Chapter 5

Neutrosophic Resolvable Sets and Neutrosophic Resolvable Functions in Neutrosophic Hyperconnected spaces

5.1 Introduction

In this Chapter, we strived to ideate a new type of set labelled as neutrosophic resolvable set in neutrosophic topological spaces. We present the neutrosophic resolvable functions between neutrosophic topological spaces by using neutrosophic resolvable sets. The characteristics of neutrosophic resolvable sets and neutrosophic resolvable functions with existing sets are examined. We wish to implement these sets and functions in neutrosophic hyperconnected spaces. This study also aims to examine the basic properties of these sets and explore the relationship between neutrosophic resolvable functions and neutrosophic hyperconnected spaces.

5.2 Neutrosophic resolvable sets

Definition 5.2.1. *A neutrosophic set P is said to be neutrosophic resolvable set in neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if $\{\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]\}$ is neutrosophic*

nowhere dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
i.e.,$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]\} = 0_N$, where $C[Q] \in \tau_{\mathcal{X}}$.

Example 5.2.2. *Consider $\mathcal{X} = \{\mu, \nu\}$ and the neutrosophic sets P_1, P_2, P_3 and P_4 in $\mathcal{X}$ as follows:*
P_1=$\{< \mu, 0.4, 0.3, 0.6 >, < \nu, 0.5, 0.3, 0.2 >; \mu, \nu \in \mathcal{X}\}$
P_2=$\{< \mu, 0.3, 0.4, 0.3 >, < \nu, 0.6, 0.5, 0.2 >; \mu, \nu \in \mathcal{X}\}$
P_3=$\{< \mu, 0.4, 0.4, 0.3 >, < \nu, 0.6, 0.5, 0.2 >; \mu, \nu \in \mathcal{X}\}$
P_4=$\{< \mu, 0.3, 0.3, 0.6 >, < \nu, 0.5, 0.3, 0.2 >; \mu, \nu \in \mathcal{X}\}$.
Then $\tau_{\mathcal{X}} = \{0_N, P_1, P_2, P_3, P_4, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$.
Now, $\tau_{\mathcal{X}}^C = \{0_N, C[P_1], C[P_2], C[P_3], C[P_4], 1_N\}$
Let R=$\{< \mu, 0.2, 0.1, 0.7 >, < \nu, 0.3, 0.1, 0.3 >; \mu, \nu \in \mathcal{X}\}$ be a neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
Then $C[R]$=$\{< \mu, 0.8, 0.9, 0.3 >, < \nu, 0.7, 0.9, 0.7 >; \mu, \nu \in \mathcal{X}\}$,
$C[P_1]$=$\{< \mu, 0.6, 0.7, 0.4 >, < \nu, 0.5, 0.7, 0.8 >; \mu, \nu \in \mathcal{X}\}$
$C[P_2]$=$\{< \mu, 0.7, 0.6, 0.7 >, < \nu, 0.4, 0.5, 0.8 >; \mu, \nu \in \mathcal{X}\}$
$C[P_3]$=$\{< \mu, 0.6, 0.6, 0.7 >, < \nu, 0.4, 0.5, 0.8 >; \mu, \nu \in \mathcal{X}\}$
$C[P_4]$=$\{< \mu, 0.7, 0.7, 0.4 >, < \nu, 0.5, 0.7, 0.8 >; \mu, \nu \in \mathcal{X}\}$
Now,
$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[P_1] \wedge A] \wedge \mathcal{N}cl[C[P_1] \wedge C[A]]\} = 0_N$
$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[P_2] \wedge A] \wedge \mathcal{N}cl[C[P_2] \wedge C[A]]\} = 0_N$
$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[P_3] \wedge A] \wedge \mathcal{N}cl[C[P_3] \wedge C[A]]\} = 0_N$
$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[P_4] \wedge A] \wedge \mathcal{N}cl[C[P_4] \wedge C[A]]\} = 0_N$.
Therefore R is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Example 5.2.3. *Consider $\mathcal{X} = \{\mu\}$ and consider the neutrosophic sets Q_1, Q_2, Q_3 in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ as follows:*
$Q_1 = \{< \mu, 0.4, 0.5, 0.6 >, \mu \in \mathcal{X}\}$
$Q_2 = \{< \mu, 0.3, 0.4, 0.8 >, \mu \in \mathcal{X}\}$
$Q_3 = \{< \mu, 0.4, 0.5, 0.8 >, \mu \in \mathcal{X}\}$
Then $\tau_{\mathcal{X}} = \{0_N, Q_1, Q_2, Q_3, 1_N\}$ is a neutrosophic topological space on $\mathcal{X}$.
Here $\tau_{\mathcal{X}}^C = \{0_N, C[Q_1], C[Q_2], C[Q_3], 1_N\}$. where

$C[Q_1] = \{< \mu, 0.6, 0.5, 0.4 >, \mu \in \mathcal{X}\}$

$C[Q_2] = \{< \mu, 0.7, 0.6, 0.2 >, \mu \in \mathcal{X}\}$

$C[Q_3] = \{< \mu, 0.6, 0.5, 0.2 >, \mu \in \mathcal{X}\}$

Take $R = \{< \mu, 0.4, 0.4, 0.6 >, \mu \in \mathcal{X}\}$

Then $C[R] = \{< \mu, 0.6, 0.6, 0.4 >, \mu \in \mathcal{X}\}$. Now,

$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[Q_1] \wedge R] \wedge \mathcal{N}cl[C[Q_1] \wedge C[R]]] = Q_1 \neq 0_N$

$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[Q_2] \wedge R] \wedge \mathcal{N}cl[C[Q_2] \wedge C[R]]] = Q_1 \neq 0_N$

$\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[Q_3] \wedge R] \wedge \mathcal{N}cl[C[Q_3] \wedge C[R]]] = 0_N$

This implies R is not a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Remark 5.2.4. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, every neutrosophic resolvable set need not be a neutrosophic open set. In example 5.2.2, R is a neutrosophic resolvable set but not neutrosophic open set.*

Proposition 5.2.5. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is a neutrosophic resolvable set, then $\mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N$ for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then for each neutrosophic closed set Q, we have $\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]]\} = 0_N$, We know that,

$$\begin{aligned} \mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]] &\geq \mathcal{N}cl[[Q \wedge P] \wedge [Q \wedge C[P]] \\ &\geq \mathcal{N}cl[Q \wedge P \wedge C[P]] \end{aligned}$$

Now,

$$\begin{aligned} \mathcal{N}int\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]] &\geq \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[[Q \wedge P] \wedge [Q \wedge C[P]]] \\ &= \mathcal{N}int\mathcal{N}cl[Q \wedge P \wedge C[P]] \end{aligned}$$

This implies $0_N \geq \mathcal{N}int\mathcal{N}cl[Q \wedge P \wedge C[P]]$. Since P is a neutrosophic resolvable set. Hence $\mathcal{N}int\mathcal{N}cl[Q \wedge P \wedge C[P]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.2.6. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $\mathcal{N}cl[P \vee C[P] \vee R] = 1_N$, for each neutrosophic open set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ Then we have, $\mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N$, for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then

$$\begin{aligned} C[\mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q]] &= 1_N \\ \implies \mathcal{N}cl\mathcal{N}int[C[P \wedge C[P] \wedge Q]] &= 1_N \\ \implies \mathcal{N}cl\mathcal{N}int[C[P] \vee P \vee C[Q]] &= 1_N \end{aligned}$$

We know that
$\mathcal{N}cl\mathcal{N}int[C[P] \vee P \vee C[Q]] \leq \mathcal{N}cl[C[P] \vee P \vee C[Q]]$
$\implies 1_N \leq \mathcal{N}cl[C[P]] \vee [P] \vee C[Q]$ and put $C[Q] = R$. Then we have $\mathcal{N}cl[P \vee C[P] \vee R] = 1_N$, for each neutrosophic open set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.2.7. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic topological space , then the neutrosophic interior of a neutrosophic closed set is neutrosophic regular open.*

Proof. Let Q be a neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and take $R = \mathcal{N}intQ$. Therefore $R \leq Q$. Since Q is closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. $\mathcal{N}cl[R] \leq \mathcal{N}cl[Q] = Q$. This implies $\mathcal{N}int\mathcal{N}cl[R] \leq \mathcal{N}int[Q] = R$. We have $R \leq \mathcal{N}cl[R]$. Now $\mathcal{N}int[R] \leq \mathcal{N}int\mathcal{N}cl[R]$. Since R is a neutrosophic open. This implies $R \leq \mathcal{N}int\mathcal{N}cl[R]$ gives $\mathcal{N}int\mathcal{N}cl[R] = R$. Therefore R is a neutrosophic regular open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.2.8. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is a neutrosophic resolvable set, then there exists a neutrosophic regular open set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $R \leq \mathcal{N}cl[P \vee C[P]]$.*

Proof. Let P be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using proposition 5.2.5, we have $\mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N$ for each closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Now, $\mathcal{N}int[P \wedge C[P] \wedge Q] \leq \mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N \implies \mathcal{N}int[P \wedge C[P] \wedge Q] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \implies \mathcal{N}int[P \wedge C[P]] \wedge \mathcal{N}int[Q] = 0_N \implies \mathcal{N}intQ \leq C[\mathcal{N}int[P \wedge C[P]]] = \mathcal{N}cl[C[P] \vee P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since Q is a neutrosophic closed set. Using Proposition 5.2.7 , $\mathcal{N}intQ$ is a neutrosophic regular open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Put $\mathcal{N}intQ = R$. Hence if P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, there exist a neutrosophic regular open set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $R \leq \mathcal{N}cl[P \vee C[P]]$. □

Proposition 5.2.9. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$ if P is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$, then $C[P]$ is also a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$ then $\mathcal{N}int\mathcal{N}cl\{[\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]]\} = 0_N$, for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. For the set $C[P]$, $\implies \mathcal{N}int\mathcal{N}cl\{[\mathcal{N}cl[Q \wedge C[P]] \wedge \mathcal{N}cl[Q \wedge C[C[P]]]]\} = 0_N$
$\implies \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge C[P]] \wedge \mathcal{N}cl[Q \wedge P]] = 0_N$.
Hence $C[P]$ is also a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. □

Proposition 5.2.10. *In a neutrosophic topological space $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$, if P is a neutrosophic closed set with $\mathcal{N}int[P] = 0_N$, then P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic closed set and $\mathcal{N}int[P] = 0_N$ in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. For a neutrosophic closed set Q in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$, we have

$$\begin{aligned}
\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]] &\leq \mathcal{N}cl[Q] \wedge \mathcal{N}cl[P] \wedge \mathcal{N}cl[C[P]] \\
&= \mathcal{N}cl[P] \wedge C[\mathcal{N}intP] \\
&= \mathcal{N}cl[P] \wedge 1_N \\
&= \mathcal{N}cl[P] \quad \text{since}(\mathcal{N}intP = 0_N)
\end{aligned}$$

Therefore $\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]] \leq P$. Since P is a neutrosophic closed set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. Thus $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]] \leq \mathcal{N}int[\mathcal{N}cl[P]] = \mathcal{N}int[P] = 0_N$. Hence P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. □

Proposition 5.2.11. *Let $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$ be the neutrosophic topological space . If P is a neutrosophic open set and neutrosophic dense in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$, then P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic open set and neutrosophic dense in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. Then $C[P]$ is neutrosophic closed and $\mathcal{N}cl[P] = 1_N$. For a neutrosophic closed set Q, we have $\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]] \leq \mathcal{N}cl[Q] \wedge \mathcal{N}cl[P] \wedge \mathcal{N}cl[Q] \wedge \mathcal{N}cl[C[P]] \implies Q \wedge C[P]$. Since $\mathcal{N}cl[P] = 1_N$, Q and $C[P]$ are neutrosophic closed set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$.

This implies

$$\begin{aligned}
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]] &\leq \mathcal{N}int\mathcal{N}cl[Q \wedge C[P]] \\
&\leq \mathcal{N}int[\mathcal{N}cl[Q] \wedge \mathcal{N}cl[C[P]]] \\
&= \mathcal{N}int[Q \wedge C[P]] \\
&= \mathcal{N}int[Q] \wedge \mathcal{N}int[C[P]] \\
&= \mathcal{N}int[Q] \wedge C[\mathcal{N}cl[P]] \\
&= \mathcal{N}int[Q] \wedge C[1_N] \\
&= \mathcal{N}int[Q] \wedge 0_N \\
&= 0_N
\end{aligned}$$

Thus $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge P] \wedge \mathcal{N}cl[Q \wedge C[P]]] = 0_N$. Hence P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.2.12. *If P is neutrosophic open and neutrosophic dense in a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $C[P]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic open set and neutrosophic dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. This implies $C[P]$ is a neutrosophic closed set and $\mathcal{N}cl[P] = 1_N$. Then

$$\begin{aligned}
C[\mathcal{N}cl[P]] &= C[1_N] \\
\mathcal{N}int[C[P]] &= 0_N
\end{aligned}$$

Using Proposition 5.2.10, $C[P]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.2.13. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic topological space . If P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $\mathcal{N}int[P \wedge C[P]] \leq \mathcal{N}cl[C[Q]]$ for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
Using Proposition 5.2.5 $\mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
We know that, $\mathcal{N}int[P \wedge C[P] \wedge Q] \leq \mathcal{N}int\mathcal{N}cl[P \wedge C[P] \wedge Q] = 0_N \implies \mathcal{N}int[P \wedge C[P] \wedge \mathcal{N}int[Q]] = 0_N \implies \mathcal{N}int[P \wedge C[P]] \leq C[\mathcal{N}intQ] = \mathcal{N}cl[C[Q]]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
□

5.3 Neutrosophic resolvable functions

Definition 5.3.1. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be any two neutrosophic topological spaces. A function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is called as neutrosophic resolvable function if $R^{-1}[P]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, for each neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Example 5.3.2. *Let the set $\mathcal{X} = \{\mu, \nu, \omega\}$ and the neutrosophic sets U_1, U_2, U_3 and U_4 defined as follows:-*

$$
\begin{aligned}
U_1 = &\ \{< \mu, 0.6, 0.7, 0.2 >, < \nu, 0.7, 0.8, 0.2 >, < \omega, 1.0, 0.6, 0.3 >, \mu, \nu, \omega \in \mathcal{X}\} \\
U_2 = &\ \{< \mu, 0.5, 0.6, 0.3 >, < \nu, 0.8, 0.7, 0.4 >, < \omega, 1.0, 0.8, 0.4 >, \mu, \nu, \omega \in \mathcal{X}\} \\
U_3 = &\ \{< \mu, 0.5, 0.6, 0.3 >, < \nu, 0.7, 0.7, 0.4 >, < \omega, 1.0, 0.6, 0.4 >, \mu, \nu, \omega \in \mathcal{X}\} \\
U_4 = &\ \{< \mu, 0.6, 0.7, 0.2 >, < \nu, 0.8, 0.8, 0.2 >, < \omega, 0.1, 0.8, 0.3 >, \mu, \nu, \omega \in \mathcal{X}\}
\end{aligned}
$$

Then $\tau_{\mathcal{X}} = \{0_N, U_1, U_2, U_3, U_4, 1_N\}$ is the neutrosophic topological spaces in $\mathcal{X}$.
Then $\tau_{\mathcal{X}}^C = \{0_N, C[U_1], C[U_2], C[U_3], C[U_4], 1_N\}$ where,

$$
\begin{aligned}
C[U_1] = &\ \{< \mu, 0.4, 0.3, 0.8 >, < \nu, 0.3, 0.2, 0.8 >, < \omega, 0.0, 0.4, 0.7 >, \mu, \nu, \omega \in \mathcal{X}\} \\
C[U_2] = &\ \{< \mu, 0.5, 0.4, 0.7 >, < \nu, 0.2, 0.3, 0.6 >, < \omega, 0.0, 0.2, 0.6 >, \mu, \nu, \omega \in \mathcal{X}\} \\
C[U_3] = &\ \{< \mu, 0.5, 0.4, 0.7 >, < \nu, 0.3, 0.3, 0.6 >, < \omega, 0.0, 0.4, 0.6 >, \mu, \nu, \omega \in \mathcal{X}\} \\
C[U_4] = &\ \{< \mu, 0.4, 0.3, 0.8 >, < \nu, 0.2, 0.2, 0.8 >, < \omega, 0.0, 0.2, 0.7 >, \mu, \nu, \omega \in \mathcal{X}\}
\end{aligned}
$$

Put $S = \{< \mu, 0.3, 0.2, 0.8 >, < \nu, 0.1, 0.2, 0.9 >, < \omega, 0.0, 0.3, 0.8 >, \mu, \nu, \omega \in \mathcal{X}\}$, So $C[S] = \{< \mu, 0.7, 0.8, 0.2 >, < \nu, 0.9, 0.8, 0.1 >, < \omega, 1.0, 0.7, 0.2 >, \mu, \nu, \omega \in \mathcal{X}\}$. Now,

$$
\begin{aligned}
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C(U_1) \wedge S] \wedge \mathcal{N}cl[C(U_1) \wedge C(S)]] = &\ 0_N \\
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C(U_2) \wedge S] \wedge \mathcal{N}cl[C(U_2) \wedge C(S)]] = &\ 0_N \\
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C(U_3) \wedge S] \wedge \mathcal{N}cl[C(U_3) \wedge C(S)]] = &\ 0_N \\
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C(U_4) \wedge S] \wedge \mathcal{N}cl[C(U_4) \wedge C(S)]] = &\ 0_N.
\end{aligned}
$$

This implies S is a neutrosophic resolvable set. Put $T = \{< \mu, 0.0, 0.3, 0.8 >, < \nu, 0.3, 0.2, 0.8 >, < \omega, 0.1, 0.2, 0.0 >, \mu, \nu, \omega \in \mathcal{X}\}$. Define a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to$

$\mathcal{N}(\mathcal{Y},\tau_{\mathcal{Y}})$ by $R(\mu)=\nu$, $R(\nu)=\omega$, $R(\omega)=\mu$. Now,

$$\begin{aligned} R^{-1}[T] = &\quad \{<\mu,0.0,0.3,0.8>,<\nu,0.3,0.2,0.8>,<\eta,0.1,0.2,0.0>,\mu,\nu,\omega\in\mathcal{X}\} \\ = &\quad \{<\mu,0.1,0.2,0.9>,<\nu,0.1,0.2,0.9>,<\omega,0.0,0.3,0.8>,\mu,\nu,\omega\in\mathcal{X}\} \\ = &\quad \{<\mu,0.3,0.2,0.8>,<\nu,0.1,0.2,0.9>,<\omega,0.0,0.3,0.8>,\mu,\nu,\omega\in\mathcal{X}\} \\ = &\quad S. \end{aligned}$$

Since S is a resolvable set. Hence R is a neutrosophic resolvable function.

Example 5.3.3. *Consider the set $\mathcal{X}=\{\mu,\nu,\omega\}$ and the neutrosophic sets V_1, V_2, V_3, V_4 and V_5 are defined in $\mathcal{X}$ as follows:-*

$$\begin{aligned} V_1 = &\quad \{<\mu,0.0,0.0,0.5>,<\nu,0.1,0.1,0.6>,<\omega,0.2,0.1,0.5>,\mu,\nu,\omega\in\mathcal{X}\} \\ V_2 = &\quad \{<\mu,1.0,0.9,0.0>,<\nu,0.9,0.8,0.1>,<\omega,0.8,0.7,0.5>,\mu,\nu,\omega\in\mathcal{X}\} \\ V_3 = &\quad \{<\mu,0.0,0.0,0.4>,<\nu,0.1,0.1,0.5>,<\omega,0.1,0.1,0.4>,\mu,\nu,\omega\in\mathcal{X}\} \\ V_4 = &\quad \{<\mu,0.2,0.2,0.4>,<\nu,0.3,0.3,0.5>,<\omega,0.3,0.2,0.5>,\mu,\nu,\omega\in\mathcal{X}\} \\ V_5 = &\quad \{<\mu,0.3,0.2,0.5>,<\nu,0.2,0.2,0.5>,<\omega,0.3,0.3,0.5>,\mu,\nu,\omega\in\mathcal{X}\} \end{aligned}$$

Then $\tau_{\mathcal{X}}=\{0_N,V_1,V_2,1_N\}$, $\tau_{\mathcal{Y}}=\{0_N,V_5,1_N\}$ are two neutrosophic topological spaces. Now we define a function $R:\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\tau_{\mathcal{Y}})$ by $R(\mu)=\nu$, $R(\nu)=\mu$ and $R(\omega)=\mu$. Now, $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_1]\wedge V_3]\wedge\mathcal{N}cl[C[V_1]\wedge C[V_3]]]=0_N \implies \mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_2]\wedge V_3]\wedge\mathcal{N}cl[C[V_2]\wedge C[V_3]]]=0_N$. This implies V_3 is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. But $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[C[V_1]\wedge V_4]]\wedge\mathcal{N}cl[C[V_1]\wedge C[V_4]]=V_1\neq 0_N$. Therefore V_4 is not a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})$. By computation, we have

$$\begin{aligned} R^{-1}[V_5] = &\quad R^{-1}[<\mu,0.3,0.2,0.5>,<\nu,0.2,0.2,0.5>,<\eta,0.3,0.3,0.5>,\mu,\nu,\omega\in\mathcal{X}] \\ = &\quad R^{-1}[<\mu,0.2,0.2,0.5>,<\nu,0.3,0.3,0.5>,<\omega,0.3,0.3,0.5>,\mu,\nu,\omega\in\mathcal{X}] \\ = &\quad R^{-1}[<\mu,0.2,0.2,0.5>,<\nu,0.3,0.3,0.5>,<\omega,0.3,0.2,0.5>,\mu,\nu,\omega\in\mathcal{X}] \\ = &\quad V_4 \end{aligned}$$

Since V_4 is not a neutrosophic resolvable set. Therefore the function $R:\mathcal{N}(\mathcal{X},\tau_{\mathcal{X}})\to\mathcal{N}(\mathcal{Y},\tau_{\mathcal{Y}})$ is not a neutrosophic resolvable function.

Proposition 5.3.4. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is a neutrosophic resolvable function, then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$*

(a) $\mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P \wedge C[P]]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic closed set Q, where $C[Q] \in \tau_{\mathcal{X}}$.

(b) For the neutrosophic closed set Q, $\mathcal{N}int[Q \wedge R^{-1}[P \wedge C[P]]] = 0_N$.

Proof. (a) Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be the neutrosophic resolvable function. Then for the neutrosophic open set $0_N \neq P$ in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, there exist the neutrosophic resolvable set $R^{-1}[P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using the definition of resolvable set, we have, $\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge R^{-1}[P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since, $\mathcal{N}cl[[Q \wedge R^{-1}[P]] \wedge [Q \wedge C[R^{-1}[P]]]] \leq \mathcal{N}cl[Q \wedge R^{-1}[P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]]$ $\implies \mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]$ is the neutrosophic nowhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
(b) Using (a), we have $\mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P \wedge C[P]]] = 0_N$, in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Now, $\mathcal{N}int[Q \wedge R^{-1}[P \wedge C[P]]] \leq \mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P \wedge C[P]]] = 0_N \implies \mathcal{N}int[Q \wedge R^{-1}[P \wedge C[P]]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.3.5. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function then $\mathcal{N}int[R^{-1}[P \wedge C[P]]] \leq \mathcal{N}cl[C[Q]]$ for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, and Q is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let us take a neutrosophic resolvable function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ using the proposition 5.3.4 (b), we have $\mathcal{N}int[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N$, here Q is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Now, $\mathcal{N}int[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]] = \mathcal{N}int[Q] \wedge \mathcal{N}int[R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N \implies \mathcal{N}int[R^{-1}[P] \wedge C[R^{-1}[P]]] \leq C[\mathcal{N}int[Q]] = \mathcal{N}cl[C[Q]]$. □

Proposition 5.3.6. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function from the neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ to neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

(a) there exist a regular open set S in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P \vee C[P]]] \geq S$.

(b) $\mathcal{N}int\mathcal{N}cl[P \vee C[P]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Proof. (a) Let R be a neutrosophic resolvable function from neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, using the Proposition 5.3.4 (b), we have $\mathcal{N}int[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, where Q is the neutrosophic closed set. This implies $\mathcal{N}int[Q] \wedge \mathcal{N}int[R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N$

$$\begin{aligned} \implies \mathcal{N}int[Q] \leq & \ C[\mathcal{N}int[R^{-1}[P]] \wedge C[R^{-1}[P]]] \\ = & \ \mathcal{N}cl[C[R^{-1}[P] \wedge C[R^{-1}[P]]]] \\ = & \ \mathcal{N}cl[C[R^{-1}[P]] \vee R^{-1}[P]] \end{aligned}$$

Since Q is the neutrosophic closed set. Using 5.2.7, $\mathcal{N}int[Q]$ is the neutrosophic regular open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Put $S = \mathcal{N}int[Q]$. Then $\mathcal{N}cl[R^{-1}[C[P] \wedge P]] \geq R$, for a neutrosophic regular open set S in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

(b) Since every neutrosophic regular open set is neutrosophic open in a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore the neutrosophic regular open set S is neutrosophic open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using (a) $S \leq \mathcal{N}cl[R^{-1}[[C[P]] \vee [P]]] \neq 0_N \implies \mathcal{N}intS = S \leq \mathcal{N}int\mathcal{N}cl[R^{-1}[C[P] \vee [P]]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.3.7. *Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic resolvable function, then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, there exists a neutrosophic regular closed set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $S \geq \mathcal{N}int[R^{-1}[P] \wedge C[R^{-1}[P]]]$.*

Proof. Consider $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic resolvable function. Using the proposition 5.3.5, for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, we have $\mathcal{N}cl[C[Q]] \geq \mathcal{N}int[R^{-1}[P \wedge C[P]]]$, where Q is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $C[Q]$ is the neutrosophic open set. Put $S = C[Q]$. This implies, $\mathcal{N}cl[S]$ is the neutrosophic regular closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $S \geq \mathcal{N}int[R^{-1}[P] \wedge C[R^{-1}[P]]]$. □

Proposition 5.3.8. *If $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function, then $\mathcal{N}cl[R^{-1}[P \vee C[P]] \vee S] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ and $S \in \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be the neutrosophic resolvable function. By Proposition 5.3.5 $\mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]] = 0_N$ for the neutrosophic open set P and neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then

$$\begin{aligned} C[\mathcal{N}int\mathcal{N}cl[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]]] = &\ 1_N \\ \implies \mathcal{N}cl\mathcal{N}int[C[Q \wedge R^{-1}[P] \wedge C[R^{-1}[P]]]] = &\ 1_N \\ \implies \mathcal{N}cl\mathcal{N}int[C[Q] \vee C[R^{-1}[P]] \vee R^{-1}[P]] = &\ 1_N \end{aligned}$$

Since $C[C[R^{-1}[P]]] = R^{-1}[P]$. Now,

$$\begin{aligned} \mathcal{N}cl\mathcal{N}int[C[Q] \vee C[R^{-1}[P]] \vee R^{-1}[P]] \leq &\ \mathcal{N}cl[C[Q] \vee C[R^{-1}[P]] \vee R^{-1}[P]] \\ \implies 1_N \leq &\ \mathcal{N}cl[C[Q] \vee C[R^{-1}[P]] \vee R^{-1}[P]] \\ \implies \mathcal{N}cl[C[Q] \vee C[R^{-1}[P] \vee R^{-1}[P]]] = &\ 1_N \\ \mathcal{N}cl[C[Q] \vee R^{-1}[P \vee C[P]]] = &\ 1_N \end{aligned}$$

Put $S = C[Q]$ then we have
$\mathcal{N}cl[S \vee R^{-1}[P \vee C[P]]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, where S is the neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.3.9. *If the function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function and P is the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then $C[R^{-1}[P]]$ is also neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ and R be a neutrosophic resolvable function from a neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into a neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. This implies $R^{-1}[P]$ is the neutrosophic resolvable set $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, using Proposition 5.2.9 $C[R^{-1}[P]]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.3.10. *Let $R_1 : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be the neutrosophic resolvable function and $R_2 : \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}}) \to \mathcal{N}(\mathcal{Z}, \tau_{\mathcal{Z}})$ be the neutrosophic continuous function, then $R_2 \circ R_1 : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Z}, \tau_{\mathcal{Z}})$ is the neutrosophic resolvable function.*

Proof. Let $0_N \neq P$ be a neutrosophic open set in $[\mathcal{Z}, \tau_{\mathcal{Z}}]$. Since R_2 is the neutrosophic continuous function from $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ into $[\mathcal{Z}, \tau_{\mathcal{Z}}]$. This implies $R^{-1}[P]$ is the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Since R_1 is the neutrosophic resolvable set from $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Therefore $R_1^{-1}[R_2^{-1}[P]]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Thus $[R_2 \circ R_1]^{-1}[P]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, for the neutrosophic resolvable function from $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Z}, \tau_{\mathcal{Z}})$. □

Proposition 5.3.11. *If $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic contra continuous function from the neutrosophic topological space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, and if $\mathcal{N}int[Q] = 0_N$, for each neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be the neutrosophic contra continuous function. Take P be the neutrosophic open set in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. This implies $R^{-1}[P]$ is the neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using the hypothesis, $\mathcal{N}int[R^{-1}[P]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. For the neutrosophic closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

$$\begin{aligned}
\mathcal{N}cl[Q \wedge [R^{-1}][P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]] \leq\ & \mathcal{N}cl[R^{-1}[P]] \wedge \mathcal{N}cl[C[R^{-1}[P]]] \\
=\ & \mathcal{N}cl[R^{-1}[P]] \wedge C[\mathcal{N}int[R^{-1}[P]]] \\
=\ & \mathcal{N}cl[R^{-1}[P]] \wedge C[0_N] \\
=\ & \mathcal{N}cl[R^{-1}[P]] \\
=\ & R^{-1}[P]
\end{aligned}$$

Since $R^{-1}[P]$ is neutrosophic closed set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $\mathcal{N}cl[Q \wedge R^{-1}[P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]] \leq R^{-1}[P]$. Now,

$$\begin{aligned}
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge R^{-1}[P] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]]] \leq\ & \mathcal{N}int\mathcal{N}cl[R^{-1}[P]] \\
=\ & \mathcal{N}int[R^{-1}[P]] \\
=\ & 0_N
\end{aligned}$$

This implies $R^{-1}[P]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function. □

5.4 Neutrosophic resolvable sets in Neutrosophic hyperconnected spaces

Proposition 5.4.1. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space. If P is neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then P is a neutrosophic dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. This implies P is neutrosophic open and neutrosophic dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.2.11 P is a neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Remark 5.4.2. *The following example shows that the converse of the above proposition need not be true in general. That is, every neutrosophic resolvable sets need not be a neutrosophic open sets in neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Example 5.4.3. *Consider $\mathcal{X} = \{\mu, \nu\}$ and the neutrosophic sets S_1, S_2, S_3 and S_4 as follows*

$S_1 = \{< \mu, 0.3, 0.4, 0.4 >, < \nu, 0.6, 0.1, 0.4 >; \mu, \nu \in \mathcal{X}\}$

$S_2 = \{< \mu, 0.2, 0.5, 0.7 >, < \nu, 0.5, 0.2, 0.0 >; \mu, \nu \in \mathcal{X}\}$

$S_3 = \{< \mu, 0.3, 0.5, 0.4 >, < \nu, 0.6, 0.2, 0.0 >; \mu, \nu \in \mathcal{X}\}$

$S_4 = \{< \mu, 0.2, 0.4, 0.7 >, < \nu, 0.5, 0.1, 0.4 >; \mu, \nu \in \mathcal{X}\}$

Then $\tau_{\mathcal{X}} = \{0_N, S_1, S_2, S_3, S_4, 1_N\}$ is a neutrosophic topological space and $\mathcal{N}cl[S_1] = 1_N$, $\mathcal{N}cl[S_2] = 1_N$, $\mathcal{N}cl[S_3] = 1_N$, $\mathcal{N}cl[S_4] = 1_N$. Therefore $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic hyperconnected space. Let $R = \{< \mu, 0.8, 0.4, 0.3 >, < \nu, 0.4, 0.8, 0.5 >; \mu, \nu \in \mathcal{X}\}$.

Then $C[R] = \{< \mu, 0.2, 0.6, 0.7 >, < \nu, 0.6, 0.2, 0.5 >; \mu, \nu \in \mathcal{X}\}$

Now,

$$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[S_1] \wedge R] \wedge \mathcal{N}cl[C[S_1] \wedge C[R]]\} = 0_N$$
$$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[S_2] \wedge R] \wedge \mathcal{N}cl[C[S_2] \wedge C[R]]\} = 0_N$$
$$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[S_3] \wedge R] \wedge \mathcal{N}cl[C[S_3] \wedge C[R]]\} = 0_N$$
$$\mathcal{N}int\mathcal{N}cl\{\mathcal{N}cl[C[S_4] \wedge R] \wedge \mathcal{N}cl[C[S_4] \wedge C[R]]\} = 0_N$$

Hence R is a neutrosophic resolvable set but not neutrosophic open set.

Proposition 5.4.4. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is any neutrosophic set in $\mathcal{X}$ with $\mathcal{N}int[P] \neq 0_N$, then*

(i) $\mathcal{N}int[P]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

(ii) The neutrosophic set P is neutrosophic semi open and then there exists a neutrosophic resolvable set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $Q \leq P$.

Proof. (i) Let $0_N \neq P$ be any neutrosophic set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ with $\mathcal{N}int[P] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is the neutrosophic hyperconnected space. Using the proposition 5.4.1 the neutrosophic open set $\mathcal{N}int[P]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

(ii) Let P be any neutrosophic subset of $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ with $\mathcal{N}int[P] \neq 0_N$. Then $\mathcal{N}cl[\mathcal{N}int[P]] = 1_N$. Therefore $P \leq \mathcal{N}cl\mathcal{N}int[P]$. This implies P is a neutrosophic semi open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. We know that $\mathcal{N}int[P] \leq P$ and take $\mathcal{N}int[P] = Q$, there exist an neutrosophic resolvable set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $Q \leq P$.

□

Proposition 5.4.5. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ be a neutrosophic hyperconnected space. If P is a neutrosophic somewhere dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then there exists a neutrosophic resolvable sets $\mathcal{N}int[P]$ and $\mathcal{N}int[C[P]]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}int[P \wedge C[P]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. By the definition $\mathcal{N}int\mathcal{N}cl[P] \neq 0_N$ and there exists a neutrosophic open set R in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $R \leq \mathcal{N}cl[P]$. Then

$$\begin{aligned} C[R] &\geq C[\mathcal{N}cl[P]] \\ \mathcal{N}int[C[R]] &\geq \mathcal{N}int[C[\mathcal{N}cl[P]]] \\ &= \mathcal{N}int[\mathcal{N}int[C[P]]] \\ \mathcal{N}int[C[R]] &\geq \mathcal{N}int[C[P]] \end{aligned}$$

$\implies \mathcal{N}int[C[R]] \neq 0_N$, Put $C[R] = S$

$\implies \mathcal{N}int[S] \neq 0_N$.

Using Proposition 5.4.1, a non empty neutrosophic open set $\mathcal{N}int\mathcal{N}cl[P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic resolvable set and for the neutrosophic closed set S,

we have

$[\mathcal{N}int\mathcal{N}cl[P] \wedge C[\mathcal{N}int\mathcal{N}cl[P]] \wedge S] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$

$\mathcal{N}int\{\mathcal{N}int\mathcal{N}cl[P] \wedge C[\mathcal{N}int\mathcal{N}cl[P]] \wedge S\} \leq \mathcal{N}int\mathcal{N}cl\{\mathcal{N}int\mathcal{N}cl[P] \wedge C[\mathcal{N}int\mathcal{N}cl[P]] \wedge S\} = 0_N$

$\implies \{\mathcal{N}int\mathcal{N}int\mathcal{N}cl[P] \wedge \mathcal{N}int[C[\mathcal{N}int\mathcal{N}cl[P]]] \wedge \mathcal{N}intS\} = 0_N$

$\implies \{\mathcal{N}int\mathcal{N}cl[P] \wedge \mathcal{N}int\mathcal{N}cl\mathcal{N}int[C[P]] \wedge \mathcal{N}intS\} = 0_N$

We have

$\mathcal{N}intP \leq \mathcal{N}int\mathcal{N}cl[P], \mathcal{N}intS \geq \mathcal{N}int[C[P]], \mathcal{N}int\mathcal{N}cl\mathcal{N}int[C[P]] \geq \mathcal{N}int[C[P]]$

$\implies \mathcal{N}int[P] \wedge \mathcal{N}int[C[P]] \wedge \mathcal{N}int[C[P]] \leq \mathcal{N}int\mathcal{N}cl[P] \wedge \mathcal{N}int\mathcal{N}cl\mathcal{N}int[P] \wedge \mathcal{N}intS = 0_N$

$\implies \mathcal{N}int[P] \wedge \mathcal{N}int[C[P]] = 0_N$

$\implies \mathcal{N}int[P \wedge C[P]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.4.1, the neutrosophic open sets $\mathcal{N}int[P]$ and $\mathcal{N}int[C[P]]$ are resolvable sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

□

Proposition 5.4.6. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is the neutrosophic somewhere dense set, then there exists a neutrosophic resolvable set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $Q \leq \mathcal{N}cl[P]$.*

Proof. Let P be a neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, then $\mathcal{N}int\mathcal{N}cl[P] \neq 0_N$ and we obtain a neutrosophic open set Q such that $Q < \mathcal{N}cl[P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.4.1, neutrosophic open set Q is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence $Q \leq \mathcal{N}cl[P]$.

□

Proposition 5.4.7. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is the neutrosophic somewhere dense set, then $\mathcal{N}cl[C[P]] \wedge \mathcal{N}cl[P] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof: *Let P be a neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is a* *neutrosophic hyperconnected space. Using Proposition 5.4.5, there exist two resolvable sets $\mathcal{N}int[P]$ and $\mathcal{N}int[C[P]]$ such that $\mathcal{N}int[P \wedge C[P]] = 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Now,

$\implies C[\mathcal{N}int[P \wedge C[P]]] = C[0_N] = 1_N$.

$\implies \mathcal{N}cl[C[P] \vee C[C[P]]] = 1_N$

$\implies \mathcal{N}cl[C[P] \wedge P] = 1_N$

$\implies \mathcal{N}clC[P] \vee \mathcal{N}cl[P] = 1_N$ *in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Proposition 5.4.8. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if $0_N \neq P$ is the neutrosophic semi pre open set, then $\mathcal{N}cl[P] \vee \mathcal{N}cl[C[P]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let $0_N \neq P$ be the neutrosophic semi pre open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then $P \leq \mathcal{N}cl\mathcal{N}int\mathcal{N}cl[P]$, this implies $\mathcal{N}int\mathcal{N}cl[P] \neq 0_N$. Suppose $\mathcal{N}int\mathcal{N}cl[P] = 0_N$ then we have $P \leq \mathcal{N}cl[0_N] = 0_N \implies P = 0_N$ which contradicts our assumption that $0_N \neq P$. Therefore P is a neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.4.7 for P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, we have $\mathcal{N}cl[P] \wedge \mathcal{N}cl[C[P]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.4.9. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if $0_N \neq P$ is the neutrosophic pre open set, then $\mathcal{N}cl[P] \wedge \mathcal{N}cl[C[P]]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof: *Let $0_N \neq P$ be the neutrosophic pre open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ then $P \leq \mathcal{N}int\mathcal{N}cl[P]$, this implies $\mathcal{N}int\mathcal{N}cl[P] \neq 0_N$. Suppose $\mathcal{N}int\mathcal{N}cl[P] = 0_N$, then we have $P \leq 0_N$, therefore, $P = 0_N$. Thus P is the neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proposition 5.4.10. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is the neutrosophic subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}int[P] \neq 0_N$, then*

(i) there exists a neutrosophic resolvable set $\mathcal{N}int(P)$ and also $\mathcal{N}int(P)$ is the neutrosophic dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$

(i) the neutrosophic set P is neutrosophic dense i.e., $\mathcal{N}cl[P] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

Proof. (i) Take P be the neutrosophic subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}int[P] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.4.4 $\mathcal{N}int[P]$ is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. This implies $\mathcal{N}int[P]$ is the neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Therefore $\mathcal{N}cl[\mathcal{N}int[P]] =$

1_N. Thus $\mathcal{N}int[P]$ is the neutrosophic dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

(ii) Using (i), we have $\mathcal{N}cl[\mathcal{N}int[P]] = 1_N$. We know that $\mathcal{N}cl[\mathcal{N}int[P]] \leq \mathcal{N}cl[P]$, this implies $1_N \leq \mathcal{N}cl[P]$. Therefore $\mathcal{N}cl[P] = 1_N$. Thus P is neutrosophic dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.

□

Proposition 5.4.11. *In a neutrosophic hyperconnected spaces $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is a neutrosophic subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}int[P] \neq 0_N$, then $\mathcal{N}int[P]$ and $C[\mathcal{N}int[P]]$ are the neutrosophic resolvable sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic subset in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, and $\mathcal{N}int[P] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then by Proposition 5.4.4 $\mathcal{N}int[P]$ is neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.2.9, $C[\mathcal{N}int[P]]$ is also neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.4.12. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, if P is the neutrosophic subset defined in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}int[P] \neq 0_N$, then $\mathcal{N}int[\mathcal{N}cl[P]]$ and $C[\mathcal{N}int[\mathcal{N}cl[P]]]$ are neutrosophic resolvable sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let P be a neutrosophic set in $\mathcal{X}$ with $\mathcal{N}int[P] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. $\mathcal{N}int[P] \leq \mathcal{N}int[\mathcal{N}cl[P]]$, this implies that $\mathcal{N}int\mathcal{N}cl[P] \neq 0_N$. Then $\mathcal{N}cl[P]$ is the neutrosophic open set in $\mathcal{X}$ with $\mathcal{N}int[\mathcal{N}cl[P]] = 0_N$. Using Proposition 5.4.4 $\mathcal{N}int[\mathcal{N}cl[P]]$ and $C[\mathcal{N}int[\mathcal{N}cl[P]]]$ are neutrosophic resolvable sets.

□

Proposition 5.4.13. *In a neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. if $0_N \neq P$ is the neutrosophic open set then P is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[P] = 1_N$ and $\mathcal{N}cl[C[P]] \neq 1_N$*

Proof. Let $0_N \neq P$ is the neutrosophic open set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Using Proposition 5.4.1, P is the neutrosophic resolvable set. Also we have, $\mathcal{N}cl[P] = 1_N$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is the neutrosophic hyperconnected space. Clearly $\mathcal{N}cl[1 - P] \neq 1_N$. □

5.5 Relationship between Neutrosophic hyperconnected spaces and Neutrosophic resolvable functions

Proposition 5.5.1. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic continuous function from the neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then the function, $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is a neutrosophic resolvable function.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic function from the neutosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then for any neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ there exist a neutosophic open set $R^{-1}[P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is the hyperconnected space, then $\mathcal{N}cl[R^{-1}[P]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. At present, for the neutrosophic hyperconnected closed set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$,

$$\begin{aligned}
\mathcal{N}cl[Q \wedge R^{-1}[P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]] &\leq \mathcal{N}cl[Q] \wedge \mathcal{N}cl[R^{-1}[P]] \wedge \mathcal{N}cl[Q] \wedge \mathcal{N}cl[C[R^{-1}[P]]] \\
&= Q \wedge 1_N \wedge Q \wedge \mathcal{N}cl[C[R^{-1}[P]]] \\
&= Q \wedge C[R^{-1}[P]]
\end{aligned}$$

Since $R^{-1}[P]$ is neutosophic open in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then

$$\begin{aligned}
\mathcal{N}int\mathcal{N}cl[\mathcal{N}cl[Q \wedge R^{-1}[P]] \wedge \mathcal{N}cl[Q \wedge C[R^{-1}[P]]]] &\leq \mathcal{N}int\mathcal{N}cl[Q \wedge C[R^{-1}[P]]] \\
&\leq \mathcal{N}int[\mathcal{N}cl[Q] \wedge \mathcal{N}cl[C[R^{-1}[P]]]] \\
&= \mathcal{N}int[Q \wedge C[R^{-1}[P]]] \\
&= \mathcal{N}intQ \wedge \mathcal{N}int[C[R^{-1}[P]]] \\
&= \mathcal{N}intQ \wedge C[\mathcal{N}cl[R^{-1}[P]]] \\
&= \mathcal{N}intQ \wedge C[1_N] \\
&= 0_N.
\end{aligned}$$

Therefore $[R^{-1}[P]]$ is the neutosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Thus the function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic resolvable function. □

Proposition 5.5.2. *If the function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic continuous function from the neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then $\mathcal{N}int\mathcal{N}cl[R^{-1}[P \vee C[P]]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.*

Proof. Let us take the function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ to be a neutrosophic continuous function from the neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Using 5.5.1 $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutosophic resolvable function. Also using 5.3.5, for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, $\mathcal{N}int\mathcal{N}cl[R^{-1}[P \vee C[P]]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.5.3. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic continuous function from the neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into the neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then there exist a neutrosophic resolvable set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P \vee C[P]]] \geq Q$, where P is neutrosophic open in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be a neutrosophic continuous function from $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Using 5.5.2, we have $\mathcal{N}int\mathcal{N}cl[R^{-1}[P \vee C[P]]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Then there exist a neutrosophic open set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P \vee C[P]]] \geq Q$.

Using 5.5.1 Q is the neutrosophic resolvable set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Hence, there exists a neutrosophic resolvable set Q in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P \vee C[P]]] \geq Q$. □

Definition 5.5.4. *Let $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ and $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be any two neutrosophic topological space. The function $S : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is called a neutrosophic somewhere continuous function if $S^{-1}[W]$ is neutrosophic somewhere dense in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for each neutrosophic open set W in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$.*

Proposition 5.5.5. *If a function $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic somewhere continuous function from the neutrosophic hyperconnected space $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into another neutrosophic topological space $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$, then for the neutrosophic open set P in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*

(i) $\mathcal{N}cl[R^{-1}[P \vee C[P]]] = 1_N$ *in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$

(ii) $\mathcal{N}int[R^{-1}[P \vee C[P]]] \neq 0_N$ *in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$

Proof. (i) Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ is the neutrosophic somewhere continuous function from $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Then $R^{-1}[P]$ is the neutrosophic somewhere dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Now, for the neutrosophic somewhere dense set $R^{-1}[P]$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$, we have $\mathcal{N}cl[R^{-1}[P] \vee C[R^{-1}[P]]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is the neutrosophic hyperconnected space. Now $R^{-1}[P] \vee R^{-1}[C[P]] = R^{-1}[P \vee C[P]]$. Then $\mathcal{N}cl[R^{-1}[[P] \vee C[P]]] = 1_N$. Thus $R^{-1}[[P] \vee C[P]]$ is neutrosophic dense set in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$.
(ii) By (i) $\mathcal{N}cl[R^{-1}[P \vee C[P]]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Now, $\mathcal{N}cl[C[R^{-1}[P \vee C[P]]]] \neq 1_N \implies C[\mathcal{N}int[R^{-1}[P \vee C[P]]]] \neq 1_N$. Hence $\mathcal{N}int[R^{-1}[P \vee C[P]]] \neq 0_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

Proposition 5.5.6. *If a function* $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ *is the neutrosophic resolvable function from the neutrosophic hyperconnected space* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ *into the neutrosophic topological space* $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$*, then there exist a neutrosophic regular open set* $0_N = S$ *in* $[\mathcal{X}, \tau_{\mathcal{H}}]$ *such that, either* $\mathcal{N}cl[R^{-1}[P \vee C[P]]] = 1_N$ *or* $\mathcal{N}cl[R^{-1}[P] \vee C[R^{-1}[P]]] \geq S$ *in* $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$*.*

Proof. Let $R : \mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}}) \to \mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$ be the neutrosophic resolvable function from $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ into $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Using 5.3.6, there exists a neutrosophic regular open set S in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P] \vee C[R^{-1}[P]]] \geq S$ for the neutrosophic open set P in $\mathcal{N}(\mathcal{Y}, \tau_{\mathcal{Y}})$. Since $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ is the hyperconnected space, we have 0_N and 1_N are the only neutrosophic regular open sets in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Thus either $S = 0_N$ or $S = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. Suppose $S = 0_N$, then there exist no neutrosophic regular open set $0_N \neq S$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$ such that $\mathcal{N}cl[R^{-1}[P] \vee C[R^{-1}[P]]] \geq S$. If $S = 1_N$, then $\mathcal{N}cl[R^{-1}[P] \vee C[R^{-1}[P]]] \geq 1_N$. This implies $\mathcal{N}cl[R^{-1}[P \vee C[P]]] = 1_N$ in $\mathcal{N}(\mathcal{X}, \tau_{\mathcal{X}})$. □

5.6 Conclusion

The study has demonstrated the concept of neutrosophic resolvable sets and neutrosophic resolvable functions in neutrosophic topological spaces by its properties. Also, the features of such sets and spaces are closely examined in neutrosophic hyperconnected spaces. Thus the view of resolvable set has been generalized according to the research fields.