A structure of open sets in quad topological spaces

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Abstract

The main aim of this paper is to present a new set, namely quad j-open set in quad topological space. We discuss the basic properties of quad j-open sets using quad j-interior and quad j-closure. In Addition we study the relationships among quad j-closed, quad b-closed and quad j-regular open in quad topological space.

Keywords:quad j-open, quad j-closed, quad pre closure, quad regular open.

1 Introduction

In 1963, J. C. Kelly [4] introduced the concept of bitopological space using quasimetrics. In 2000, Martin Kovar [5] finding the new concept of tri topological spaces using three topologies. In 2013, the concept with four topologies called as quad topological spaces(4-tuple)was initiated by D. V. Mukundan [8]. In 2017, U.D.Tapi and RanuSharma[13] initiate semi open sets and pre open sets in quad topological space. In the year 2018, Ranu Sharma, BhagyasriA.Deole, Smit Verma [9] label Fuzzy qb-open sets and qb-separation axioms in Fuzzy quad topological space.Tri topological spaces, quad topological spaces are the generalization of bitopological spaces. The idea of N-topological space related to ordinary topological spaces was introduced and studied, in 2011, by Tawfiq and Majeed [15]. In 2017, M. LellisThivagar, V. Ramesh and M. ArokiaDasan [6] defined the new structure of N- topology using quasi pseudo metrics. In 2013, I. Arokiarani and D. Sasikala introduced the new type of open set [10]namlely j-closed in topological spaces. Using this notion we introduce the set namely quad j-open sets in quad topological spaces.

2 Preliminaries

Inthis section we discuss some preliminaries about quad sets and quad topological spaces. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a quad topological space. It is simply denoted by $(X, 4\tau)$. The member of $(X, 4\tau)$ is quad open set and its complement is quad closed set.

Definition 2.1[10]

A subset A of X is called asj-openif $A \subseteq \text{int}(pcl(A))$ and j-closed if $\text{int}(pcl(A)) \subseteq A$.

Definition 2.2[6]

Let X be a non-empty set $\tau_1, \tau_2, \tau_3, \tau_4$ are four arbitrary topologies on X and the collection 4τ is defined by

$$4\tau = \{Q \subset X; Q = (A_1 \cup A_2 \cup A_3 \cup A_4) \cup (B_1 \cap B_2 \cap B_3 \cap B_4) /$$
$$A_1, B_1 \in \tau_1, A_2, B_2 \in \tau_2, A_3, B_3 \in \tau_3, A_4, B_4 \in \tau_4\}$$

satisfying the following axioms

i. $x, \varphi \in 4\tau$

ii. $\bigcup_{i=1}^{\infty} Q_i \in 4\tau$ for all $Q_i \in 4\tau$

iii. $\bigcap_{i=1}^{n} Q_i \in 4\tau$ for all $Q_i \in 4\tau$

Then the pair $(X, 4\tau)$ is called quad topological space.

Example 2.3

Let X ={p,q,r,s} and τ_1 ={φ,X,{p,q}}, τ_2 = {φ ,X, {q,s}}, τ_3 ={φ, X, {p,r}}, τ_4 ={φ, X, {s}}. Then $(X, 4\tau)$ ={φ ,X, {s}, {p,q}, {q,s}, {p,r}, {p,q,s}, {p,r,s}, {p,q,r}, {q}, {p}, {p,s}}.Here quad open sets are in (X, 4τ) and quad closed sets are X,φ, {p,q,r}, {r,s}, {p,r}, {q,s}, {r}, {q}, {s}, {p,r,s}, {q,r,s}, {q,r}. We denote the sets X,φ, {p,r}, {q,s}, {q}, {s}, {p,r,s} are both quad open and quad closed.

Remark 2.4

(i) Intersection of two quad topology is also a quad topology

(ii) Union of twoquad topology need not be a quad topology.

Definition 2.5[13]

Let A be a subset of a quad topological space $(X, 4\tau)$ then the union of all q-open sets of X contained in A is called q-interior of A and it is simply denoted by q-int(A). The intersection of all q- closed set of X containing A is called q-closure of A and it is simply denoted by q-cl(A).

Definition 2.6[13]

A subset A of $(X, 4\tau)$ is said to be a quad pre open set if $A \subseteq q\,\text{int}(qcl(A))$ and complement of quad pre open set is quad pre closed set . It is defined by $qcl(q\,\text{int}\,A) \subseteq A$.

Definition 2.7[8]

A subset A of $(X, 4\tau)$ is said to be a quad b open set if $A \subseteq q\,\text{int}(qcl(A)) \cup qcl(q\,\text{int}(A))$ and complement of quad b open set is quad b closed set .

3. Quad j-open sets in quad topological space

Definition 3.1

In a quad topological space $(X, 4\tau)$, a subset A of X is said to be quad j-open if $A \subseteq q\,\text{int}(qpcl(A))$. It is simply denoted by q j-open.

Example 3.2

Consider X = {p,q,r,s} . Let τ_1={φ, X}, τ_2 ={φ, X,{q}}, τ_3 = {φ, X, {p,r}}, τ_4 ={φ, X,{q,r}}. Then 4τ ={ φ, X, {q}, {q,r}, {p,r}, {p,q,r}, {r}}, $4\tau^c$ ={X,φ, {p,r,s}, {p,s}, {q,s},{s}, {p,q,s}}.We have the qj-open sets are φ, X, {q}, {r}, {p,r}, {q,r}, {p,q,r}, {q,r,s} and qj-closed sets are X, φ, {p,r,s}, {p,q,s}, {q,s}, {p,s}, {s}, {p}.

Definition 3.3

Let A be a subset of quad topological space $(X,4\tau)$ then, the union of all qj-open sets of X contained in A is called qj- interior of A and it is denoted by $q\text{int}_j(A)$.

Definition3.4

Let A be a subset of quad topological space $(X,4\tau)$ then the intersection of all qj-closed set of X containing A is called qj-closure of A and it is denoted by $qcl_j(A)$.

Definition 3.5

Let A be a subset of a quad topological space $(X,4\tau)$ then the union of all q-pre open sets of X contained in A is called q-pre interior of A and it is simply denoted by q-pint(A). The intersection of all q- pre closed sets of X containing A is called q-pre closure of A and it is simply denoted by q-pcl(A).

Definition 3.6

A subset A of $(X,4\tau)$ is said to be a quad regular open set if $A = q\text{int}(qcl(A))$ and complement of quad regular open set is quad regular closed set .

Definition 3.7

A subset A of $(X,4\tau)$ is said to be a quad j-regular open set if $A = q\text{int}(qpcl(A))$ and complement of quad j-regular open set is quad j-regular closed set .

Theorem 3.8

Every quad open set is quad pre-open.

Proof

For any set A, we have $A \subseteq qcl(A)$. Since $A \subseteq B \Rightarrow q\text{int}\,A \subseteq q\text{int}\,B$. Therefore $q\text{int}\,A \subseteq q\text{int}(qcl(A))$.But$A$ is quad open. That implies $A = q\text{int}\,A$..Hence $A \subseteq q\text{int}(qcl(A))$. Therefore every quad open set is quad pre open.

Converse of the above theorem need not be true by the following example.

Example 3.9

Let X= $\{a,b,c,d\}$, τ_1 =$\{\varphi, X\}$, τ_2 =$\{\varphi, X, \{c\}\}$, τ_3 = $\{\varphi, X, \{d\}\}$ and τ_4 =$\{\varphi, X, \{c\}, \{d\}, \{c,d\}\}$. Then 4τ =$\{\varphi, X,\{c\}, \{d\}, \{c,d\}\}$ and $4\tau^c$ = {X,φ, {a,b,d}, {a,b,c}, {a,b}}. The sets $\{a,c,d\}$ and $\{b,c,d\}$ are quad pre open but not open.

Theorem 3.10

Every quad open set is qj-open.

Proof

For any set A, we have $A \subseteq cl(A)$. Since $A \subseteq B \Rightarrow q\,\text{int}\, A \subseteq q\,\text{int}\, B.$ Therefore $q\,\text{int}\, A \subseteq q\,\text{int}(qcl(A))$. Since $qpcl(A) \subseteq qcl(A)$. That implies $q\,\text{int}\, A \subseteq q\,\text{int}(qpcl(A)) \subseteq q\,\text{int}(qcl(A))$ But A is quad open. That implies $A = q\,\text{int}\, A.$.Hence $A \subseteq q\,\text{int}(qpcl(A))$.Therefore every quad open set is quad j- open.

Converse of the above theorem need not be true from the example 3.2. A set {q,r,s} is qj-open but not quad open.

Theorem 3.11

Every quad j-open is q pre-open.

Proof

If a set A is quad j-open , then $A \subseteq q\,\text{int}(qpcl(A))$. Since $qpcl(A) \subseteq qcl(A)$. That implies $A \subseteq q\,\text{int}(qpcl(A)) \subseteq q\,\text{int}(qcl(A))$ That implies $A \subseteq q\,\text{int}(qcl(A))$ Hence every quad j-open set is quad pre open.

Converse of the above theorem need not be true as shown by the following example.

Example 3.12

Let $X = \{p,q,r,s\}$. Let τ_1 =$\{\varphi, X\}$, τ_2 =$\{\varphi, X,\{p,s\}\}$, τ_3 = $\{\varphi, X, \{p,q,r\}\}$, τ_4 =$\{\varphi, X,\{q,r,s\}\}$. Then 4τ ={ φ, X, {p,s}, {p,q,r}, {q,r,s},{p}, {s},{q,r}}}, $4\tau^c$ ={X,φ, {q,r}, {s}, {p},{q,r,s}, {p,q,r},{p,s}}.Here $\{p,q,s\}$ is quad pre open but not qj-open.

Theorem 3.13

Let us take the collection $\{P_\alpha : \alpha \in J\}$ ofqj-open sets in quad topological space $(X, 4\tau)$. Then $\bigcup_{\alpha \in J} P_\alpha$ is also qj-open in $(X, 4\tau)$.

Proof

Since P_α is the arbitrary collection of qj-open sets. Therefore P_α is qj-open.(ie) $P_\alpha \subseteq q\operatorname{int}(qpcl(P_\alpha))$ which implies $\bigcup_{\alpha\in J} P_\alpha \subseteq \bigcup_{\alpha\in J} q\operatorname{int}(qpcl(P_\alpha)) \subseteq q\operatorname{int}(qpcl(\bigcup_{\alpha\in J} P_\alpha)$. Hence $\bigcup_{\alpha\in J} P_\alpha$ is also qj-open in X.

Theorem 3.14

In a quad topological space $(X,4\tau)$, arbitrary intersection of qj-closed sets is also qj-closed.

Proof

Let $\{Q_\alpha : \alpha \in I\}$ be a family of qj-closed sets in X. let $P_\alpha = Q_\alpha{}^c$ then $\{P_\alpha : \alpha \in I\}$ is a family of qj-open sets in X.. Using the previous theorem $\bigcup P_\alpha$ is qj-open and $(\bigcup P_\alpha)^c$ is qj-closed. $\bigcap P_\alpha{}^c$ isqj-closed. Hence $\bigcap Q_\alpha$ is qj-closed.

Theorem 3.15

Let $(X,4\tau)$ be a quad topological space, if a singleton subset of $(X,4\tau)$ is qj-open if and only if it is q-open.

Proof:

Let *{p}* be a q j-open subset of X. Then *{p}*$\subseteq$*q int(qpcl{p})*. Since each singleton subset of any space X is q j -closed that implies p cl{p} $\subseteq$ cl{p} $\subseteq$ {p}.Thus{p} $\subseteq$ int{p}. Hence{p} is q-open.

Theorem 3.16

Let $(X,4\tau)$ be a quad topological space and $P,Q \subseteq X$ .Then

i. $q\operatorname{int}_j(P)$ is the largest qj open set contained in P.
ii. $P \subseteq Q$ then $q\operatorname{int}_j(P) \subseteq q\operatorname{int}_j(Q)$.
iii. P is qj-open if and only if $q\operatorname{int}_j(P) = P$ In particular $q\operatorname{int}_j(\phi) = \phi$. and $q\operatorname{int}_j(X) = X$.
iv. $q\operatorname{int}_j(P \cup Q) \supseteq q\operatorname{int}_j(P) \cup q\operatorname{int}_j(Q)$.
v. $q\operatorname{int}_j(P \cap Q) = q\operatorname{int}_j(P) \cap q\operatorname{int}_j(Q)$.
vi. $q\operatorname{int}_j(q\operatorname{int}_j(P)) = q\operatorname{int}_j(P)$.

Proof

i. If a quad topology space arbitrary union of qj-open sets is again qj-open, then $q\operatorname{int}_j(P)$ is qj-open, using the definition of qj-interior of A, we have $q\operatorname{int}_j(P) \subseteq P$ .Let Q be any qj-open set which contained in P then $Q \subseteq \cup\{F : F \subseteq P$ and F is qj-open$\}= q\operatorname{int}_j(P)$ in P and also $q\operatorname{int}_j(P)$ is the largest qj-open set which contained in P .

ii. Assume $P \subseteq Q$ then $X - Q \subseteq X - P$ implies $qcl_j X - Q \subseteq qcl_j X - P \Rightarrow q\operatorname{int}_j(P) \subseteq q\operatorname{int}_j(Q)$

iii. Let P be an qj-open set if and only if $X - P$ is qj closed set, then $qcl_j X - P = X - P$ if and only if $X - qcl_j(X - P) = P$ if and only if $q\operatorname{int}_j(P) = P$. Also ϕ and X are qj open sets then $q\operatorname{int}_j(\phi) = \phi$ and $q\operatorname{int}_j(X) = X$

iv. Let $x \in q\operatorname{int}_j(P) \cup q\operatorname{int}_j(Q)$.,then $x \notin X - (q\operatorname{int}_j(P) \cup q\operatorname{int}_j(Q))$ $x \notin qcl_j(X - P) \cap qcl_j(X - Q)$.which implies $x \notin qcl_j(X - (P \cup Q) \Rightarrow x \in q\operatorname{int}_j(P \cup Q)$ Hence $q\operatorname{int}_j(P \cup Q) \supseteq q\operatorname{int}_j(P) \cup q\operatorname{int}_j(Q)$.

v. Assume $x \in q\operatorname{int}_j(P \cap Q)$.then $x \notin X - q\operatorname{int}_j(P \cap Q)$ which implies $x \notin qcl_j(X - P) \cup qcl_j(X - Q)$ then $x \in (X - qcl_j(X - P)) \cap (X - qcl_j(X - Q))$ Then $x \in q\operatorname{int}_j(P) \cap q\operatorname{int}_j(Q)$ Thus $q\operatorname{int}_j(P \cap Q) \subseteq q\operatorname{int}_j(P) \cap q\operatorname{int}_j(Q)$.Similarly we can prove $q\operatorname{int}_j(P) \cap q\operatorname{int}_j(Q) \subseteq q\operatorname{int}_j(P \cap Q)$. Hence $q\operatorname{int}_j(P \cap Q) = q\operatorname{int}_j(P) \cap q\operatorname{int}_j(Q)$.

vi. Since $q\operatorname{int}_j(P)$ is a qj open set, Then $q\operatorname{int}_j(q\operatorname{int}_j(P)) = q\operatorname{int}_j(P)$..

Theorem 3.17

Let $(X, 4\tau)$ be a quad topological space on X and $P, Q \subseteq X$, Then

i. $qcl_j(P)$ is the smallest quad closed set which contains P .
ii. If $P \subseteq Q$ then $qcl_j(P) \subseteq qcl_j(Q)$.
iii. $qcl_j(P \cup Q) = qcl_j(P) \cup qcl_j(Q)$.
iv. $qcl_j(P \cap Q) = qcl_j(P) \cap qcl_j(Q)$.
v. $qcl_j(qcl_j(P)) = qcl_j(P)$.

Proof

i. Since intersection of arbitrary collection of quad j-closed set is quad j-closed ,then $qcl_j(P)$ is a quad closed set . Therefore $P \subseteq qcl_j(P)$. Let Q be quad j closed set which contains P .This implies $qcl_j(P) \subseteq \cap\{G : P \subseteq G$ and G is qj-closed$\} \subseteq Q$ Hence P is the smallest quad j closed set which containing P .

ii. Assume $P \subseteq Q$, since $Q \subseteq qcl_j(Q)$ that implies $P \subseteq qcl_j(Q)$.We know that $qcl_j(P)$ is the smallest closed set which containing P . Hence $qcl_j(P) \subseteq qcl_j(Q)$.

iii. Since $P \subseteq P \cup Q$ and $Q \subseteq P \cup Q$. Using (ii) we have $qcl_j(P) \cup qcl_j(Q) \subseteq qcl_j(P \cup Q)$ Using(i) $P \cup Q \subseteq qcl_j(P) \cup qcl_j(Q)$ Since $qcl_j(P \cup Q)$ is the smallest quad j closed set which containing $P \cup Q$ .Therefore $qcl_j(P \cup Q) = qcl_j(P) \cup qcl_j(Q)$. Hence $qcl_j(P \cup Q) = qcl_j(P) \cup qcl_j(Q)$.

iv. Since $P \cap Q \subseteq P, P \cap Q \subseteq Q$ that implies $qcl_j(P \cap Q) = qcl_j(P) \cap qcl_j(Q)$..

v. Since $qcl_j(P)$ is q j closed set then $qcl_j(qcl_j(P)) = qcl_j(P)$..

Theorem 3.18

Let $(X, 4\tau)$ be a quad topological space and $P \subset X$ then

$(a)[qcl_j(P)]^c = q\operatorname{int}_j(P^c)$

$(b)[q\operatorname{int}_j(P)]^c = qcl_j(P^c)$

Proof

Given $P \subset X$ in quad topological space $(X, 4\tau)$

$[qcl_j(P)] = \cap\{Q : P \subset Q \text{and Q is qj - closedset}\}$

$[qcl_j(P)]^c = [\cap\{Q : P \subset Q \text{and Q is qj - closedset}\}]^c = \cup\{Q^c : Q^c \subset P^c \text{and } Q^c \text{ is qj - open set}\}$

Hence $[qcl_j(P)]^c = q\operatorname{int}_j(P^c)$

(b)Similarly we can prove $[q\operatorname{int}_j(P)]^c = qcl_j(P^c)$

Theorem 3.19

The relationship between the concepts of q-closed set ,qj-closed set and qb-closed is

q-closed set $\Rightarrow$qj-closed set$\Rightarrow$qb-closed

Proof

First we prove**q-closed set $\Rightarrow$ qj-closed set**

Let $P \subseteq X$ be a q-closed set. Therefore P^c is q-open. Since $P^c \subset qcl(P^c)$ $qpclP^c \subset qcl(P^c)$ which implies $q\,\text{int}\,P^c \subset q\,\text{int}(qpcl(P^c))$. Therefore P^c is qj-open. Hence P is qj-closed set.

Next we prove that **qj-closed set $\Rightarrow$ qb-closed**

Let P be qj-closed set of X . Then P^c is qj- open, ie. $P^c \subseteq q\,\text{int}\,qpcl(P^c) \subseteq q\,\text{int}\,qcl(P^c) \Rightarrow P^c \subseteq q\,\text{int}\,qcl(P^c) \Rightarrow P^c \subseteq q\,\text{int}\,qcl(P^c) \cup qclq\,\text{int}(P^c)$ Therefore P^c is qb-open. Hence P is qb-closed set.

Definition 3.20

A subset P of a quad topological space $(X, 4\tau)$ is said to be quad regular open if $P = q\,\text{int}(qcl(P))$ and its complement is called quad regular closed set.

Theorem 3.21

Every quad regular open set is quad open.

Proof

A set A is quad regular open, if $A = q\,\text{int}(qcl(A)), q\,\text{int}(A) = q\,\text{int}(q\,\text{int}(qcl(A))) = q\,\text{int}(A)$ $q\,\text{int}(A)$ is always open. Hence every quad regular open set is quad open.

Converse of the above theorem need be true as shown by the following example.

Example 3.2

Let $X = \{1,2,3,4\}$. Let $\tau_1 =\{\varphi, X\}$, $\tau_2 =\{\varphi, X,\{1\}\}$, $\tau_3 = \{\varphi, X, \{2\}\}$, $\tau_4 =\{\varphi, X,\{3\}\}$. Then 4τ ={ φ, X, {1}, {2}, {3},{1,2}, {1,3},{2,3},{1,2,3}}, Here *{1,2,3}* is quad open but not quad regular open.

Theorem 3.23

Every quad regular open set is quad j-regular open.

Proof

By theorem 3.21, every quad regular open set is quad open and using the theorem 3.10 every quad open set is qj-open. Hence every quad regular open set is quad j-regular open.

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