >, < r, 0, 1, 1 >; q, r \in \mathcal{X}\}$.
Here we obtain $N_1 \vee N_2 = \{< q, 1, 1, 0 >, < r, 1, 1, 0 >; q, r \in \mathcal{X}\} = 1_N$ and $N_1 \wedge N_2 = \{< q, 0, 0, 1 >, < r, 0, 0, 1 >; q, r \in \mathcal{X}\} = 0_N$ where N_1 and N_2 are neutrosophic open sets. Therefore, N_1, N_2 are neutrosophic semi j-open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-disconnected space.

Theorem 4.5.5. *For a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, the following statements hold:*

(i) *$\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-connected.*

(ii) *The only neutrosophic semi j-open and neutrosophic semi j-closed sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ are 0_N and 1_N.*

Proof. $(i) \implies (ii)$
Assume (i). Let P be any neutrosophic subset in $\mathcal{X}$, which is both neutrosophic semi j-

open and neutrosophic semi j-closed. Then P and $C(P)$ are disjoint neutrosophic semi j-open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then, we have $1_N = P \vee C(P)$. This implies $P = 0_N$ or $C(P) = 0_N$. Hence either $P = 0_N$ or $P = 1_N$.

$(ii) \implies (i)$

Assume $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-disconnected space. Then, we obtain $1_N = P_1 \vee P_2$ and $P_1 \wedge P_2 = 0_N$, where P_1 and P_2 are nonempty neutrosophic semi j-open sets. Since $P_1 = C(P_2)$. Therefore, P_1 is neutrosophic semi j-closed set which contradicts (ii). Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-connected. □

Theorem 4.5.6. *A neutrosophic subset P_1 of a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-connected iff P_1 is not the union of any two neutrosophic semi j-separated sets.*

Proof. Let S_1 and S_2 be neutrosophic semi j-separated sets such that $P_1 = S_1 \vee S_2$ and $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be neutrosophic semi j-connected space. Therefore, $\mathcal{N}cl_{sj}(S_1) \wedge S_2 = 0_N$ and $S_1 \wedge \mathcal{N}cl_{sj}(S_2) = 0_N$. Since $S_1 \leq \mathcal{N}cl_{sj}(S_1)$, therefore $S_1 \wedge S_2 \leq \mathcal{N}cl_{sj}(S_1) \wedge S_2 = 0_N$. Thus $S_1 \wedge S_2 = 0_N$. Also, $\mathcal{N}cl_{sj}(S_1) \leq C(S_1) = S_1$ and $\mathcal{N}cl_{sj}(S_2) \leq C(S_2) = S_2$.Hence $NCl_{s_j}(S_1) = S_1$ and $NCl_{s_j}(S_2) = S_2$. Therefore S_1 and S_2 are neutrosophic semi j-closed sets. Thus $S_1 = C(S_2)$ and $S_2 = C(S_1)$ are disjoint neutrosophic semi j-open sets. This implies P_1 is not neutrosophic semi j-connected, which contradicts P_1 is neutrosophic semi j-connected. Hence P_1 is not the union of any two neutrosophic semi j-separated sets.

Conversely, Assume P_1 is not the union of any two neutrosophic semi j- separated sets. Let $(\mathcal{X}, \tau_{\mathcal{X}})$ be neutrosophic semi j-disconnected space. Then, $P_1 = S_1 \vee S_2$, where S_1 and S_2 are non-empty neutrosophic semi j-open sets in $(\mathcal{X}, \tau_{\mathcal{X}})$ such that $S_1 \wedge S_2 = 0_N$ Since $S_1 \leq C(S_2)$ and $S_2 \leq C(S_1)$, $NCl_{s_j}(S_1) \wedge S_2 = C(S_2) \wedge S_1 = 0_N$. $S_1 \wedge \mathcal{N}cl_{s_j} \leq S_1 \wedge C(S_1) = 0_N$. This implies S_1 and S_2 are neutrosophic semi j-separated which contradicts our assumption. Hence P_1 is neutrosophic semi j-connected. □

Theorem 4.5.7. *In a neutrosophic topological space $(\mathcal{X}, \tau_{\mathcal{X}})$ if P is a neutrosophic semi j-connected set, then $\mathcal{N}cl_{s_j}(P)$ is also neutrosophic semi j-connected.*

Proof. Let P be a neutrosophic semi j-connected set. Assume $\mathcal{N}cl_{s_j}(P)$ is neutrosophic semi j-disconnected. Then, we obtain $\mathcal{N}cl_{s_j}(P) = S_1 \vee S_2$, where S_1 and S_2 are neutrosophic semi j-separated sets in $(\mathcal{X}, \tau_{\mathcal{X}})$. Since P is neutrosophic semi j-connected and $P \leq \mathcal{N}cl_{s_j}(P) = S_1 \vee S_2$. We have $P \leq S_1$ or $P \leq S_2$. Since $P \leq S_1$, therefore $\mathcal{N}cl_{s_j}(P) \leq \mathcal{N}cl_{s_j}(S_1)$. This implies $\mathcal{N}cl_{s_j}(P) \wedge S_2 \leq \mathcal{N}cl_{s_j}(S_1) \wedge S_2 = 0_N$. Therefore $S_2 \leq C(\mathcal{N}cl_{s_j}(P))$. Also $S_2 \leq S_1 \vee S_2 = \mathcal{N}cl_{s_j}(P)$. This implies $S_2 \leq C(\mathcal{N}cl_{s_j}(P)) \wedge \mathcal{N}cl_{s_j}(P) = 0_N$ which contradicts $S_2 \neq 0_N$. Similarly for $P \leq S_2$, we get a contradiction to $S_1 \neq 0_N$. Therefore $\mathcal{N}cl_{s_j}(P)$ is neutrosophic semi-connected. □

Theorem 4.5.8. *If P is a neutrosophic semi j-connected set and S_1, S_2 are the neutrosophic semi j-separated sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P \leq S_1 \vee S_2$, then either $P \leq S_1$ or $P \leq S_2$.*

Proof. Assume $P \nleq S_1$ and $P \nleq S_2$. Put $P_1 = S_1 \wedge P$ and $P_2 = S_2 \wedge P$. Then P_1 and P_2 are non-empty neutrosophic subsets and $P_1 \vee P_2 = (S_1 \wedge P) \vee (S_2 \wedge P) = (S_1 \vee S_2) \wedge P = P$, $P \leq S_1 \vee S_2$, since $P_1 \leq S_1, P_2 \leq S_2, S_1, S_2$ are neutrosophic semi j-separated sets, $\mathcal{N}cl_{s_j}(P_1) \wedge (P_2) \leq \mathcal{N}cl_{s_j}(S_1) \wedge S_2 = 0_N$ and $P_1 \wedge \mathcal{N}cl_{s_j}(P_2) \leq S_1 \wedge \mathcal{N}cl_{s_j}(S_2) = 0_N$. This implies P_1 and P_2 are neutrosophic semi j-separated sets such that $P = P_1 \vee P_2$. Therefore P is a neutrosophic semi j-disconnected set, which is a contradiction. Hence either $P \leq S_1$ or $P \leq S_2$. □

Theorem 4.5.9. *Let P be a neutrosophic semi j-connected set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. If $P \leq Q \leq \mathcal{N}cl_{s_j}(P)$, then Q is also neutrosophic semi j-connected.*

Proof. Assume Q is neutrosophic semi j-disconnected in $(\mathcal{X}, \tau_{\mathcal{X}})$. Then we have $Q = S_1 \vee S_2$, where S_1 ans S_2 are non empty neutrosophic semi j-separated sets. Since $P \leq Q = S_1 \vee S_2$. Therefore, by theorem 4.5.8, $P \leq S_1$ or $P \leq S_2$.
Let $P \leq S_1$, then $\mathcal{N}cl_{s_j}(P) \leq \mathcal{N}cl_{s_j}(S_1)$. Now $\mathcal{N}cl_{s_j}(P) \wedge S_2 \leq \mathcal{N}cl_{s_j}(S_1) \wedge S_2 = 0_N$. Thus, $\mathcal{N}cl_{s_j}(P) \wedge S_2 = 0_N$. Also $S_1 \vee S_2 = Q$ also $Q \leq \mathcal{N}cl_{s_j}, S_2 \leq Q \leq \mathcal{N}cl_{s_j}(P)$. Therefore, $\mathcal{N}cl_{s_j}(P) \wedge S_2 = S_2$. Thus $S_2 = 0_N$ which contradicts $S_2 \neq 0_N$. Hence S_2 is neutrosophic semi j-connected. □

Theorem 4.5.10. *If P and Q are neutrosophic semi j-connected spaces in a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ with $P \wedge Q \neq 0_N$, then $P \vee Q$ is neutrosophic semi j-connected space in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let $P \vee Q$ be neutrosophic semi j-disconnected, then there exist two neutrosophic semi j-separated sets S_1 and S_2 such that $P \vee Q = S_1 \vee S_2$. Now $S_1 \wedge S_2 \leq \mathcal{N}Cl_{s_j}(S_1) \wedge S_2 = 0_N$. Since $P \leq P \vee Q = S_1 \vee S_2, Q \leq P \vee Q = S_1 \vee S_2$ and P, Q are neutrosophic semi j-connected. Using theorem 4.5.8, $P \leq S_1$ or $P \leq S_2$ and $Q \leq S_1$ or $Q \leq S_2$.
(i) If $P \leq S_1$ and $Q \leq S_1$, then $P \vee Q \leq S_1$. Thus $P \vee Q \leq S_1$, since $S_1 \wedge S_2 = 0_N$, we have $S_2 = 0_N$, which contradicts $S_2 \neq 0_N$. Similarly, if $P \leq S_2$ and $Q \leq S_2$, we get $S_1 = 0_N$ which is a contradiction to $S_1 \neq 0_N$.
(ii) If $P \leq S_1$ and $Q \leq S_2$, then $P \wedge Q \leq S_1 \wedge S_2 = 0_N$. Therefore $P \wedge Q = 0_N$, which is a contradiction to $P \wedge Q \neq 0_N$. Similarly, we have a contradiction for $P \leq S_2$ and $Q \leq S_1$. Hence $P \vee Q$ is neutrosophic semi j-connected in $(\mathcal{X}, \tau_{\mathcal{X}})$. □

4.6 Neutrosophic hyperconnected spaces

Definition 4.6.1. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, is said to be neutrosophic hyperconnected if for every non empty neutrosophic open subsets of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic dense in $\mathcal{X}$. Equivalently, $\mathcal{N}cl(P) = 1_N$, for every neutrosophic open set P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Example 4.6.2. *Consider $\mathcal{X} = \{s_1, s_2\}$ with $\tau_{\mathcal{X}} = \{0_N, 1_N, P_1, P_2, P_3, P_4\}$, where*
$P_1 = \{< s_1, 0.2, 0.4, 0.3 >, < s_2, 0.5, 0.1, 0.4 >, s_1, s_2 \in \mathcal{X}\}$,
$P_2 = \{< s_1, 0.1, 0.5, 0.6 >, < s_2, 0.4, 0.2, 0.0 >, s_1, s_2 \in \mathcal{X}\}$,
$P_3 = \{< s_1, 0.2, 0.5, 0.3 >, < s_2, 0.5, 0.2, 0.0 >, s_1, s_2 \in \mathcal{X}\}$,
$P_4 = \{< s_1, 0.1, 0.4, 0.6 >, < s_2, 0.4, 0.1, 0.4 >, s_1, s_2 \in \mathcal{X}\}$.
For this $\tau_{\mathcal{X}}$, we have $\mathcal{N}cl[P_1] = 1_N$,
$\mathcal{N}cl[P_2] = 1_N$,
$\mathcal{N}cl(P_3) = 1_N$,
$\mathcal{N}cl(P_4) = 1_N$,

$\mathcal{N}cl(1_N) = 1_N$.

Here every non empty neutrosophic open sets $P_1, P_2, P_3, P_4, 1_N$ *are neutrosophic dense in* $\mathcal{X}$. *ie.,*

Therefore $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is neutrosophic hyperconnected space.*

Definition 4.6.3. *A neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is called as neutrosophic extremally disconnected if the neutrosophic closure of each neutrosophic open set is neutrosophic open in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Theorem 4.6.4. *In a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*, every neutrosophic hyperconnected space is neutrosophic extremally disconnected.*

Proof. Let us take $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be neutrosophic hyperconnected space. Then for any neutrosophic open set P, we have $\mathcal{N}cl[P] = 1_N$. This implies that $\mathcal{N}cl[P]$ is neutrosophic open, for every P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic extremally disconnected. □

Remark 4.6.5. *The following example shows that the converse of the above theorem need not be true.*

Example 4.6.6. *Let* $\mathcal{X} = \{s\}$ *with* $\tau_{\mathcal{X}} = \{0_N, P_1, P_2, P_3, P_4, 1_N\}$*, where*

$P_1 = \{< s, 0.5, 0.3, 0.2 >; s \in \mathcal{X}\}$,

$P_2 = \{< s, 0.2, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$P_3 = \{< s, 0.3, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$P_4 = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$.

Here $\mathcal{N}cl[P_1] = \{< s, 0.5, 0.3, 0.2 >; s \in \mathcal{X}\}$,

$\mathcal{N}cl[P_2] = \{< s, 0.2, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$\mathcal{N}cl(P_3) = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$,

$\mathcal{N}cl(P_4) = \{< s, 0.5, 0.3, 0.5 >; s \in \mathcal{X}\}$.

Thus $\mathcal{N}cl[P_1]$, $\mathcal{N}cl[P_2]$, $\mathcal{N}cl(P_3)$ *and* $\mathcal{N}cl(P_4)$ *are neutrosophic open but not neutrosophic dense, therefore* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is not neutrosophic hyperconnected.*

Theorem 4.6.7. *In a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*, the following properties are equivalent.*

(a) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected.

(b) In $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, the only neutrosophic regular open sets are 0_N and 1_N.

Proof. (a) $\implies$ (b)

Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space. If P is a non-empty neutrosophic regular open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then by the definition, $P = \mathcal{N}int[\mathcal{N}cl[P]]$. This implies $[\mathcal{N}int[\mathcal{N}cl[P]]]^C = [1_N - \mathcal{N}int[\mathcal{N}cl[P]]] = \mathcal{N}cl[1_N - \mathcal{N}cl[P]] = \mathcal{N}cl(C[P]) = C[P] \neq 1_N$. Since $P \neq 0_N$. This is a contradiction to the assumption. Hence, the only neutrosophic regular open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ are 0_N and 1_N.

(b) $\implies$ (a)

Assume that 0_N and 1_N are the only neutrosophic regular open subsets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Suppose that $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is not neutrosophic hyperconnected. Then there exist a non empty neutrosophic open subset P of $\mathcal{X}$ such that $\mathcal{N}cl[P] \neq 1_N$. This implies $\mathcal{N}cl[\mathcal{N}int[P]] \neq 1_N$. Therefore, we have $\mathcal{N}cl[\mathcal{N}int[P]] = 0_N$. This gives $\mathcal{N}cl[P] = 0_N$. It contradicts our assumption that $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is not neutrosophic hyperconnected . Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space., □

Theorem 4.6.8. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, $\mathcal{N}pcl(P) = 1_N$ for every subset $\emptyset \neq P \in NSO(\mathcal{X})$.*

Proof. Let $P \in NSJO(\mathcal{X})$. Then P contains a neutrosophic open set C. Therefore $C < P \implies \mathcal{N}int(C) \leq \mathcal{N}int(P) \implies \mathcal{N}cl[\mathcal{N}int(c)] \leq \mathcal{N}cl[\mathcal{N}int(P)]$ since $\mathcal{N}int(C) = C$ and $\mathcal{N}cl(C) = 1_N$. This implies $\mathcal{N}cl[\mathcal{N}int(C)] = 1_N$. Therefore, $I_N \leq \mathcal{N}cl[\mathcal{N}int(P)]$. Thus $\mathcal{N}cl[\mathcal{N}int(P)] = 1_N$. Hence $\mathcal{N}pcl[P] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Theorem 4.6.9. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected if and only if for every neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is either neutrosophic dense or neutrosophic nowhere dense.*

Proof. Suppose $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space and let P be any neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $P \leq 1_N$. Assume P is not neutrosophic nowhere dense. This implies $1_N - [\mathcal{N}int[\mathcal{N}cl[P]]] \neq 1_N$. Then $\mathcal{N}int[\mathcal{N}cl[P]] \neq 0_N$.

Therefore $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}cl[P]]] = 1_N$. Since $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}cl[P]]] = 1_N \leq \mathcal{N}cl[P]$. Thus $\mathcal{N}cl[P] = 1_N$. Hence P is neutrosophic dense set.

For the converse part, let $0_N \neq R$ be any neutrosophic open subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $P_1 < \mathcal{N}int[\mathcal{N}cl[P_1]]$ which implies that P_1 is not neutrosophic nowhere dense set. By hypothesis, P_1 is neutrosophic dense. □

Proposition 4.6.10. *If $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic hyperconnected space, then the intersection of any two neutrosophic semi open sets is also neutrosophic semi open.*

Proof. Let P_1 and P_2 be the two non empty neutrosophic semi open sets in a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then, we have $P_1 \leq \mathcal{N}cl[\mathcal{N}int[P_1]]$ and $P_2 \leq \mathcal{N}cl[\mathcal{N}int[P_2]]$. It follows that, $\mathcal{N}cl[P_1] = \mathcal{N}cl[\mathcal{N}int[P_1]] = 1_N$ & $\mathcal{N}cl[P_2] = \mathcal{N}cl[\mathcal{N}int[P_2]] = 1_N$. Also we have $P_1 \wedge P_2 \neq 0_N$. Therefore, $\mathcal{N}cl[\mathcal{N}int[P_1 \wedge P_2]] = \mathcal{N}cl[\mathcal{N}int[P_1]] \wedge \mathcal{N}cl[\mathcal{N}int[P_2]] = 1_N$. This implies $P_1 \wedge P_2 \leq \mathcal{N}cl[\mathcal{N}int[P_1]] \wedge \mathcal{N}cl[\mathcal{N}int[P_2]] = \mathcal{N}cl[\mathcal{N}int[P_1 \wedge P_2]]$. Hence $P_1 \wedge P_2$ is neutrosophic semi open set. □

Theorem 4.6.11. *If $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic hyperconnected space, then for any neutrosophic subset P of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi open if $\mathcal{N}int(P) \neq 0_N$.*

Proof. Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space and P be any neutrosophic open subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, also $\mathcal{N}int(P) \neq 0_N$. This implies $\mathcal{N}cl[\mathcal{N}int(P)] = 1_N$. Therefore, $P \leq \mathcal{N}cl[\mathcal{N}int(P)]$. Hence every neutrosophic open subset P is neutrosophic semi open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Theorem 4.6.12. *For a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, the following statements are equivalent*

(i) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space.

(ii) Each neutrosophic preopen set is neutrosophic dense set.

Proof. (i) $\Longrightarrow$ (ii)

Let $0_N \neq P$ be any neutrosophic pre open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $P \leq \mathcal{N}int[\mathcal{N}cl(P)]$.

We have $\mathcal{N}cl(P) = \mathcal{N}cl[\mathcal{N}int(\mathcal{N}cl(P))] = 1_N$. This implies P is a neutrosophic dense set.

(ii) $\implies$ (i)

Let P be any neutrosophic preopen set and also neutrosophic dense set. Then $\mathcal{N}cl(P) = \mathcal{N}cl[\mathcal{N}int(\mathcal{N}cl(P))] = 1_N$. This implies that $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected. □

Definition 4.6.13. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is called as neutrosophic door space if and only if each subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is either neutrosophic $\tau_{\mathcal{X}}$ open or neutrosophic $\tau_{\mathcal{X}}$ closed.*

Theorem 4.6.14. *If a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic door and neutrosophic hyperconnected, then the intersection of any two non empty neutrosophic semi open set P_1 and P_2 is also non empty (i.,e) $P_1 \wedge P_2 \neq 0_N$.*

Proof. Let us take P_1 and P_2 be two non empty neutrosophic open subsets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Assume that $P_1 \wedge P_2 = 0_N$ for some open sets P_1 and P_2. This implies that $P_1 \wedge P_2 \in NSO(\mathcal{X})$. Thus $P_1 \wedge P_2 \neq 0_N$, since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space. Let $P_1 < P_2$ and $P_1 \in NSO(\mathcal{X})$. If $P_2 \in NSO(\mathcal{X})$, then $\mathcal{N}cl(P_2) \in SO(\mathcal{X})$, since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic door space. Thus $P_1 \wedge \mathcal{N}cl(P_2) = 0_N$ which is a contradiction. Hence $P_2 \in NSO(\mathcal{X})$ and $P_1 \wedge P_2 \neq 0_N$. □

Theorem 4.6.15. *Let $(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space. If $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic feebly continuous onto function then $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic hyperconnected.*

Proof. Let $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic onto function and $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space. Assume that $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is not neutrosophic hyperconnected space, then there exist two non empty disjoint neutrosophic open sets P_1 and P_2 such that $P_1 \wedge P_2 = 0_N$. This implies $f^{-1}(P_1) \wedge f^{-1}(P_2) = 0_N$. Since $f^{-1}(P_1) \neq 0_N$ and $f^{-1}(P_2) \neq 0_N$, then $\mathcal{N}int[f^{-1}(P_1)] \neq 0_N$ and $\mathcal{N}int[f^{-1}(P_2) \neq 0_N]$. We have $f^{-1}(P_1) \wedge f^{-1}(P_2) = 0_N$. This implies $\mathcal{N}int[f^{-1}(P_1)] \wedge \mathcal{N}int[f^{-1}(P_2)] = 0_N$.

Therefore $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is not neutrosophic hyperconnected which is prohibitive to our assumption. Hence $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic hyperconnected. □

Theorem 4.6.16. *Let $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic hyperconnected space. If $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic one-to-one function, then $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected.*

Proof. Let P_1 and P_2 be the two non empty neutrosophic open subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since f is neutrosophic open, then $f(P_1)$ and $f(P_2)$ are non empty neutrosophic open sets in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. By hypothesis $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic hyperconnected space. Therefore $f^{-1}(P_1) \wedge f^{-1}(P_2) \neq 0_N$. Since f is one-to-one, we have $0_N \neq f(P_1) \wedge f(P_2) = f(P_1 \wedge P_2)$. Therefore $P_1 \wedge P_2 \neq 0_N$. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space. □

Proposition 4.6.17. *Let $f : (\mathcal{X}, \tau_{\mathcal{X}}) \to (\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic semi continuous function from a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into neutrosophic hyperconnected space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then f is neutrosophic almost continuous function.*

Proof. Let P be any neutrosophic open subset in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then $f^{-1}(P)$ is semi open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. By the hypothesis, $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic hyperconnected space. This implies 0_N and 1_N are the only neutrosophic regular open sets in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. It gives that $f^{-1}(1_N) = 1_N$ and $f^{-1}(0_N) = 0_N$. Therefore, inverse image of every neutrosophic regular open set in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence f is neutrosophic almost continuous. □

Theorem 4.6.18. *Let $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic almost continuous function from a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, $\mathcal{N}int f^{-1}(\mathcal{Y}) \neq 0_N$ for every neutrosophic open subset $0_N \neq P$ in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then f is neutrosophic semi continuous.*

Proof. Since f is neutrosophic almost continuous function, then for each neutrosophic regular open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, there exists an inverse image $f^{-1}(P)$ which is regular open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. By the hypothesis, $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space with $\mathcal{N}intf^{-1}(P) \neq 0_N$ for any neutrosophic open set $0_N \neq P \in \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. This implies $\mathcal{N}cl[\mathcal{N}intf^{-1}(P)] = 1_N$. Therefore, we have $f^{-1}[P] \leq \mathcal{N}cl[\mathcal{N}int[f^{-1}(P)]]$. Thus $f^{-1}[P]$ is neutrosophic semi open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic open set P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.Hence f is neutrosophic semi continuous function. □

Proposition 4.6.19. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if $0_N \neq P$ and $0_N \neq Q$ are any two neutrosophic open subsets of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $P \wedge Q \neq 0_N$ for every pair of neutrosophic open sets P and Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Suppose $P \wedge Q = 0_N$, for any two neutrosophic open subsets P and Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. This implies $\mathcal{N}cl(P) \wedge Q = 0_N$ and $\mathcal{N}cl(P) \neq 1_N$. Since P is neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore, $P \leq \mathcal{N}cl(P) \implies \mathcal{N}int(P) \leq \mathcal{N}int[\mathcal{N}cl(P)] \implies P \leq \mathcal{N}int[\mathcal{N}cl(P)]$ and $P \neq 0_N$, which implies $\mathcal{N}cl(P) \neq 1_N$. This contradicts our assumption. Hence $P \wedge Q \neq 0_N$. □

Remark 4.6.20. *The reverse of the above proposition need not be true as seen in the following example.*

Example 4.6.21. *Let $\mathcal{X} = \{s, t\}$ and the neutrosophic subsets in $\mathcal{X}$ as follows,*
$S_1 = \{< s, 0.6, 0.4, 0.5 >, < t, 0.5, 0.3, 0.4 >, s, t \in \mathcal{X}\}$
$S_2 = \{< s, 0.3, 0.6, 0.5 >, < t, 0.4, 0.5, 0.6 >, s, t \in \mathcal{X}\}$
$S_1 \vee S_2 = \{< s, 0.6, 0.6, 0.5 >, < t, 0.5, 0.5, 0.4 >, s, t \in \mathcal{X}\}$
$S_1 \wedge S_2 = \{< s, 0.3, 0.4, 0.5 >, < t, 0.4, 0.3, 0.6 >, s, t \in \mathcal{X}\}$
Put $\tau_{\mathcal{X}} = \{0_N, 1_N, S_1, S_2, S_1 \vee S_2, S_1 \wedge S_2\}$ be a neutrosophic topological space on $\mathcal{X}$. Here we have $\mathcal{N}cl(S_1 \wedge S_2) = \{< s, 0.4, 0.4, 0.5 >, < t, 0.5, 0.5, 0.6 >; s, t \in \mathcal{X}\} \neq 1_N$. Clearly, $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is not neutrosophic hyperconnected space.

Proposition 4.6.22. *Let $0_N \neq P$ and $0_N \neq Q$ be any two neutrosophic subsets in a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $\mathcal{N}cl[P \wedge Q] = \mathcal{N}cl[P] \wedge \mathcal{N}cl[Q]$.*

Proof. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space. Therefore, $\mathcal{N}cl[P] = 1_N$ and $\mathcal{N}cl[Q] = 1_N$. This implies $\mathcal{N}cl(P) \wedge \mathcal{N}cl(Q) = 1_N$ —— 1
Also, $P \wedge Q \neq 0_N$ implies $\mathcal{N}cl(P \wedge Q) = 1_N$ ——2
From 1 and 2, hence $\mathcal{N}cl(P \wedge Q) = \mathcal{N}cl(P) \wedge \mathcal{N}cl(Q)$. □

Theorem 4.6.23. *Neutrosophic feebly continuous function preserves neutrosophic hyperconnectedness.*

Proof. Let $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic feebly continuous onto function. Take $(\mathcal{X}, \tau_{\mathcal{X}})$ as a neutrosophic hyperconnected space. Assume $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is not neutrosophic hyperconnected space. Then, we obtain neutrosophic open sets $0_N \neq P_1$ and $0_N \neq P_2$ such that $P_1 \wedge P_2 = 0_N$ in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, which implies $f^{-1}(P_1) \neq 0_N$ and $f^{-1}(P_2) \neq 0_N$. This implies $\mathcal{N}int[f^{-1}(P_1)] \neq 0_N$ and $\mathcal{N}int[f^{-1}(P_2)] \neq 0_N$. But we obtain, $f^{-1}(P_1) \wedge f^{-1}(P_2) = 0_N$ and $\mathcal{N}int[f^{-1}(P_1)] \wedge \mathcal{N}int[f^{-1}(P_2)] = 0_N$. This implies $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is not neutrosophic hyperconnected. It contradicts our assumption. Hence $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is neutrosophic hyperconnected space. □

Proposition 4.6.24. *If $f : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is a neutrosophic feebly continuous function from a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then f is neutrosophic semi continuous function.*

Proof. Let f be a neutrosophic feebly continuous function. Assume P is a neutrosophic open subset in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ such that $f^{-1}(P) \neq 0_N$, which implies $\mathcal{N}int[f^{-1}(P)] \neq 0_N$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic hyperconnected space, we have $\mathcal{N}cl[\mathcal{N}int[f^{-1}(P)] = 1_N$. This implies $f^{-1}(P) \leq \mathcal{N}cl[\mathcal{N}int[f^{-1}(P)]]$. Thus $f^{-1}(P)$ is neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Hence f is neutrosophic semi continuous function. □

4.7 Neutrosophic semi j-hyperconnected spaces

Definition 4.7.1. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is said to be neutrosophic semi j-hyperconnected space if for each nonempty neutrosophic semi j-open subset P of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. ie., $\mathcal{N}cl_{sj}(P) = 1_N$ for every P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Example 4.7.2. *Let $\mathcal{X} = \{s_1, s_2, s_3\}$ and the neutrosophic subsets P_1, P_2, P_3 in $\mathcal{X}$ as follows,*

$P_1 = \{< s_1, 0.1, 0.3, 0.2 >, < s_2, 0.4, 0.1, 0.3 >, < s_3, 0.3, 0.1, 0.2 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$P_2 = \{< s_1, 0.1, 0.4, 0.5 >, < s_2, 0.3, 0.1, 0.0 >, < s_3, 0.2, 0.0, 0.1 >; s_1, s_2, s_3 \in \mathcal{X}\}$,
$P_3 = \{< s_1, 0.2, 0.4, 0.2 >, < s_2, 0.4, 0.1, 0.0 >, < s_3, 0.3, 0.1, 0.0 >; s_1, s_2, s_3 \in \mathcal{X}\}$.
Put $\tau_{\mathcal{X}} = \{0_N, P_1, 1_N\}$. *Then the collection of neutrosophic semi j-open sets are* 0_N, 1_N, $P_1 \vee P_2$ *and* $P_1 \vee P_3$. *i.e.,* $P_1 \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P_1]]]$, $P_1 \vee P_2 \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P_1 \vee P_2]]]$ *and* $P_1 \vee P_3 \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P_1 \vee P_2]]]$. *Here every non empty neutrosophic semi j-open sets are neutrosophic semi j-dense in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. *i.e.,* $\mathcal{N}cl_{sj}[P_1] = 1_N$, $\mathcal{N}cl_{sj}[P_1 \vee P_2] = 1_N$, $\mathcal{N}cl_{sj}[P_1 \vee P_3] = 1_N$ *and* $\mathcal{N}cl_{sj}[1_N] = 1_N$. *Therefore a neutrosophic topological space* $\tau_{\mathcal{X}} = \{0_N, P_1, 1_N\}$ *is neutrosophic semi j-hyperconnected space.*

Theorem 4.7.3. *In a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, *every neutrosophic hyperconnected space is neutrosophic semi j-hyperconnected.*

Proof. Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space and P be a neutrosophic open subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $\mathcal{N}cl[P] = 1_N$. Since every neutrosophic open sets is neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. This implies $\mathcal{N}cl_{sj}(P) = 1_N$ for each neutrosophic open set P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-hyperconnected space. □

Theorem 4.7.4. *Let* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *be a neutrosophic topological space, then each of the following statements are equivalent.*

(a) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is neutrosophic semi j-hyperconnected.*

(b) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *has no two proper neutrosophic semi j-regular open or proper neutrosophic semi j-regular closed subset.*

(c) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *has no proper disjoint neutrospohic semi j-open subsets* P *and* Q *such that* $\mathcal{N}cl_{sj}[P] \vee Q = P \vee \mathcal{N}cl_{sj}[Q] = 1_N$.

(d) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *has no proper neutrosophic semi j-closed subsets* S *and* T *such that* $\mathcal{X} = S \vee T$ *and* $\mathcal{N}int_{sj}[S] \wedge T = S \wedge \mathcal{N}int_{sj}(T) = 0_N$.

Proof. (a) $\Longrightarrow$ (b) Let $0_N \neq P$ be neutrosophic semi j-regular open subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $P = \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic semi j-hyperconnected space. then $\mathcal{N}cl_{sj}[P] = 1_N$. This implies $P = \mathcal{N}int_{sj}[1_N] = 1_N$. Clearly P is not a proper neutrosophic semi j-regular open subset of $\mathcal{X}$. Similarly, $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ cannot have a proper neutrosophic semi j-regular closed subset.

(b) $\implies$ (c) Suppose P and Q are the neutrosophic subsets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $P \wedge Q = 0_N$ such that $\mathcal{N}cl_{sj}[P] \vee Q = P \vee \mathcal{N}cl_{sj}[Q] = 1_N$. This implies $0_N \neq \mathcal{N}cl_{sj}[P]$ is the neutrosophic semi j-regular closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since $P \wedge Q = 0_N$ and $\mathcal{N}cl_{sj}[P] \wedge Q = 0_N \implies \mathcal{N}cl_{sj}[P] \neq 1_N$ which implies $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ has a proper neutrosophic semi j-regular closed subset P. This contradicts (b).

(c) $\implies$ (d) Suppose, there exist two proper neutrosophic semi j-closed subset, $0_N \neq S$ and $0_N \neq T$ in $\mathcal{X}$ such that $\mathcal{X} = S \vee T, \mathcal{N}int_{sj}(S) \wedge T = S \wedge \mathcal{N}int_{sj}(T) = 0_N$. Then, we take $P = 1_N - S$ and $Q = 1_N - T$ are the two non-empty neutrosophic semi j-open sets. Then $\mathcal{N}cl_{sj}[P] \vee Q = \mathcal{N}cl_{sj}(1_N - S) \vee Q = [1_N - \mathcal{N}int_{sj}(S)] \vee Q = 1_N. \implies \mathcal{N}cl_{sj}[P] \vee Q = 1_N$ which contradicts (c).

(d) $\implies$ (a) Suppose there exist a proper neutrosophic semi j-open set $0_N \neq P$ of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl_{sj}[P] \neq 1_N$. Then $\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]] \neq 1_N$. Take $S = \mathcal{N}cl_{sj}[P]$ and $T = 1_N - \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]$. This implies $S \vee T = \mathcal{N}cl_{sj}[P] \vee [1_N - \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]] = \mathcal{N}cl_{sj}[P] \vee \mathcal{N}cl_{sj}[1_N - \mathcal{N}cl_{sj}[P]] \implies \mathcal{N}cl_{sj}[P] \vee \mathcal{N}cl_{sj}[C(S)] \implies S \vee C(S) = 1_N$. Then $\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]] \wedge [1_N - \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]] = 0_N. \implies \mathcal{N}cl_{sj}[P] \wedge \mathcal{N}int_{sj}[1_N - \mathcal{N}int_{sj}[\mathcal{N}cl_{sj}[P]]] = S \wedge \mathcal{N}int_{sj}\mathcal{N}cl_{sj}[1_N - \mathcal{N}cl_{sj}[P]] = S \wedge \mathcal{N}int_{sj}\mathcal{N}cl_{sj}\ [C(S)] = S \wedge C(S) = 0_N$. Since $C(S)$ is neutrosophic semi j-open. Thus $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ has two proper neutrosophic semi j-closed sets S and T such that $\mathcal{X} = S \vee T$ and $\mathcal{N}int_{sj}S \wedge T = S \wedge \mathcal{N}int_{sj}C[T] = 0_N$. This is a contradiction to (d). □

Theorem 4.7.5. *In a neutrosophic semi j-hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Let $0_N \neq P$ and $0_N \neq Q$ be the two neutrosophic semi j-open subsets in $\mathcal{X}$, then $P \wedge Q \neq 0_N$.*

Proof. Suppose $P \wedge Q = 0_N$, for any neutrosophic semi j-open sets $0_N \neq P$ and $0_N \neq Q$ in $\mathcal{X}$. Then $\mathcal{N}cl_{sj}[P] \wedge Q = 0_N$. This implies P is not neutrosophic semi j-dense. We have P is neutrosophic semi j-open then $P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$ and

P is not neutrosophic semi j-dense which is a contradiction to our assumption that $P \wedge Q = 0_N$. Hence $P \wedge Q \neq 0_N$. □

Theorem 4.7.6. *In a neutrosophic semi j-hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, intersection of any two neutrosophic semi j-open sets are also neutrosophic semi j-open.*

Proof. Let $0_N \neq P$, $0_N \neq Q$ be the two neutrosophic semi j-open sets in a neutrosophic semi j-hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $P \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]]$ and $Q \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[Q]]]$. We have $\mathcal{N}cl_{sj}[P] = 1_N$ and $\mathcal{N}cl_{sj}[Q] = 1_N$. This implies $\mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]] = \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[Q]]] = 1_N$, also we have $P \wedge Q \neq 0_N$. It follows that $P \wedge Q \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P]]] \wedge \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[Q]]] = \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P \wedge Q]]] = 1_N$. Therefore $P \wedge Q \leq \mathcal{N}cl[\mathcal{N}int[\mathcal{N}pcl[P \wedge Q]]] = 1_N$. Hence $P \wedge Q$ is also neutrosophic semi j-open. □

Definition 4.7.7. *A neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is called as neutrosophic extremally semi j-disconnected iff neutrosophic semi j-closure of every neutrosophic semi j-open set is neutrosophic semi j-open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Theorem 4.7.8. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, every neutrosophic semi j-hyperconnected space is neutrosophic extremally semi j-disconnected.*

Proof. Assume $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic semi j-hyperconnected space. Then we have, $\mathcal{N}cl_{sj}[P] = 1_N$. This implies $\mathcal{N}cl_{sj}(P)$ is neutrosophic semi j-open for every P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic extremally semi j-disconnected space. □

Remark 4.7.9. *The reverse of the above theorem may not be true as shown by the following example.*

Example 4.7.10. *Let $\mathcal{X} = \{s\}$ and the neutrosophic subsets W_1, W_2 and W_3 defined as follows,* $W_1 = \{< s, 0.5, 0.4, 0.3 >, s \in \mathcal{X}\}$

$W_2 = \{< s, 0.5, 0.6, 0.7 >, s \in \mathcal{X}\}$

$W_3 = \{< s, 0.4, 0.4, 0.5 >, s \in \mathcal{X}\}$

Taking $\tau_{\mathcal{X}} = \{0_N, 1_N, W_1, W_2, W_1 \vee W_2\}$ is a neutrosophic topological space on $\mathcal{X}$. For this $\tau_{\mathcal{X}}$, we have $0_N, 1_N, W_1, W_2, W_1 \vee W_2, W_1 \vee W_3, W_2 \vee W_3$ are the neutrosophic semi *j-open sets. Here we obtain,* $\mathcal{N}cl_{sj}(0_N) = 0_N$, $\mathcal{N}cl_{sj}(1_N) = 1_N$, $\mathcal{N}cl_{sj}(W_1) = W_1$, $\mathcal{N}cl_{sj}(W_2) = W_2$,$\mathcal{N}cl_{sj}(W_1 \vee W_2) = 1_N$,$\mathcal{N}cl_{sj}(W_1 \vee W_3) = W_1$ *and* $\mathcal{N}cl_{sj}(W_2 \vee W_3) = 1_N$. *Clearly* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is neutrosophic extremally semi j-disconnected but not neutrosophic semi j-hyperconnected.*

Theorem 4.7.11. *In a neutrosophic topological space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*, the following statements are equivalent.*

(i) $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is neutrosophic semi j-hyperconnected space.*

(ii) For each neutrosophic subset P of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *is either neutrosophic semi j-dense or neutrosophic semi j-nowhere dense set in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Proof. $(i) \implies (ii)$
Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic semi j-hyperconnected space and P be any nonempty neutrosophic subset such that $P \leq 1_N$. Assume P is not a neutrosophic semi j-nowhere dense set. Then we have $\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}(P)] \neq 0_N$. This implies $\mathcal{N}cl_{sj}[\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}(P)]] = 1_N$. But we have $\mathcal{N}cl_{sj}[\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}(P)]] = 1_N \leq \mathcal{N}cl_{sj}(P)$. Thus $\mathcal{N}cl_{sj}(P) = 1_N$. Hence P is neutrosophic semi j-dense set.
$(ii) \implies (i)$
Let $\emptyset \neq P_1$ be any neutrosophic semi j-open subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then we have $P_1 \leq \mathcal{N}int_{sj}(\mathcal{N}cl_{sj}(P_1))$. Therefore, $\mathcal{N}int_{sj}[\mathcal{N}cl_{sj}(P_1)] \neq 0_N$. By hypothesis P_1 is neutrosophic semi j-dense set. Thus $\mathcal{N}cl_{sj}(P_1) = 1_N$ for every P_1 in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is neutrosophic semi j-hyperconnected space. □

4.8 Conclusion

This chapter conveyed the characteristics of neutrosophic semi j-open sets, neutrosophic semi j-closed sets, neutrosophic hyperconnectedness and neutrosophic semi j-hyperconnectedness. Recently, neutrosophic sets have began to play a vital role by helping in the analysis of real life situations. In future, neutrosophic hyperconnected spaces will assist in determining solutions in each situations where indeterminancy occurs as the main crisis.