ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Soft $\hat{\beta}$- Generalized Closed Sets and Soft $\hat{\beta}$- Generalized Open Sets in Soft Topological Spaces

C.R.Parvathy[1], M.Deepa[2]

Assistant Professor, Department of Mathematics, PSGR Krishnammal College For Women, Peelamedu, Coimbatore, India[1]

M.Phil Scholar, Department of Mathematics, PSGR Krishnammal College For Women, Peelamedu, Coimbatore, India[2]

ABSTRACT: In this paper a new class of soft sets called Soft $\hat{\beta}$ - generalized closed sets and Soft $\hat{\beta}$ - generalized open sets in Soft Topological spaces are introduced and studied. This new class is defined over an initial universe and with a fixed set of parameters. Some basic properties of this new class of soft sets are investigated. This new class of Soft $\hat{\beta}$ - generalized closed sets and Soft $\hat{\beta}$ - generalized open sets widening the scope of Soft Topological spaces and its applications.

KEYWORDS: Soft sets, Soft Topological Spaces, $\hat{\beta}$ -g closed sets, Soft $\hat{\beta}$ -g closed sets, $\hat{\beta}$ -g open sets, Soft $\hat{\beta}$ -g open sets.

I. INTRODUCTION

Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Here the researchers have succeeded in their knowledge building effort by introducing a new class of soft sets called Soft $\hat{\beta}$ - generalized closed sets and open sets in Soft Topological spaces.

The concept of soft sets was first introduced by Molodtsov [1] in 1999 who began to develop the basics of corresponding theory as a new approach to modeling uncertainties. In [1, 2], Molodtsov successfully applied the soft theory in several directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. In recent years, an increasing number of papers have been written about soft sets theory and its applications in various fields [3, 4]. Shabir and Naz [5] introduced the notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameters. In addition, Maji et al. [6] proposed several operations on soft sets, and some basic properties of these operations have been revealed so far.

II. PRELIMINARIES

Definition 2.1[7]

Let X be an initial Universe set and E be the set of parameters. Let $P(X)$ denote the power set of X. For $A \subseteq E$, the pair (F, A) is called a Soft set over X, where F is a mapping given by $F : A \rightarrow P(X)$.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

In other words, a soft set over X is parameterized family of subsets of the universe X. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε - approximate elements of the soft set (F, A).

Definition 2.2 [7]

i) A soft set (F, A) over X is said to be Null Soft set denoted by ϕ if for all $e \in A, F(e) = \phi$.

ii) A soft set (F, A) over X is said to be Absolute Soft set denoted by $\tilde{A}$ if for all $e \in A, F(e) = X$.

Definition 2. 3 [7]

The Union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \bigcup B$, and for all $e \in C, H(e) = F(e)$, if $e \in A / B, H(e) = G(e)$ if $e \in B / A \,\&\, H(e) = F(e) \bigcup G(e)$ if $e \in A \bigcap B$ and is denoted as $(F, A) \bigcup (G, B) = (H, C)$.

Definition 2.4 [7]

The Intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \bigcap B$ and $H(e) = F(e) \bigcap G(e)$ for all $e \in C$ and is denoted as $(F, A) \bigcap (G, B) = (H, C)$.

Definition 2.5 (7)

The Relative Complement of (F, A) is denoted by $(F, A)^C$ and is defined by $(F, A)^C = (F^C, A)$ where $F^C : A \to P(X)$ is a mapping given by $F^C(e) = X - F(e)$, for all $e \in A$.

Definition 2.6 (7)

The Difference (H, E) of two sets $(F, E) \,\&\, (G, E)$ over X, denoted by $(F, E) / (G, E)$ is defined as $H(e) = F(e) / G(e)$ for all $e \in A$

Definition 2.7 (7)

Let (F, A) and (G, B) be soft sets over X, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A, F(e)$ and $G(e)$ are identical approximations. $(F, A) \subseteq (G, B)$

Definition 2.8 (7)

Let τ be the collection of soft sets over X with the fixed set of parameters. Then τ is called a soft topology on X if

(i) $\phi, X \in \tau$.

(ii) The union of any number of sets in τ belongs to τ.

(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called soft topological space over X. The members of τ are called soft open sets in X and complements of them are soft closed sets over X.

Definition 2.9 (7)

Let be the Soft Topological Spaces over X. The Soft interior of (F, E) denoted by Int (F, E) is the union of soft open subsets of (F, E). Clearly (F, E) is the largest soft open set over X which is contained in (F, E).

The soft closure of (F, E) denoted by cl (F, E) is the intersection of closed sets containing (F, E). Clearly (F, E) is the smallest soft closed set containing (F, E).

Int $(F, E) = \bigcup \{(O, E) : (O, E)$ is soft open and $(O, E) \subseteq (F, E)\}$.

cl $(F, E) = \bigcap \{(O, E) : (O, E)$ is soft closed and $(F, E) \subseteq (O, E)\}$.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Definition 2.10 (14)

A subset A of a topological space (X,τ) is called

i) A semi open set if $A \subseteq cl(Int(A))$ and a semi closed set if $Int(cl(A)) \subseteq A$.

ii) A pre open set if $A \subseteq Int(cl(A))$ and a pre closed set if $cl(Int(A)) \subseteq A$.

iii) An α -open set if $A \subseteq Int(cl(Int(A)))$ and α -closed set if $cl(Int(cl(A))) \subseteq A$.

iv) A regular open set if $A = Int(cl(A))$ and a soft regular closed if $A = cl(Int(A))$

v) A generalized α -closed set (briefly α g-closed) if $cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $\alpha -$ open in (X,τ).

vi) A subset A of a topological space (X,τ) is called $\hat{\beta}$ g-closed set if $cl(Int(cl(A))) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

vii) A subset A of a topological space (X,τ) is called $\hat{\beta}$ g-open in X if A^C is $\hat{\beta}$ g-closed in X.

Definition 2.11 (14)

In a Soft Topological spaces (X,τ,E), a soft set (F,E) over is called

i) A soft semi open set if $(F,E) \subseteq cl(Int(F,E))$ and a soft semi closed set if $Int(cl((F,E))) \subseteq (F,E)$.

ii) A soft pre open set if $(F,E) \subseteq Int(cl(F,E))$ and soft pre closed set if $cl(Int(F,E)) \subseteq (F,E)$.

iii) A soft α -open set if $(F,E) \subseteq Int(cl(Int(F,E)))$ and soft α -closed set if $cl(Int(cl(F,E))) \subseteq (F,E)$.

iv) A soft regular open set if $(F,E) = Int(cl(F,E))$ and a soft regular closed if $(F,E) = cl(Int(F,E))$

v) A soft generalized α -closed set (briefly soft α g-closed) if $cl(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft $\alpha -$ open in (X,τ,E).

III. SOFT $\hat{\beta}$ G-CLOSED SETS IN SOFT TOPOLOGICAL SPACES.

Soft $\hat{\beta}$ g-closed sets:

Definition 3.1

A subset (F,E) of a soft topological space (X,τ,E) is called soft $\hat{\beta}$ g-closed set if $cl(Int(cl(F,E))) \subseteq (U,E)$ whenever $(F,E) \subseteq (U,E)$ and (U,E) is open in X.

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Example: 3.2 Let us consider $X=\{a,b\}, E=\{e_1,e_2\}$.

$$\begin{array}{ll}
(F_1,E)=\{(e_1,\{\phi\}),(e_2,\{\phi\})\} & (F_9,E)=\{(e_1,\{b\}),(e_2,\{\phi\})\} \\
(F_2,E)=\{(e_1,\{\phi\}),(e_2,\{a\})\} & (F_{10},E)=\{(e_1,\{b\}),(e_2,\{a\})\} \\
(F_3,E)=\{(e_1,\{\phi\}),(e_2,\{b\})\} & (F_{11},E)=\{(e_1,\{b\}),(e_2,\{b\})\} \\
(F_4,E)=\{(e_1,\{\phi\}),(e_2,\{a,b\})\} & (F_{12},E)=\{(e_1,\{b\}),(e_2,\{a,b\})\} \\
(F_5,E)=\{(e_1,\{a\}),(e_2,\{\phi\})\} & (F_{13},E)=\{(e_1,\{a,b\}),(e_2,\{\phi\})\} \\
(F_6,E)=\{(e_1,\{a\}),(e_2,\{a\})\} & (F_{14},E)=\{(e_1,\{a,b\}),(e_2,\{a\})\} \\
(F_7,E)=\{(e_1,\{a\}),(e_2,\{b\})\} & (F_{15},E)=\{(e_1,\{a,b\}),(e_2,\{b\})\} \\
(F_8,E)=\{(e_1,\{a\}),(e_2,\{a,b\})\} & (F_{16},E)=\{(e_1,\{a,b\}),(e_2,\{a,b\})\}
\end{array}$$

Here $(F_1,E),(F_2,E),(F_3,E),(F_4,E)..........\ ...(F_{16},E)$ are soft sets over X.

$$\tau=\{(F_1,E),(F_5,E),(F_7,E),(F_8,E),(F_{16},E)\}$$
$$\tau^C=\{(F_1,E),(F_9,E),(F_{10},E),(F_{12},E),(F_{16},E)\}$$

Put $(F,E)=\{(e_1,\{b\}),(e_2,\{a\})\}$ clearly, $ci(Int(cl(F,E)))\subseteq(U,E)$ whenever $(F,E)\subseteq(U,E)$ and (U,E) is open (X,τ,E).

Theorem: 3.3

The union of two soft $\hat{\beta}$ -g closed subsets of a soft topological space X is also a soft $\hat{\beta}$ g closed subset of X.

Proof:

Assume that (F,E) and (G,E) are soft $\hat{\beta}$ g-closed sets in X. Let (U,E) is soft open in X such that $(F,E)\cup(G,E)\subset(U,E)$.Then $(F,E)\subset(U,E)$ and $(G,E)\subset(U,E)$. Since (F,E) and (G,E) are soft $\hat{\beta}$ g-closed sets, $cl(Int(cl(F,E)))\subset(U,E)$ and $cl(Int(cl(G,E)))\subset(U,E)$. Hence $cl(Int(cl(F,E)\cup(G,E)))=cl(Int(cl(F,E)))\cup cl(Int(cl(G,E)))\subset(U,E)$ That is $cl(Int(cl(F,E)\cup(G,E)))\subset(U,E)$. Therefore $(F,E)\cup(G,E)$ is soft $\hat{\beta}$ g-closed sets in X.

Remark:3.4

The intersection of two soft $\hat{\beta}$ g closed sets in X is generally not soft $\hat{\beta}$ g closed set in X.

Example:3.5

From 3.2, If we take $(F_6,E)=\{(e_1,\{a\}),(e_2,\{a\})\}$ and $(F_{13},E)=\{(e_1,\{a,b\}),(e_2,\{\phi\})\}$. Then (F_6,E) and (F_{13},E) are two soft $\hat{\beta}$ g closed sets in X, but $(F_6,E)\cap(F_{13},E)=(F_5,E)$ is not soft $\hat{\beta}$ g closed set in X.

Theorem: 3.6

If a soft subset (F,E) of X is soft $\hat{\beta}$ g-closed set in X. Then $cl(Int(cl(F,E)))-(F,E)$ does not contain any non empty soft open set in X.

IJIRSET

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

Proof:

Suppose that (F,E) is soft $\hat{\beta}$ g-closed set in X. We prove the result by contradiction. Let (U,E) is soft open in X such that $cl(Int(cl(F,E)))-(F,E)\supset(U,E)$ and $(U,E)=(\phi,E)$. Now $(U,E)\subset cl(Int(cl(F,E)))-(F,E)$,Therefore $(U,E)\subset X-(U,E)$. Since (U,E) is soft open set, $X-(U,E)$ is also soft open in X . Since (F,E) is soft $\hat{\beta}$ g-closed set in X , by definition we have $cl(Int(cl(F,E)))\subset X-(U,E)$. So $(U,E)\subset X-cl(Int(cl(F,E)))$. Also $(U,E)\subset cl(Int(cl(F,E)))$, $(U,E)\subset cl(Int(cl(F,E)))\cap(X-cl(Int(cl(F,E))))=(\phi,E)$ This shows that $(U,E)=(\phi,E)$ which is a contradiction. Hence $cl(Int(cl(F,E)))-(F,E)$ does not contain any non empty soft open set in X .

Theorem: 3.7

If (F,E) is soft regular closed in (X,τ,E) then (F,E) is soft $\hat{\beta}$ -g closed subset of (X,τ,E).

Proof:

Suppose that $(F,E)\subseteq(U,E)$ and (U,E) is open in X .Now $(U,E)\subset X$ is soft open iff (U,E) is the union of a soft semi open set and soft pre-open set. Let (F,E) be a soft regular closed subset of (X,τ,E) .So $(F,E)=cl(\text{int}(ci(F,E)))$ .Hence $cl(\text{int}(ci(F,E)))\subseteq(U,E)$ .whenever (U,E) is open in X .Therefore (F,E) is soft $\hat{\beta}$ -g closed in X .This theorem is verified by the following example.

Example:3.8

Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ Let $\tau=\{(F_1,E),(F_7,E),(F_{10},E),(F_{16},E)\}$ and $\tau^c=\{(F_1,E),(F_{10},E),(F_7,E),(F_{16},E)\}$. Put (F_{10},E), then $cl(\text{int}(cl(F_{10},E)))=(F_{10},E)$ .Clearly (F_{10},E) is soft $\hat{\beta}$ -g closed subset of (X,τ,E).

Remark:3.9

The converse of the above theorem need not be true as seen from the following example.

Example:3.10

Consider $X=\{a,b\}$, $E=\{e_1,e_2\}$ with $\tau=\{(F_1,E),(F_5,E),(F_7,E),(F_8,E),(F_{16},E)\}$ and $\tau^C=\{(F_1,E),(F_9,E),(F_{10},E),(F_{12},E),(F_{16},E)\}$ Let $(F_{10},E)=\{(e_1,\{b\}),(e_2,\{a\})\}$ $cl(Int(cl(F_{10},E)))=(F_1,E)\neq(F_{10},E)$ .Therefore (F_{10},E) is not soft regular closed, but (F_{10},E) is soft $\hat{\beta}$ g closed set in X .

Theorem: 3.11

For any soft set $(F,E)\in X$, the set $X-(F,E)$ is soft $\hat{\beta}$ g closed set or soft $\hat{\beta}$ g open.

Proof:

Suppose $X-(F,E)$ is not soft open. Then X is the only soft open set containing $X-(F,E)\Rightarrow cl(Int(cl(X-(F,E)))\subset X$ .Hence $X-(F,E)$ is soft $\hat{\beta}$ g closed set in X .

ISSN(Online): 2319-8753
ISSN (Print): 2347-6710

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 11, November 2015

IV. SOFT $\hat{\beta}$ GENERALIZED OPEN SETS IN SOFT TOPOLOGICAL SPACES.

Definition: 4.1

A subset (F,E) in X is called Soft $\hat{\beta}$ generalized open set (briefly $\hat{\beta}$ g-open) in X if $(F,E)^C$ is soft $\hat{\beta}$ g-closed set in X.

Example:4.2

Let $(F_7,E)=\{(e_1,\{a\}),(e_2,\{b\})\}$, then $(F_7,E)^C=\{(e_1,\{b\}),(e_2,\{a\})\}$

Clearly $(F_7,E)^C$ is soft $\hat{\beta}$ g closed set in X.

Theorem: 4.3

If (F,E) and (G,E) are soft $\hat{\beta}$ g-open sets in X. Then $(F,E)\cap(G,E)$ is also soft $\hat{\beta}$ g-open set in X.

Proof:

Let (F,E) and (G,E) are soft $\hat{\beta}$ g-open sets in X. Then $(F,E)^C$ and $(G,E)^C$ are soft $\hat{\beta}$ g-closed sets in X. By a theorem (3.3) $(F,E)^C\cup(G,E)^C$ is also soft $\hat{\beta}$ g-closed sets in X. That is $(F,E)^C\cup(G,E)^C=((F,E)\cap(G,E))^C$ is a soft $\hat{\beta}$ g-closed set in X. Therefore $(F,E)\cap(G,E)$ is also soft $\hat{\beta}$ g-open sets in X.

V.CONCLUSION

In the present work, a new class of sets called Soft $\hat{\beta}$ g-Closed sets in Soft Topological Spaces is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Fuzzy Soft Topological Spaces and also in Ideal Topological Spaces..

REFERENCES

[1] D. Molodtsov, "Soft set theory—first results," *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[2] D. Molodtsov, V. Y. Leonov, and D. V. Kovkov, "Soft sets technique and its application," *Nechetkie Sistemyi Myagkie Vychisleniya*, vol. 1, no. 1, pp. 8–39, 2006.

[3] P. K.Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.

[4] I. Zorlutuna, M. Akdag, W. K. Min, and S. Atmaca, "Remarks on soft topological spaces," *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 171–185, 2011.

[5] M. Shabir and M. Naz, "On soft topological spaces," *Computers and Mathematics with Applications*, vol. 61, no. 7, pp. 1786–1799, 2011.

[6] P. K. Maji, R.Biswas, and A.R.Roy, "Soft set theory," *Computers and Mathematics with Applications*, vol. 45,no. 4-5, pp. 555–562, 2003.

[7] A. Kalaiselvi, and T. Nandhini, "Soft $\hat{g}$ -closed sets in Soft Topological Spaces", International journal of Innovative Research in Science, Engineering and Technology, Vol.3, Issue 7,July 2014.

[8] K.Kannan and N.Nagaveni, "On $\hat{\beta}$ Generalized closed sets and open sets in Topological Spaces", Int.Journal of Math.Analysis, Vol. 6,2012, no.57, 2819-2828.